MARKING SCHEME
PRE-BOARD EXAMINATION (2023-24)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

Instructions:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks be absorbed to them.

2. If a student has attempted an extra question answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale of marks (0-80) has to be used. Please do not hesitate toward full marks if the answer deserves it.

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SECTION-A

1. (c) $-\frac{\sqrt{3}}{2}$ 1

2. (a) 34 1

3. (a) $\begin{bmatrix} 3 & -6 \\ 6 & 15 \end{bmatrix}$ 1

4. (d) 169 1

5. (d) –cot x 1

6. (a) $(-\infty, \infty)$ 1

7. (b) 4π b

8. (a) 0 1

9. (b) 6 1

10. (d) –1 1

11. (c) 4 1

12. (c) 45° 1

13. (b) 90° 1

14. (d) $\frac{7}{10}$ 1

15. (c) 64 1

16. (a) 9 1

17. (c) (3, 4) 1

18. (a) a = 2b 1

19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). 1

20. (d) Assertion (A) is false, but Reason (R) is true. 1

SECTION-B

21. Put x = tan θ ½

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$ ½

$$=\tan^{-1}\left(\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\sin\frac{\cancel{\theta}}{2}\cos\frac{\theta}{2}}\right)=\frac{1}{2}\theta=\frac{1}{2}\tan^{-1}x$$ 1

22. (a) $\int e^{2\log\sec x}.\tan dx=\int\sec^2 x.\tan dx$ ½

$\{$Put tan x = t $\Rightarrow$ $\sec^2 x\, dx = dt\}$ ½

$$=\int t\,dt=\frac{1}{2}t^2+c=\frac{1}{2}\tan^2 x+c$$ 1

OR

(b) $\int e^x.\left(1+\frac{1}{x}-\frac{1}{x^2}\right)dx=\int e^x dx+=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ ½

$$=e^x+e^x.\frac{1}{x}+c$$ 1½

23. (a) $y = x^y \Rightarrow \log_e y = y.\log x$ ½

On diff. w.r.t. x , $\frac{1}{y}.\frac{dy}{dx} = y \times \frac{1}{x} + \log x.\frac{dy}{dx}$ ½

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$ 1

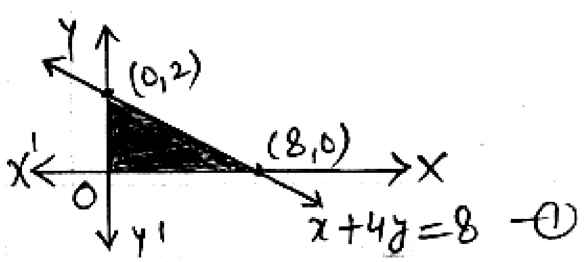
OR

(b) $y = \sin^{-1} x + \cos^{-1} x$ 1

$\Rightarrow y = \frac{\pi}{2}$ ½

$\Rightarrow \frac{dy}{dx} = 0$ ½

24. ½



Required area $\int_0^2 x \ dy = \int_0^2 (8 - 4y)\ dy$ ½

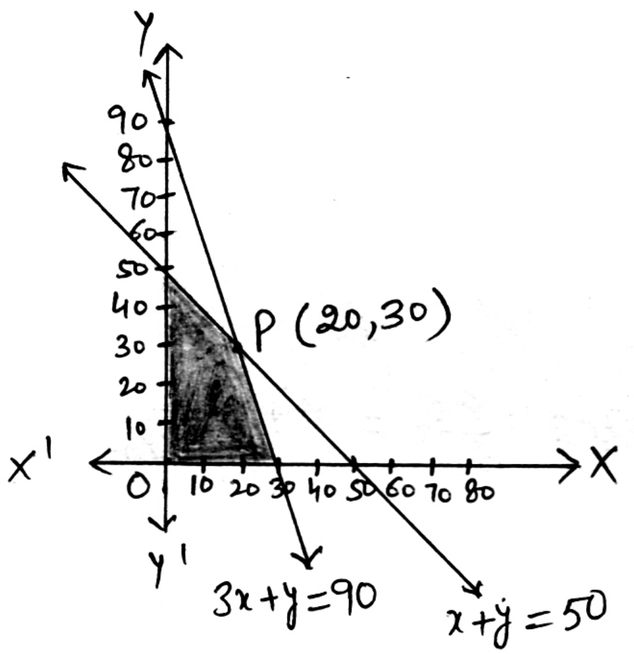
$= \left[8y - 2y^2\right]_0^2 = 8$ sq.units 1

