Pre-Board I (2023 – 24)

Class: X

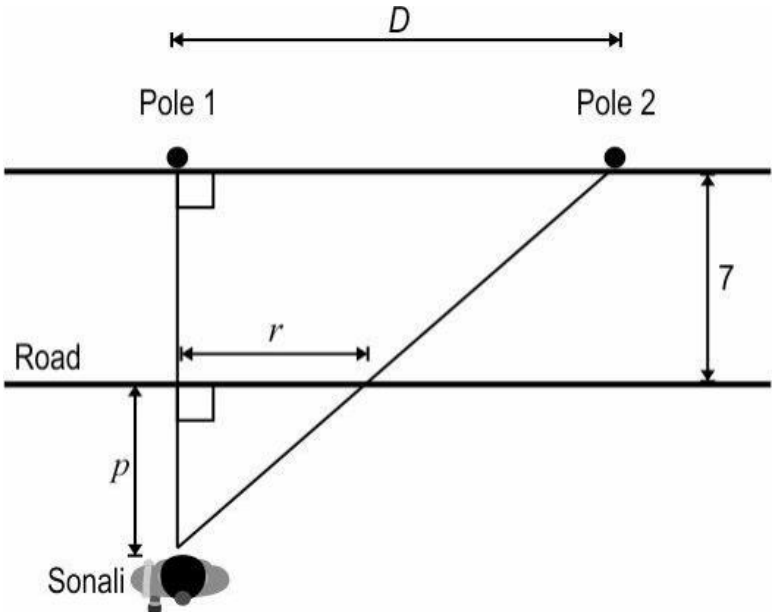
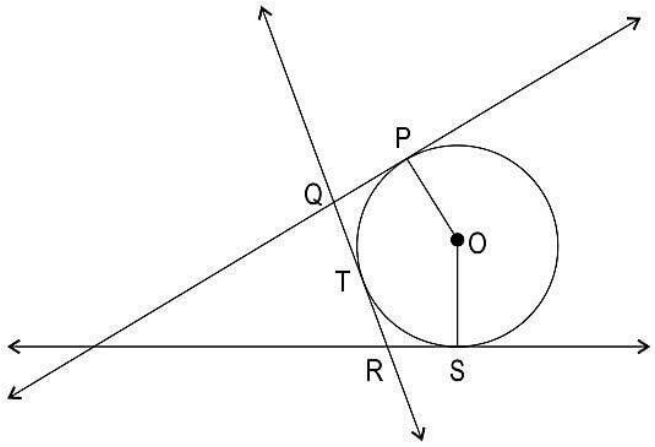
Subject: Mathematics

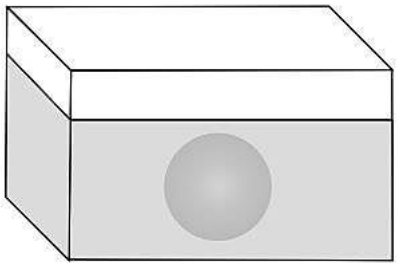
M.M. 80 **Time: 3 Hours**

General Instructions:

1. **This Question Paper has 5 Sections A-E.**
2. **Section A has 20 MCQs carrying 1 mark each.**
3. **Section B has 5 questions carrying 2 marks each.**
4. **Section C has 6 questions carrying 3 marks each.**
5. **Section D has 4 questions carrying 5 marks each.**
6. **Section E has 3 case based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.**
7. **All Questions are compulsory. However, an internal choice in 2 questions of 5 marks, 2 questions of 3 marks and 2 questions of 2 marks has been provided. An internal choice has been provided in the 2 questions of 2 marks of Section E.**
8. **Draw neat figures wherever required. Take π as $22/7$ wherever required, if not stated.**

