DELHI PUBLIC SCHOOL, VIJAYAWADA

PRE-BOARD EXAM : 2023-24

CLASS- XII

MATHEMATICS

PB 1 - 041 / A

Time : 3 Hrs **Maximum Marks: 80**

General Instructions:

1. This paper contains five sections A, B, C, D, and E. Each section is compulsory. Howeverthere are internal choices in some questions.
2. Section A has 18MCQ's questions and 2 Assertion and Reasoning questions of 1 mark each.
3. Section B has 5 VERY SHORT ANSWER (VSA)-type questions of 2 marks each.
4. Section C has 6 SHORT ANSWER (SA)-type questions of 3 marks each.
5. Section D has 4 LONG ANSWER (LA)-type questions of 5 marks each.
6. Section E has 3 Source Based/Case based/Passage based/Integrated Units of assessment (4 marks each) with sub parts.

SECTION – A

1. Let A = $\{x \in Z : 0 \leq x \leq 15\}$ and a relation A × A is defined by
R = {(a, b) : a, b ∈ A , $|a-b|$ is divisible by 5 } Then the equivalence class of [4] is [ ]
A) {4, 9 , 14} B) {3, 5, 6} C) { 5, 6, 8} D) { 5, 7, 10}

2. The domain of the function $\cos^{-1}(x\text{-}1)$ is [ ]
A) [0, 2] B) (-3 , 0) C) (-2, 0) D) [0, 1]

3. Sin $\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is equal to [ ]
A) 2 B) $\frac{1}{2}$ C) 1 D) $\frac{1}{3}$

4. The matrix X in the equation $\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$ X = $\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$ is given by [ ]
A) $\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}$ B) $\begin{bmatrix}0 & -1\\ -3 & 1\end{bmatrix}$ C) $\begin{bmatrix}1 & -3\\ 0 & 1\end{bmatrix}$ D) $\begin{bmatrix}1 & 0\\ -3 & 1\end{bmatrix}$

5. If A is a non – singular 3 × 3 matrix and B is its adjoint such that $|B| = 64$ then $|A|$ is [ ]
A) 16 B) ± 8 C) 64 D) ± 4

6. If A and B are square matrices each of order 3 and $|A| = 5$, $|B| = 3$ then the value of $|3\,A\,B|$ is [ ]
A) 45 B) 15 C) 405 D) 135

7. If $x = p^2$, $y = p^3$ then $\frac{d^2y}{dx^2}$ is [ ]
A) $\frac{3}{2}$ B) $\frac{3}{4p}$ C) $\frac{3}{2p}$ D) $\frac{-3}{2p}$

8. Diameter of a sphere is $\frac{3}{2}(2x+5)$ the rate of change of its surface area with respect to x is [ ]

A) $18\pi\,(2x+5)$ B) $\frac{3}{2}$ C) $9\pi\,(2x+5)$ D) $\frac{3}{4}$

9. The function $f(x) = \tan x - x$ [ ]

A) always increases B) never increases C) always decreases

D) sometimes increases and some times decreases

10. $\int \frac{e^{\sqrt{x}}\cos e^{\sqrt{x}}}{\sqrt{x}}dx$ [ ]

A) $\sin e^{\sqrt{x}} + c$ B) $\frac{1}{2}\sin e^{\sqrt{x}} + c$ C) $\frac{2}{\sqrt{x}}\sin e^{\sqrt{x}} + c$ D) $2\sin e^{\sqrt{x}} + c$

11. The value of $\int_{-1}^{1}(x-[x])\,dx$ is [ ]

A) -1 B) 0 C) 1 D) 2

12. The area of parabola $y^2 = 4ax$ bounded by its latus rectum is [ ]

A) $\frac{4}{3}$sq.units B) $\frac{4}{3}a^2$sq.units C) $\frac{8}{3}a^2$sq.units D) $\frac{8}{3}$ sq.units

