No. of pages - 24

(E)

PRE-BOARD EXAMINATION (2023-24)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours | **Maximum Marks : 80**

समय : 3 घंटे | **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड क में प्रश्न संख्या 1 से 18 तक बहु-विकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड ख में प्रश्न संख्या 21 से 25 तक अति लघु उत्तरीय(VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड ग में प्रश्न संख्या 26 से 31 तक लघु उत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच- पाँच अंकों के प्रश्न हैं।
7. खंड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में, खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, one internal choice has been provided in 2 questions in section B, 3 questions in section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The value of $\cos\left[\frac{\pi}{6}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$ is : 1

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{2}$

2. If matrix $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2a-4 & 0 \\ 0 & 0 & 3b-7 \end{bmatrix}$ is scalar matrix, then the value of $4a + b^2$ is : 1

(a) 34 (b) 24

(c) 20 (d) 14

3. If $A = [A_{ij}]$, is square matrix of order 2 such that : 1

$a_{ij} = \begin{cases} i+j \text{ यदि } i=j \\ i-j \text{ यदि } i\neq j \end{cases}$ A^2, then A^2 is :

(a) $\begin{bmatrix} 3 & -6 \\ 6 & 15 \end{bmatrix}$ (b) $\begin{bmatrix} 15 & 6 \\ 6 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 15 & -6 \\ 6 & 3 \end{bmatrix}$

4. If $A = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, then $|A^2|$ is : 1

(a) 13 (b) 100

(c) 138 (d) 169

The derivative of sin x w.r.t. cos x is :

(a) tan x (b) cot x

(c) –tan x (d) –cot x

Choose the interval in which the function $f(x) = 2x + \cos x$ is strictly increasing : 1

(a) $(-\infty, \infty)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $(0, \pi)$ (d) $(\pi, 2\pi)$

The value of $\int_{-\pi/2}^{\pi/2} (x\cos x + x^5 + 4 - \tan^3 x)\, dx$ is : 1

(a) 2π (b) 4π

(c) 8π (d) 10π

The derivative of $\sin^{-1} x + \sec^{-1}\left(\frac{1}{x}\right)$ w.r.t. x is : 1

(a) 0 (b) 1

(c) x (d) x^2

The product of order and degree of the differential equation $\left(\frac{dy}{dx} + x\right)^5 = \left(\frac{d^2y}{dx^2}\right)^3$ is : 1

(a) 3 (b) 6

(c) 10 (d) 15

If two vectors $\vec{a} = 3\hat{i} + m\hat{j} + n\hat{k}$ and $\vec{b} = -6\hat{i} - 8\hat{j} + 10\hat{k}$ are collinear, then m + n is : 1

(a) 9 (b) 0

(c) 1 (d) –1

The projection of $3\hat{i}+2\hat{j}+4\hat{k}$ in the direction of $2\hat{i}-\hat{j}+2\hat{k}$ is : 1

(a) 2 (b) 3

(c) 4 (d) 5

If a line makes angles of 90°, 135° and θ with X, Y and Z-axis respectively, then the value of θ is : 1

(a) 15° (b) 30°

(c) 45° (d) 60°

The angle between the lines $\frac{x-5}{2}=\frac{y-3}{4}=\frac{15-z}{5}$ and $\frac{x}{4}=\frac{y}{3}=\frac{z}{4}$ is : 1

(a) 45° (b) 90°

(c) 60° (d) 30°

For two independent events A and B, $P(A)=\frac{1}{2}$ and $P(B)=\frac{2}{5}$, then $P(A \cup B)$ is : 1

(a) $\frac{4}{5}$ (b) $\frac{9}{10}$

(c) $\frac{3}{5}$ (d) $\frac{7}{10}$

If A is a matrix of order 3 × 3 and |A| = 8, then the value of |adj A| is : 1

(a) 8 (b) 32

(c) 64 (d) 512

If matrix $\begin{bmatrix} 2 & 3k-5 \\ k+13 & 9 \end{bmatrix}$ is symmetric, then the value of k is : 1

(a) 9 (b) 10

(c) 11 (d) 12

If the objective function for a LPP is $Z = 5x + 7y$ and the corner points of the bounded feasible region are (0, 0), (7, 0), (3, 4) and (0, 6), then the maximum value of Z occurs at: 1

(a) (0, 0) (b) (7, 0)

(c) (3, 4) (d) (0, 5)

The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 7). If the maximum value of $Z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0); then : 1

(a) $a = 2b$ (b) $2a = b$

(c) $a = 5b$ (d) $3a = b$

Question number 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below :

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Assertion (A) : $\int_{\pi/4}^{3\pi/4} \log \tan x \ dx = 0$ 1

Reason (R) : $\int_a^b f(x)\ dx = \int_a^b f(a+b-x)\ dx$

Assertion (A) : The resultant of $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, and $\overrightarrow{BC}$ is zero vector. 1

Reason (R) : $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

SECTION-B

This section comprises of very short answer (VSA) type questions of 2 marks each.

Convert $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ in simplest form. 2

(a) Evaluate : $\int e^{2\log \sec x} . \tan x \, dx$ 2

OR

(b) Evaluate : $\int e^x . \left(1 + \frac{1}{x} - \frac{1}{x^2}\right) dx$

(a) If $y = x^y$, then find $\frac{dy}{dx}$. 2

OR

(b) If $y = \sin^{-1} x + \sin^{-1}(\sqrt{1-x^2})$, then find $\frac{dy}{dx}$.