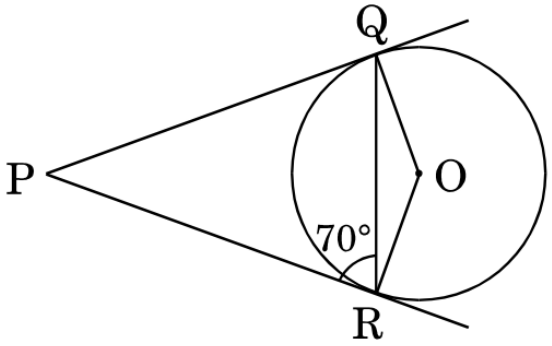
	SECTION – A	
Q.1	The LCM of two numbers is 2400. Which of the following CANNOT be their HCF? (A) 300 (B) 400 (C) 500 (D) 600	[1]
Q.2	The number of polynomials having zeroes -3 and 5 is: (A) only 1 (B) infinite (C) exactly 2 (D) at most 2	[1]
Q.3	If one root of the equation $2x^2 - 5x + (\lambda - 4) = 0$ be the reciprocal of the other, then the value of λ is: (A) 5 (B) 4 (C) 6 (D) 8	[1]
Q.4	The value of p, for which (2p + 1), 10 and (5p + 5) are three consecutive terms of an AP, is: (A) -1 (B) -2 (C) 1 (D) 2	[1]
Q.5	The sum of first n terms of an AP is $-\frac{5}{2}n^2 + \frac{3}{2}n$, then its common difference is: (A) -5 (B) $-\frac{5}{2}$ (C) 5 (D) $\frac{5}{2}$	[1]
Q.6	The perpendicular bisector of a line segment A$(-8, 0)$ and B(8, 0) passes through a point (0, k). The value of k is: (A) 0 only (B) 0 or 8 only (C) any real number (D) any non-zero real number	[1]

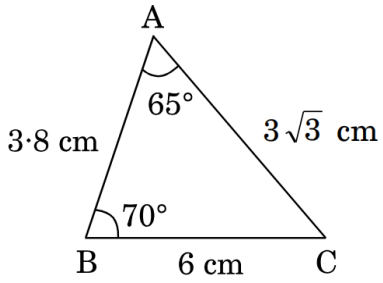
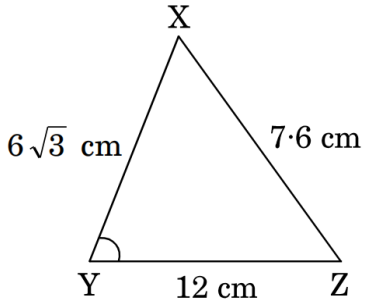
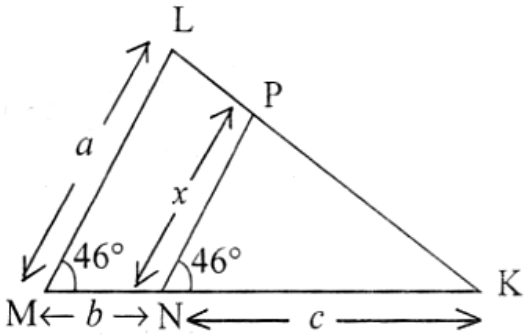
Q.7	The area of a square that can be inscribed in a circle of radius 6 cm is: (A) 36 cm^2 (B) 72 cm^2 (C) 18 cm^2 (D) $36\sqrt{2}$ cm^2	[1]
Q.8	If ΔABC ~ ΔDEF, AB = 6 cm, DE = 9 cm, EF = 6 cm and FD = 12 cm, then the perimeter of ΔABC is: (A) 28 cm (B) 28·5 cm (C) 18 cm (D) 23 cm	[1]
Q.9	Sonali is standing on one side of a 7 m wide road as shown below. She wants to estimate the distance *(D)* between two light poles on the other side without crossing the road.  Which of the following expressions represent D in terms of p and r? (A) $\frac{7r}{p}$ m (B) $\frac{pr}{7}$ m (C) $\frac{pr}{p+7}$ m (D) $\frac{r(p+7)}{p}$ m	[1]
Q.10	Shown below is a circle with centre O having tangents at points P, T and S.  If QR = 12 cm and the radius of the circle is 7 cm, what is the perimeter of the polygon PQTRSO? (A) 26 cm (B) 31 cm (C) 38 cm (D) (cannot say with the given information.)	[1]