25. $\frac{dy}{dx} = e^{5x-3y} \Rightarrow e^{3y}.dy = e^{5x}.dx$ 1

Integrating both sides, $\frac{e^{3y}}{3} = \frac{e^{5x}}{5} + c$ 1

SECTION-C

26. 1½



By graph,

Corner points	z=20x+30y
(0,0)	0
(30,0)	600
(20,30)	1300
(0,50)	1500 → (Max.)

$Z_{max.}$ = 1500 at point (0, 50) ½

27. (a) $\Sigma P(X) = 1 \Rightarrow 30k^2 + 7k = 1$ 1

On solving, $\boxed{k = \frac{1}{10}}, \left(k \neq -\frac{1}{3}\right)$ 1

Mean $= \Sigma X.P(X) = 37k^2 + 14k = 1.77$ 1

OR

(b) $P(A \cap B) = P(A) \times P(B) = 0.12$ ½

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.58$ ½

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = \frac{3}{10}$ or 0.3 ½

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{0.12}{0.3} = 0.4$ or $\frac{2}{5}$ ½

$P\left(\frac{\bar{A}}{\bar{B}}\right) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})} = \frac{1 - 0.58}{1 - 0.4} = \frac{0.42}{0.6} - 0.7$ 1

28. (a) Put $y = vx$ and $\frac{dy}{dx} = v + x.\frac{dv}{dx}$ ½

$\frac{dy}{dx} = \frac{y^2}{xy - x^2} \Rightarrow v + x\frac{dv}{dx} = \frac{v^2}{v-1}$ ½

$\Rightarrow x\frac{dv}{dx} = \frac{v}{v-1} \Rightarrow \int\left(1 - \frac{1}{v}\right)dv = \int\frac{dx}{x}$ 1

$\Rightarrow v - \log v = \log x + \log c$

$\Rightarrow v = \log(cy) \Rightarrow cy = e^{y/x}$ 1

OR

(b) $\frac{dy}{dx} - \frac{1}{x}y = x^4$ ½

$P = -\frac{1}{x}, Q = x^4$

$\text{I.F.} = e^{\int P\,dx} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ 1

General solution is given by :

$y(\text{I.F.}) = \int Q.(\text{I.F.})dx + C$ ½

$\Rightarrow \frac{y}{x} = \int x^3 dx + C$

$\Rightarrow \frac{y}{x} = \frac{x^4}{4} + C$

$\Rightarrow y = \frac{x^5}{4} + Cx$ 1

29. $\lim\limits_{x\to 0}\left[f(x)\right]=\lim\limits_{x\to 0}\left(\dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}\right)$

$=\lim\limits_{x\to 0}\left(\dfrac{(\cancel{1}+kx)-(\cancel{1}-kx)}{x(\sqrt{1+kx}+\sqrt{-kx})}\right)$

$=\dfrac{2k}{2}=k$ 2

f(0) = –1 ½

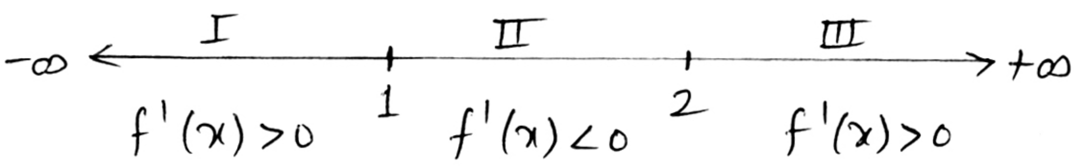
$\because$ f is continuous at x = 0

$\therefore$ $\boxed{k=-1}$ ½

30. $f'(x) = 6x^2 - 18x + 12 = 6\,(x^2 - 3x + 2)$

$= 6\,(x - 1)\,(x - 2)$

Put $f'(x) = 0 \Rightarrow \boxed{x=1,2}$ 1



Hence, f is st. ↑ on $(-\infty,1)\cup(2,\infty)$

And f is st. ↓ on (1, 2)