13. The order and Degree of the equation $x^2\ \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$ [ ]

A) 1 , 2 B) 1 , 1 C) 2 , 1 D) 2 , 2

14. The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ [ ]

A) $e^{\sqrt{x}}$ B) $2\sqrt{x}$ C) $e^{2\sqrt{x}}$ D) $\sqrt{x}$

15. If $\left|\vec{a}\right| = 10$, $\left|\vec{b}\right| = 2$ *and* $\vec{a}.\vec{b} = 12$ then the value of $\left|\vec{a}\times\vec{b}\right|$ is [ ]

A) 5 B) 10 C) 15 D) 20

16. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. The direction cosines of the line are [ ]

A) $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ B) $\frac{-12}{15}, \frac{4}{15}, \frac{-6}{15}$ C) $\frac{1}{7}, \frac{2}{7}, \frac{-3}{7}$ D) 6 , 2 , -3

17. The Cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ [ ]

A) $\frac{x-2}{3} = \frac{y+4}{-5} = \frac{z+5}{6}$ B) $\frac{x+2}{-3} = \frac{y-4}{5} = \frac{z+5}{-6}$

C) $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ D) $\frac{x+2}{-3} = \frac{y-5}{6} = \frac{z+5}{-5}$

18. Given two independent events A and B such that P (A) = 0.3 & P (B) = 0.6.
Then $P(A^1 \cap B^1)$ is [ ]

A) 0 . 47 B) 0. 18 C) 0.35 D) 0.28

Assertion – Reason based Questions

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

19. Assertion (A) : Function $f(x)\begin{cases}\frac{x^2+3x-10}{x-2} & , \ x\neq 2 \\ K & x=2\end{cases}$ is continuous for k = 7 [ ]

Reason (R) : *f(x)* is continuous at $x = a$

If $\lim\limits_{x\to a^-} f(x) = f(a)$

20. Assertion (A) : Projection vector of $\vec{a}$ along $\vec{b}$ is given by $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ [ ]

Reason (R) : Projection vector of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a}\,.\,\vec{b}}{|\vec{b}|}$

SECTION – B

21. If $f : R \to R$ be the function defined by $f(x) = 4x^3 + 7$ show that f is a bijection.

22. Find $\int\sqrt{3-2x-x^2}\ dx$

23. Find λ and μ if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{i}+\mu\hat{j})=0$

24. Find the values of P so that the lines $\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$

are at right angles

25. A coin is tossed thrice. Let event E be "the first throw results in head" and event F be "the last throw results in tail". Find whether event E and F are independent.

SECION – C

26. Show that the relations in the set $A = \{x \in z\ ;\ 0 \le x \le 12\}$ given by
$S = \{(a, b)\ ;\ a, b \in z\ ,\ |\ a - b\ |$ is divisible by 4 $\}$ is an equivalence relation.
Find the set of all elements related to 1.

27. If X and Y are 2 × 2 matrices then solve the following matrix equations of X and Y.

$$2X + 3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}, \quad 3X + 2Y = \begin{bmatrix}-2 & 2\\ 1 & -5\end{bmatrix}$$

28. X = a (θ – sinθ) y = a (1+cosθ) find $\frac{d^2y}{dx^2}$

(OR)

If $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & , \text{ when } x<0 \\ a & , \text{ when } x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , \text{ when } x>0\end{cases}$

function is continuous at x = 0. Find the value of a.

29. Find the intervals in which the function given by $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is

i) is strictly increasing

ii) strictly decreasing

30. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$

(OR)

Using integration find the area of the triangular region whose sides have equations *y=2x+1*, *y=3x+1* and *x*=4.

31. Solve the differential equation $x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}+x-y\sin\left(\frac{y}{x}\right)=0$ given that x=1 when y = π/2.

(OR)

Solve the differential equation ($\tan^{-1} x$ - *y*) *dx* = (1 + x^2) *dy*

SECTION –D

32. Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations *x – y + z = 4* , *x – 2y – 2z = 9* , *2x + y + 3z* = 1

(OR)

If A = $\begin{bmatrix}2 & 3 & 1\\ 1 & 2 & 2\\ -3 & 1 & -1\end{bmatrix}$ find A^{-1} and hence solve the system equations

2x + y – 3z = 13

3x + 2y + z = 4

x + 2y – z = 8

33. Evaluate $\int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}\, dx$

(OR)

Evaluate $\int_0^4 \left[|x| + |x-2| + |x-4| \right] dx$

34. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{3}$ Also write the equation of the line joining the given point and its image and find the length of the line segment joining the given point and its image.