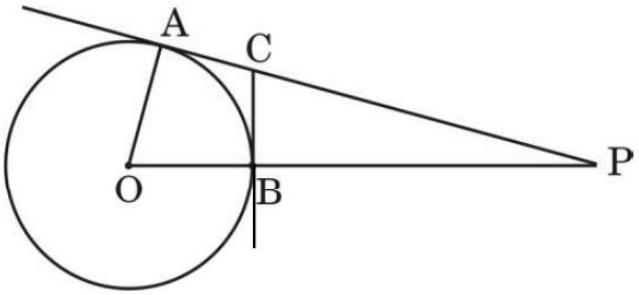
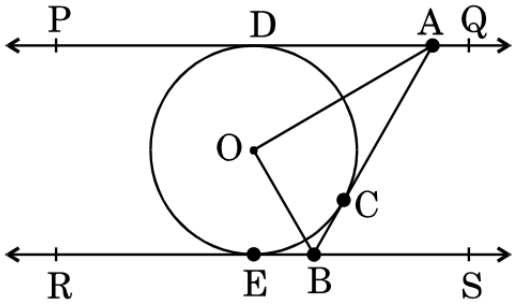
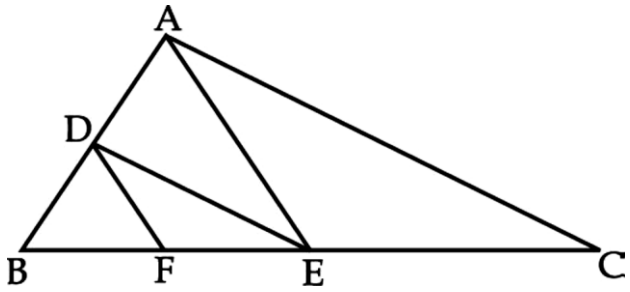
Q.11	If 2 tan A = 3, then the value of $\frac{4\sin A+3\cos A}{4\sin A-3\cos A}$ is: (A) $\frac{7}{\sqrt{13}}$ (B) $\frac{1}{\sqrt{13}}$ (C) 3 (D) does not exist	[1]
Q.12	Probability of happening an event is denoted by p and probability of non-happening of the event is denoted by q. Relation between p and q is: (A) p + q = 1 (B) p = 1, q = 1 (C) p = q −1 (D) p + q + 1 = 0	[1]
Q.13	The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is: (A) 2496 cm^3 (B) 1248 cm^3 (C) 1664 cm^3 (D) 416 cm^3	[1]
Q.14	A cuboid of base area P sq units is filled with water upto a height of Q units. A sphere of volume R cu units is dropped into the cuboid such that it is completely submerged. A representation of the submerged sphere is shown below.  Which of these represents the increase in the height of water? (A) 0 units (B) $\frac{R}{P}$ units (C) R units (D) Q + $\frac{R}{P}$ units	[1]
Q.15	The coordinates of the point A, where AB is the diameter of the circle whose centre is (3, −2) and B (7, 4) is: (A) (−1, −8) (B) (−1, 8) (C) (1, 8) (D) (1, −8)	[1]
Q.16	The value of k for which the pair of linear equations kx = y + 2 and 6x = 2y + 3 has infinitely many solutions, is: (A) 3 (B) −3 (C) 4 (D) does not exist	[1]
Q.17	The pair of equations x = a and y = b graphically represents lines which are: (A) parallel (B) intersecting at (b, a) (C) coincident (D) intersecting at (a, b)	[1]
Q.18	If the mean and the mode of a distribution are 15 and 18 respectively, then the median of the distribution is: (A) 17 (B) 15 (C) 16 (D) 18	[1]