31. (a) $\dfrac{5x+3}{x^3-4x} = \dfrac{-3}{4x} - \dfrac{7}{8(x+2)} + \dfrac{13}{8(x-2)}$ 2

$\displaystyle\int \frac{5x+3}{x^3-4x}dx = -\frac{3}{4}\log|x| - \frac{7}{8}\log|x+2| + \frac{13}{8}\log|x=2| + C$ 1

OR

(b) Let $I = \displaystyle\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}dx$ ...(1)

Applying property $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$,

$I = \displaystyle\int_0^{\pi} \frac{\pi - x}{a^2\cos^2 x + b^2\sin^2 x}dx$ ...(2)

On adding equations (1) and (2),

$2I = \pi\displaystyle\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = 2\pi\int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2\tan^2 x}$ 1½

$\left\{\text{Put } b\tan x = t \Rightarrow \sec^2 x dx = \dfrac{dt}{b}\right\}$

$2I = \dfrac{2\pi}{b}\displaystyle\int_0^{\infty} \frac{dt}{a^2+t^2} = \frac{2\pi}{ab}\left[\tan^{-1}\left(\frac{t}{a}\right)\right]_0^{\infty}$ 1

$\Rightarrow I = \dfrac{\pi^2}{2ab}$ ½

SECTION-D

32. One-One : Let $x_1, x_2 \in A$

Suppose $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 3}{x_1 - 2} = \frac{x_2 - 3}{x_2 - 2}$$

$$\Rightarrow \cancel{x_1 x_2} - 2x_1 - 3x_2 + \cancel{6} = \cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6}$$

$$\Rightarrow x_1 = x_2$$

$\therefore$ f is one-one

Onto : $y = \frac{x-3}{x-2} \Rightarrow x = \frac{2y-3}{y-1}$

$\therefore$ Range=R-{1} = B = co-domain

$\therefore$ f is onto 2

$f[\pi] = \frac{[\pi]-3}{[\pi]-2} = \frac{3-3}{3-2} = \frac{0}{1} = 0$ 1

33. (a) In matrix form, given equations can be written as

$$\begin{bmatrix} 2 & 3 & -1 \\ 3 & -1 & 2 \\ 5 & 2 & 3 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 17 \end{bmatrix}$$

$\Rightarrow A.X = B$

$\Rightarrow X = A^{-1}.B \quad ...(1)$ 1

$|A| = -22$ 1

$$\text{adj}(A) = \begin{bmatrix} -7 & 1 & 11 \\ -11 & 11 & 11 \\ 5 & -7 & -11 \end{bmatrix}' = \begin{bmatrix} -7 & -11 & 5 \\ 1 & 11 & -7 \\ 11 & 11 & -11 \end{bmatrix}$$ 1

$$A^{-1} = \frac{1}{-22} \begin{bmatrix} -7 & -11 & 5 \\ 1 & 11 & -7 \\ 11 & 11 & -11 \end{bmatrix}$$ 1

By equation (1),

$$\left.\begin{array}{l} x = 3 \\ y = 1 \\ z = 0 \end{array}\right\}$$ 1

(b) $A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$ 1

$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 33 & 0 & 55 \end{bmatrix}$ 1½

Put the values of k A, A^2 and A^3 in

$A^3 - 6A^2 + 7A + kI_3 = O$ ....(1)