(OR)

Show that the lines . $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i}+2\hat{j}+2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i}+2\hat{j}+6\hat{k})$

are intersecting. Hence find the point of inter section.

35. Solve the following LPP graphically

Minimise Z = 5*x* + 10y

Subject to the constraints $x + 2y \leq 120$

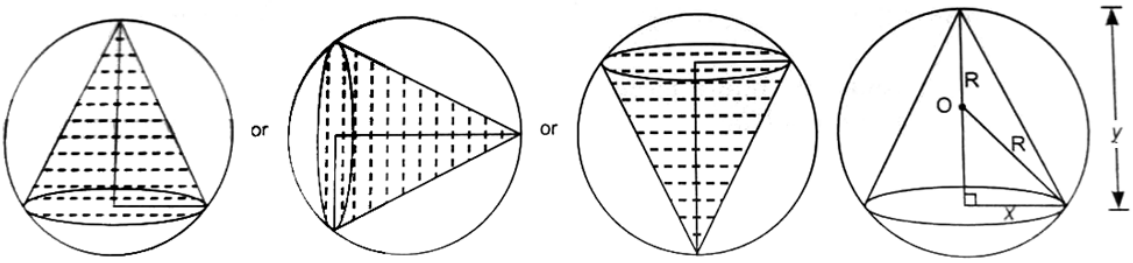
$x + y \geq 60$

$x - 2y \geq 0$

& $x, y \geq 0$

SECTION – E

36. A company dealing in gift items decided to make three - dimensional gift items and a corse team was given the responsibility for the same. After discussions core team suggested to insert a right circular cone in a sphere touching the surface of a sphere and at the same time gets illuminated when taking rounds. They presented the different dimensions also keeping the cost factor into account. Gift item looks like as shown.



(i) If a cone of base radius x and hight y is inscribed in sphere of radius R then establish relation between *x, y* and R

(ii) What is the volume of a cone in terms of y alone

(iii) What should be length of altitude of a cone so that the volume of cone inscribed is maximum.

(OR)

Find the maximum volume of a cone inscribed in a sphere.

37. An insurance company insured 9000 persons of which 2000 are schooter are scooter drivers, 4000 are truck drivers and rest are car drivers. The probability of a scooter ; a truck and car driver meeting with an accident is 0.01, 0.04 and 0.02 respectively with the above information answer the following.

(i) What is the probability of being a car driver

(ii) What is the conditional probability of accident of a car driver.

(iii) What is the probability that truck driver meets with an accident?

(OR)

What is the probability of meeting with an accident?

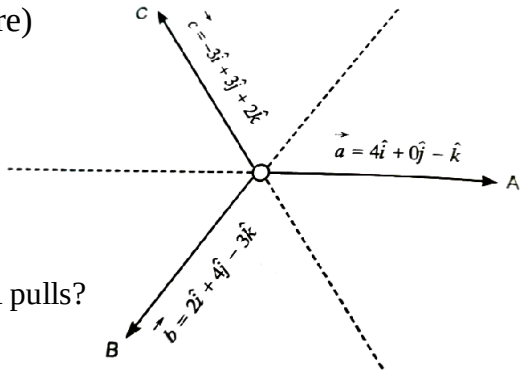
38. Teams A, B, C went for playing a tug of war game

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown in the figure)

Team A pulls along vector $\vec{a} = 4\hat{i} - \hat{k}$

Team B pulls along vector $\vec{b} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

Team C pulls along vector $\vec{c} = 3\hat{i} + 3\hat{j} + 2\hat{k}$



(i) What are the direction cosines of line along which team A pulls?

(ii) What is the magnitude of teams taken together.

(iii) Find the unit vector $\perp^r$ to both $\vec{b}$ and $\vec{c}$

(OR)

Find the area of the triangle formed by $\vec{a}$ & $\vec{b}$

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