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(i) Both A and R are true and R is the correct explanation of A.
(ii) Both A and R are true but R is not the correct explanation of A.
(iii) A is true, but R is false
(iv) A is false, but R is true

Q.19	**Assertion (A):** For $0° < \theta \leq 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other. **Reason (R):** $\operatorname{cosec}^2\theta - \cot^2\theta = 1$. (A) (i) (B) (ii) (C) (iii) (D) (iv)	[1]
Q.20	**Assertion (A):** The probability that a leap year has 53 Sundays is $\frac{2}{7}$. **Reason (R):** The probability that a non-leap year has 53 Sundays is $\frac{1}{7}$. (A) (i) (B) (ii) (C) (iii) (D) (iv)	[1]
	SECTION – B	
Q.21	There are 156, 208 and 260 students in groups A, B and C respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired if the same number of students of same group should be accommodated in each bus.	[2]
Q.22	If $6x = \sec\theta$ and $\frac{6}{x} = \tan\theta$, find the value of $9\left(x^2 - \frac{1}{x^2}\right)$.	[2]
Q.23	If one zero of the polynomial $3x^2 - 8x + (2k + 1)$ is seven times the other, find the value of k. **OR** If p and q are the zeroes of the quadratic polynomial $f(x) = 6x^2 + 8x - 2$, then find the value of $\frac{1}{p} + \frac{1}{q} - pq$.	[2]
Q.24	In Figure, PQ and PR are tangents drawn to a circle with centre O from an external point P. If ∠PRQ = 70°, then find ∠OQR and ∠QPR. 	[2]

Q.25	In figure, Δ ABC and Δ XYZ are shown. If AB = 3.8 cm, AC = $3\sqrt{3}$ cm, BC = 6 cm, XY = $6\sqrt{3}$ cm, XZ = 7.6 cm, YZ = 12 cm, ∠A = 65°, ∠B = 70°, then find the value of ∠Y.   **OR** In figure, ∠M = ∠N = 46°. Express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively. 	[2]
	SECTION – C	
Q.26	Prove that $3-2\sqrt{7}$ is an irrational number, it is given that $\sqrt{7}$ is irrational.	[3]
Q.27	The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves ₹ 2,000 yearly, find their annual incomes.	[3]
Q.28	If $\frac{1}{\sin\theta-\cos\theta}=\frac{\operatorname{cosec}\theta}{\sqrt{2}}$, prove that $\left(\frac{1}{\sin\theta+\cos\theta}\right)^2=\frac{\sec^2\theta}{2}$. **OR** Prove that $\tan^2\theta+\cot^2\theta+2=\sec^2\theta.\operatorname{cosec}^2\theta$.	[3]
Q.29	A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8:5, then find the ratio between radius of their bases to their height.	[3]
Q.30	Two dice are thrown simultaneously. What is the probability that (i) 5 will not come up on either of them? (ii) 5 will come up on at least once? (iii) 5 will come up exactly one time?	[3]

Q.31	In figure, O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP = 13 cm, then find the length of tangents PA and BC.  **OR** In figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that ∠AOB = 90°.  	[3]
	Section – D	
Q.32	State and prove basic proportionality theorem. Using the above theorem, find the length of EC in the given figure, where DE ∥ AC, DF ∥ AE, BF = 4 cm and FE = 5 cm.  	[5]
Q.33	The angle of elevation of the top of a vertical tower from a point X on the ground is 60°. From another point Y, 40 m vertically above the point X, its angle of elevation is 30°. Find: (i) The height of the tower. (ii) The distance of the point X from the foot of the tower. (iii) The distance of the point X from the top of the tower.	[5]

Q.34 A survey regarding the heights (in cm) of 50 girls of class X of a school was conducted and the following data was obtained: [5]

Height	120 - 130	130 - 140	140 - 150	150 - 160	160 - 170
Number of girls	2	8	12	20	8

Find the mean and mode of the above data.

