ERODE SOHODAYA SCHOOLS COMPLEX

PRE-BOARD EXAMINATION 2023-24

CLASS XII **MATHEMATICS** **TIME: 3 hrs**

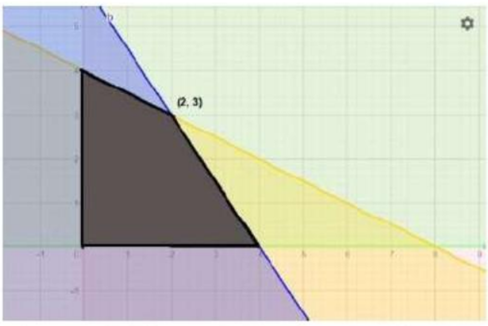
SET - B **Max Marks: 80**

MARKING SCHEME.

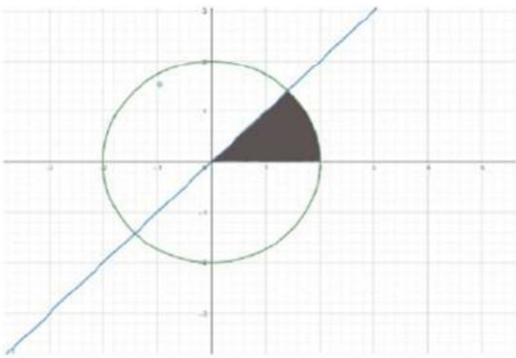
Q.NO	EXPECTED ANSWER	MARKS
1	(d) 2 and 4	1
2	(c) (2,3)	1
3	(c) 2	1
4	(b) x^2	1
5	(c) R	1
6	(d) $\frac{1}{36}$	1
7	(b) I	1
8	(b) $A^2 = I$	1
9	(d) $\frac{1}{12}$	1
10	(a) $\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$	1
11	(d) 1.5	1
12	(a) y	1
13	(d) Use midpoint formulae	1
14	(d) $\overrightarrow{PQ} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$; so unit vector $=\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$	1
15	(c) $\pi/2$	1
16	(b) 2	1
17	(c) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$	1
18	(a) 0	1
19	(c) Assertion is correct, reason is incorrect	1
20	(a) $f(x)$ has a minimum at $x = 2$ as $\frac{d}{dx}(f(x)) < 0, \forall x \in (2-h, 2)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (2, 2+h)$, where $'h'$ is an infinitesimally small positive quantity.	1
21	At any instant t, let r be the radius, V the volume and S the surface area of the balloon. Then, $\frac{dV}{dt} = 20\text{cm}^3$ /sec ... (given) ... (i)	2

	Now, V = $\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $\Rightarrow 20 = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$ $\Rightarrow 20 = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ $\Rightarrow \frac{dr}{dt} = \frac{5}{\pi r^2}$ .... (ii) $\therefore$ S = $4\pi r^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$ $= \frac{d}{dr}\left(4\pi r^2\right) \cdot \frac{5}{\pi r^2}$ $= \left(8\pi r \times \frac{5}{\pi r^2}\right) = \frac{40}{r}$ $\Rightarrow \left[\frac{dS}{dt}\right]_{r=8\text{cm}} = \left(\frac{40}{8}\right)\text{cm}^2 \text{/sec} = 5 \text{ cm}^2 \text{/sec}$ Hence, the rate of change of surface area at the instant when r = 8 cm is 5 cm^2/sec. OR We have Local max. value is 251 at x = 8 and local min. value is -5 at x = 0 Also $F'(x) = -3x^2 + 24x = 0$ $\Rightarrow -3x(x - 8) = 0$ $\Rightarrow x = 0, 8$ $F''(x) = -6x + 24$ $F''(0) > 0$, 0 is the point of local min. $F''(8) < 0$, 8 is the point of local max. F(8) = 251 and f(0) = -5	
22	π/3+2π/6+π/3 π **OR** $\tan^{-1}\left[2\sin\left(\frac{2\pi}{6}\right)\right]$, $\tan^{-1}\left[2\frac{\sqrt{3}}{2}\right]$ **π/3**	$1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
23	It is given that f(x) = |x + 2| - 1 Now, we can see that |x + 2| $\geq$ 0 for every x $\in$ R $\Rightarrow f(x) = \|x + 2\| - 1 \geq -1$ for every x $\in$ R Clearly, the minimum value of f is attained when |x + 2| = 0 i.e, |x + 2| = 0 $\Rightarrow$ x = -2 Then, Minimum value of f = f(-2) = |-2 + 2| - 1 = -1 Therefore, function f does not have a maximum value.	2