$\Rightarrow \boxed{k = 2}$ 1½

Multiplying equation (1) by A^{-1},

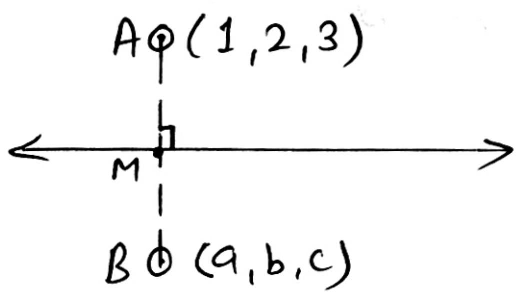
$A^2 - 6A + 7I + kA^{-1} = O$

$\Rightarrow kA^{-1} = -A^2 + 6A - 7I$

$\Rightarrow 2A^{-1} = \begin{bmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}$ 1

34. (a)



$\vec{r} = (6\hat{i} + 7\hat{j} + 7\hat{k}) + \lambda = (3\hat{i} + 2\hat{j} - 2\hat{k})$

Let arbitrary point on line be

$M \equiv (6 + 3\lambda, 7 + 2\lambda, 7 - 2\lambda)$ 1

$\overrightarrow{AM} = (5 + 3\lambda)\hat{i} + (5 + 2\lambda)\hat{j} + (4 - 2\lambda)\hat{i}$

$\because$ AM $\perp$ Line

$\therefore \overrightarrow{AM}.(3\hat{i} + 2\hat{j} - 2\hat{k}) = 0$ 1

$\Rightarrow 3(5 + 3\lambda) + 2(5 + 2\lambda) - 2(4 - 2\lambda) = 0$

$\Rightarrow \boxed{\lambda = -1}$ 1

$\therefore M \equiv (3,5,9)$ 1

Let coodinate of image B is (a, b, c).

Then, M is the mid-point of AB.

Hence, $\left(\frac{a+1}{2}, \frac{b+2}{2}, \frac{c+3}{2}\right) = (3,5,9)$

$\Rightarrow a = 5,\ b = 8,\ c = 15$

Image of point A = (5, 8, 15) 1

OR

(b) $\vec{c}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -1\\ 1 & -4 & 5\end{vmatrix}=\hat{i}-16\hat{j}-13\hat{k}$ 1

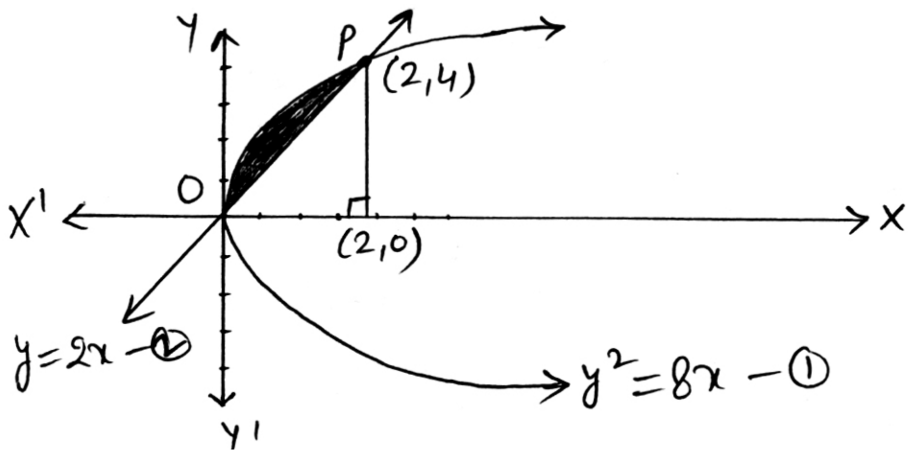
$\because \vec{d}\,||\,\vec{c}\times\vec{b}$ 1

Let $\vec{d}=\lambda(\vec{c}\times\vec{b})=\lambda(\hat{i}-16\hat{j}-13\hat{k})$ 1