OR

The median of the given frequency distribution is 46 and the sum of the frequencies is 228. Find the missing frequencies x and y.

Class	Frequency
10 – 20	12
20 – 30	30
30 – 40	x
40 – 50	65
50 – 60	y
60 – 70	25
70 – 80	18

Q.35 A train travels at a certain average speed for a distance of 54 km and then travel a distance of 63 km at a speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed? [5]

OR

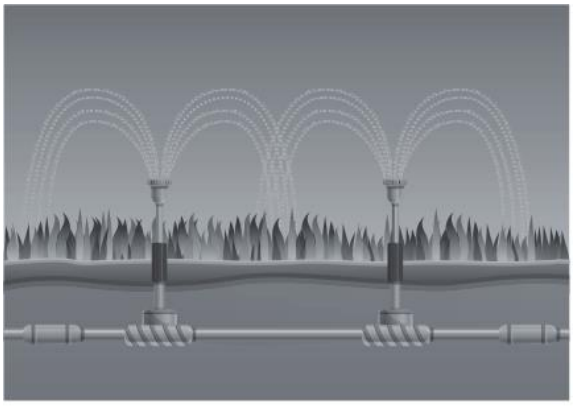
If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Section - E

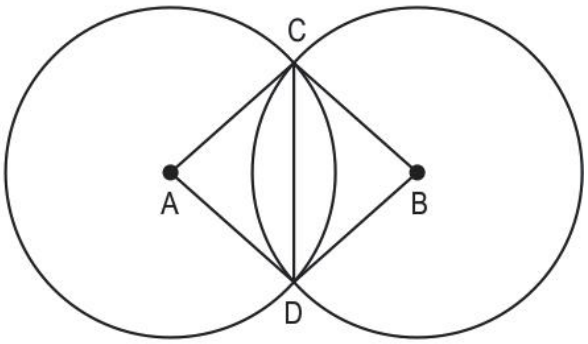
Q.36 **Case Study 1**

Sprinklers are crop irrigation equipment which rotate around a centre and spray water on the crops in the circular region.

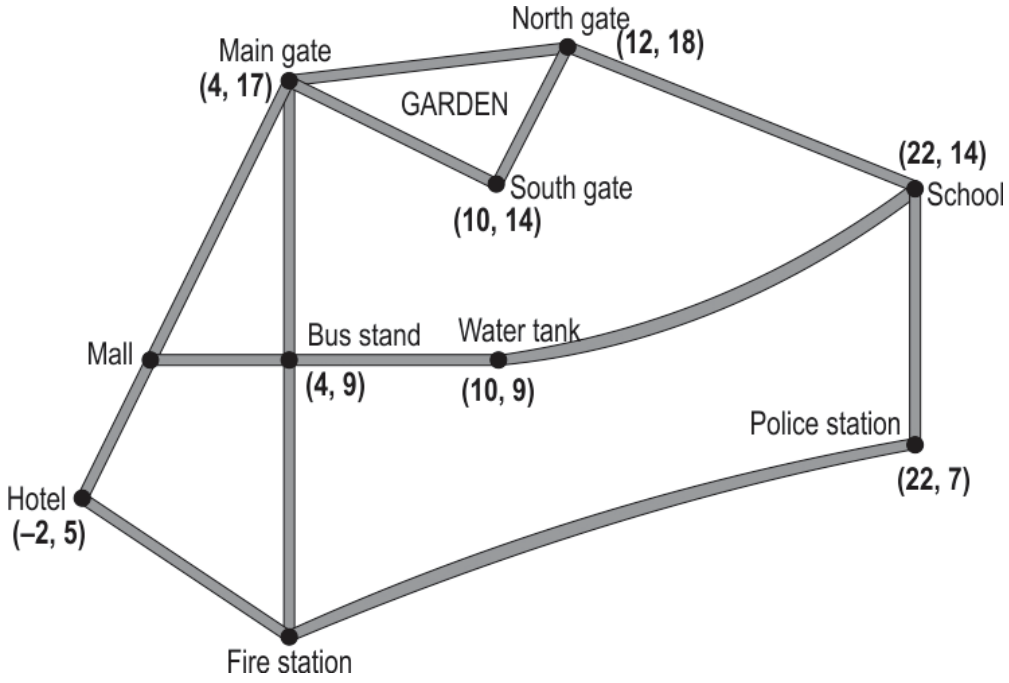
Two such high power sprinklers, occupying negligible area are installed in a straight line in a field such that they spray water on a common area. Shown below are the side and top views where points A and B are the sprinklers.