Sketch the graph and find the area bounded by the line x + 4y = 8, x-axis and y-axis, using integration. 2

Find general solution of differential equation $\frac{dy}{dx} = e^{5x-3y}$. 2

SECTION-C

This section comprises short answer (SA) type questions of 3 marks each.

Solve the following linear programming problem graphically : 3

Maximise $Z = 20x + 30y$

Subject to the constraints :

$x + y \leq 50$

$3x + y \leq 90$

$x, y \geq 0$

(a) The probability distribution function of a random variable X is given by : 3

X	0	1	2	3
P(X)	k^2	$25k^2$	$7k$	$4k^2$

Find the value of k and also find mean of random variable X.

OR

(b) If A and B are independent events with P(A) = 0.3 and P(B) = 0.4; then find

$P(A \cap B)$, $P(A \cup B)$, $P\left(\frac{A}{B}\right)$, $P\left(\frac{B}{A}\right)$ and $P\left(\frac{\overline{A}}{B}\right)$. 3

8. (a) Find general solution of differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$. 3

OR

(b) Find general solution of differential equation $x\frac{dy}{dx} = y + x^3$. 3

29. Find the value of k, for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } x \neq 0 \\ \frac{3x+1}{2x-1} & \text{if } x = 0 \end{cases}$

is continuous $x = 0$. 3

30. Find intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly increasing and strictly decreasing. 3

31. (a) Evaluate : $\int \frac{5x+3}{x^3 - 4x} dx$ 3

OR

(b) Integrate : $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

SECTION-D

This section comprises long type answer (LA) type questions of 5 marks each.

32. Let $A = R - \{2\}$ and $B = R - \{1\}$. 5

Consider the function $f : A \to B$ defined by $f(x) = \frac{x-3}{x-2}$. Check whether f is one-one and onto. Justify your answer.

Also, find the value of $f([\pi])$, where $[x]$ represents greatest integer less than or equals to x.

33. (a) Solve the system of linear equations by matrix method : 5

$$2x+3y-z=9$$
$$3x-y+2z=8$$
$$5x+2y+3z=17$$

OR

(b) If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$ and $A^3-6A^2+7A+kI_3=0$

then find the value of k and hence find A^{-1}.

34. (a) Find the image of the point (1, 2, 3) in the line $\vec{r}=(6\hat{i}+7\hat{j}+7\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$. 5

OR

(b) Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$, which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $.\vec{d}.\vec{a}=21-\vec{d}$. Also, find unit factor in the direction of $\vec{d}$.

35. Draw the graph of parabola $y^2=8x$ and line $2x-y=0$. Also find the area of bounded region using integration. 5

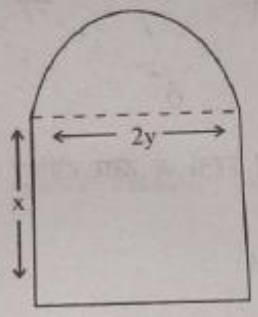
SECTION-E

This section comprises of 3 case based questions of 4 marks each. First two case study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two sub-parts (I) and (II) of marks 2 each.

Case Study-1

36. There are many windows in the house of Rudvik for ventilation.

A window is in form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metre. Let, length and breadth of rectangular part is x and 2y respectively, as shown in figure.



Based on the above information, answer the following questions :

(I) Find the value of x in terms of y. 1

(II) Find the area (A) of window in terms of y alone. 1

(III) Find the maximum area (A_{max}) of window so as to admit maximum light through the whole opening. 2

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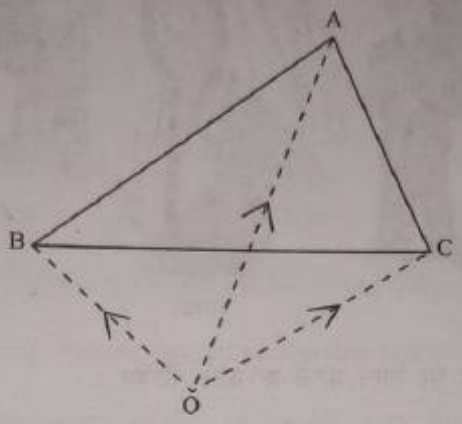
OR

(III) Find the values of x and y for which $\frac{d^2A}{dy^2}+\frac{dA}{dy}=0$.

Case Study-2

37. A farmer Devendra has a triangular farm ABC, as shown in figur. The position vectors of points A, B and C are $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+4\hat{j}-\hat{k}$ and $2\hat{j}+\hat{k}$ respectively.



Based on the above information, answer the following questions :

(I) Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$. 1

(II) Find the length of projection of $\overrightarrow{BA}$ in the direction of $\overrightarrow{BC}$. 1

(III) Find area of ΔABC. 2

OR

(III) Find $\angle ABC$.

Case Study-3

18. An insurance company has insured 5000 teachers, 7000 doctors and 8000 lawyers. The chances of a teacher, doctor and lawyer dying before the age of 60 years is 0.02, 0.03 and 0.04 respectively.

TEACHER

DOCTOR

LAWYER

Based on the above information, answer the following questions :

(I) Find the probability that an insured person dies before 60 years. 2

(II) If one of the insured person dies before 60 years, find the probability that he is a teacher. 2