$\because \vec{d}.\vec{a}=21\Rightarrow \boxed{\lambda=-\frac{1}{3}}$ 1

Hence, $\vec{d}=\frac{-\hat{i}+16\hat{j}+13\hat{k}}{3}$ 1

35. 1



Area of shaded region $=\int_1^2 y_1 dx-\int_0^2 y_2 dx$ 1½

$=2\sqrt{2}\int_0^2\sqrt{x}\ dx-2\int_0^2 x\ dx$ 1½

$=\frac{16}{3}-4=\frac{4}{3}$sq.units 1

SECTION-E

Case Study-I

36. (I) ATQ, Total perimeter of window = 10 met

$\Rightarrow x + 2y + x + \pi y = 10$

$\Rightarrow 2x + (\pi + 2)y = 10$

$\Rightarrow \boxed{x = \frac{10 - (\pi + 2)y}{2}}$ 1

(II) Area of window = $2xy + \frac{\pi y^2}{2}$

$\Rightarrow A = y\left[10 - (\pi + 2)y\right] + \frac{\pi y^2}{2}$

$\Rightarrow A = 10y - \left(\frac{\pi}{2} + 2\right)y^2$ 1

(III) $\frac{dA}{dy} = 10 - (\pi + 4)y$ ½

Put $\frac{dA}{dy} = 0 \Rightarrow y = \frac{10}{\pi + 4}$ ½

$\frac{d^2A}{dy^2} = -(\pi + 4)$

$\therefore$ A is maximum at $y = \frac{10}{\pi + 4}$ ½

$$A_{max.} = \frac{100}{\pi+4} - \left(\frac{\pi+4}{2}\right)\left(\frac{10}{\pi+4}\right)^2$$

$$= \frac{50}{\pi+4}\text{sq.units}$$ ½

OR

(III) $\frac{dA}{dy} = 10-(\pi+4)y, \frac{d^2A}{dy^2} = -(\pi+4)$ 1

$$\because \frac{d^2A}{dy^2} + \frac{dA}{dy} = 0$$

$$\Rightarrow 10-(\pi+4)y-(\pi+4) = 0$$

$$\Rightarrow \boxed{y = \frac{6-\pi}{\pi+4}}$$ ½

$$x = \frac{10-(\pi+2)\left(\frac{6-\pi}{\pi+4}\right)}{2} = \frac{10\pi+40-(\pi+2)(6-\pi)}{2(\pi+4)}$$

$$\Rightarrow \boxed{x = \frac{\pi^2+6\pi+28}{2(\pi+4)}}$$ ½

Case Study-II

37. (I) $\overrightarrow{BA} = \hat{i} - 2\hat{j} + 4\hat{k}$

$\overrightarrow{BC} = -2\hat{i} - 2\hat{j} + 2\hat{k}$ ½+½

(II) Length of projection = $\left|\dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BC}|}\right|$

$= \dfrac{7\sqrt{3}}{3}$ units 1

(III) $\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 4 \\ -2 & -2 & 2 \end{vmatrix} = 4\hat{i} - 6\hat{j} - 2\hat{k}$ 1

$\text{ar}(\Delta ABC) = \dfrac{1}{2}|\overrightarrow{BA} \times \overrightarrow{BC}| = \sqrt{14}$ sq.units 1

OR

(III) $\angle ABC = \cos^{-1}\left(\dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}|.|\overrightarrow{BC}|}\right)$ ½

$= \cos^{-1}\left(\dfrac{\sqrt{7}}{3}\right)$ 1½

Case Study-III

38. (I) Let A = To die an insured person before 60 years

E_1 = He is a teacher

E_2 = He is a doctor

E_3 = He is a lawyer.

Using total probability theorem,

$$\text{Required prob.} = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right)$$

$$= \frac{5000}{20000} \times 0.02 + \frac{7000}{20000} \times 0.03 + \frac{8000}{20000} \times 0.04$$

$$= \frac{1}{4} \times 0.02 + \frac{7}{20} \times 0.03 + \frac{2}{5} \times 0.04$$

$= 0.0315$ 2

(II) Required probability $= \dfrac{P(E_1).P\left(\dfrac{A}{E_1}\right)}{0.0315}$

$$= \frac{0.005}{0.0315}$$

$= \dfrac{10}{63}$ 2