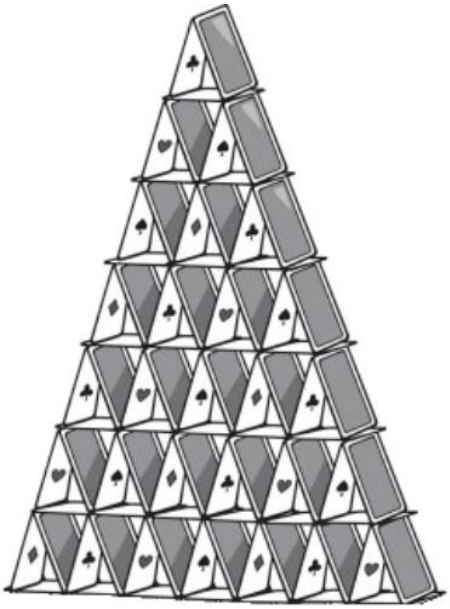


Side view of the sprinklers



Top view of the region sprayed

	Both the sprinklers spray over an equal area. It is given that, CD = 400 m and ∠CAD = ∠CBD = 90°. (i) Find the radius of the circular region sprayed by the sprinkler. (ii) Find the perimeter of the region sprayed by both the sprinklers. (Use π = 3.14) (iii) Find the area of the overlapping region. (Use π = 3.14) **OR** (iii) Find the total area of the major sectors with centres at A and B.	[1] [1] [2]
Q.37	**Case Study 2** Shown below is the map of Giri's neighbourhood.  Giri did a survey of his neighbourhood and collected the following information: • The hotel, mall and the main gate of the garden lie in a straight line. • The distance between the hotel and the mall is half the distance between the mall and the main gate of the garden. • The bus stand is exactly midway between the main gate of the garden and the fire station. • The mall, bus stand and the water tank lie in a straight line. Based on the above information, answer the following questions: (i) What are the coordinates of the fire station? (ii) What is the shortest distance between the main gate and the north gate? (iii) What is the x-coordinate of the mall's location? **OR** (iii) How much more is the shortest distance of the school from the Water tank than the distance of the school from the police station?	[1] [1] [2]

Q.38	**Case Study 3** Shown below is a house of cards, a structure created by stacking playing cards on top of each other in the shape of a pyramid. Each small triangle is made using 3 cards and each layer has 1 less triangle than the layer below it.  Ankit and his friends were having a sleepover and wanted to do something fun. One of the friends suggested that they could make a house of cards. (i) Ankit and his friends want to use 3 cards in the top layer and 18 in the bottom layer. Form an A.P. showing the number of cards in each layer starting from the top layer. (ii) Ankit is planning to make a pyramid with the top and bottom layer containing 15 and 138 cards respectively. How many layers will such a pyramid have? Show your work. (iii) They have a total of 360 cards with them. Find the maximum number of layers that Ankit and his friends can make using the cards they have, if they want to have 1 triangle at the top layer. **OR** (iii) They are planning to make a pyramid with 15 layers. If they have a total of 450 cards with them, then find the number of triangles in the top layer.	[1] [1] [2]

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