24	$I = \int_0^{\pi} \frac{1}{1+e^{\cos x}} dx ----(1)$ Applying $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ $$I = \int_0^{\pi} \frac{1}{1+e^{\cos(\pi-x)}} dx = \int_0^{\pi} \frac{1}{1+e^{-\cos x}} dx$$ $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+1} dx ----(2)$ Adding (1) and (2) $$2I = \int_0^{\pi} \frac{e^{\cos x}+1}{e^{\cos x}+1} dx = \int_0^{\pi} dx$$ $$\therefore 2I = \pi \Rightarrow I = \frac{\pi}{2}$$	1/2 1/2 1/2 1/2
25	Given:- $f(x) = x^9 + 4x^7 + 11$ $f'(x) = \frac{d}{dx}\left(x^9 + 4x^7 + 11\right)$ $f'(x) = 9x^8 + 28x^6$ $f'(x) = x^6(9x^2 + 28)$ as given in question $x \in R$, $\Rightarrow x^6 > 0$ and $9x^2 + 28 > 0$ $\Rightarrow x^6(9x^2 + 28) > 0$ $\Rightarrow f'(x) > 0$ Hence, condition for f(x) to be increasing Thus f(x) is increasing on interval $x \in R$	2
26.	P=2/x Q=x I.F=x^2 Solution is $$yx^2 = \frac{x^4}{4} + c$$ **OR** $$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$$ Put y = vx , $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$ $$\log\left\|v + \sqrt{1+v^2}\right\| = \log\|x\| + c$$ $$\log\left\|\frac{y}{x} + \sqrt{1+(\frac{y}{x})^2}\right\| = \log\|x\| + c$$	½ 1 1½ ½ ½ ½ 1 ½

27	$I=\int_{-5}^{5}\frac{x^2}{1+e^x}dx$ ….. (i), using property $\int_{-5}^{5}f(x)dx=\int_{-5}^{5}f(-x)dx$ $I=\int_{-5}^{5}\frac{x^2}{1+e^{-x}}dx$ …..(ii) From i and ii , adding $2I=\int_{-5}^{5}x^2dx \Rightarrow \quad I=\frac{125}{3}$ OR $\int_{0}^{4}\left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_{0}^{1}(3-2x)dx+\int_{1}^{2}dx+\int_{2}^{4}(2x-3)dx$ =2+1+6 = 9	1/2 1 + 1/2 1/2+1/2 +1/2 1 1/2
28	$I=\int\frac{2x}{\left(x^2+1\right)\left(x^2+2\right)}dx$ let $x^2 = t$ => $2xdx = dt$ $\frac{1}{(t+1)(t+2)}=\frac{A}{t+1}+\frac{B}{t+2}$ Solving A = 1, B = -1 Correct integration and solution $I=\log\left(\frac{x^2+1}{x^2+2}\right)$	1/2 1/2 1/2 + 1/2 1/2 + 1/2
29	For correct differentiation of LHS 1 marks Right hand side with simplification 1 marks Proving 1 marks	1 1 1
30	 Drawing each line and shading the feasible region Corner points(0,0) (4,0)(2,3)(0,4) Minimum value=-12 obtained at x=2 , y=3	1½ 1 ½

31	S ={BB,BG,GB,GG} A= both are girls = {GG} B= youngest is a girl = {BG,GG} C= atleast one is a girl ={BG,GB,GG} P(A/B)=1/2 P(A/C)=1/3 OR $P(\overline{A}) = 1/2 \quad P(\overline{B}) = 2/3$ a.P(problem is solved)= 1 - $P(\overline{A})\,P(\overline{B}) = \frac{2}{3}$ b.P(exactly one of them solves)= P(A) $P(\overline{B})$ + $P(\overline{A})P(B) = \frac{1}{2}$	1 1 1 1 1 1
32	$\|A\| = -1$ adj A = $\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix}$ $X = A^{-1}B$ Getting x=1, y=2, z=3	1 2 ½ 1½
33	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ *and* $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ $\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\overrightarrow{a_2} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ $\overrightarrow{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\overrightarrow{b_2} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ $\overrightarrow{a_2} - \overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ $\overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\|\overrightarrow{b_1} \times \overrightarrow{b_2}\| = \sqrt{6}$ $(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = 1$ S.D. = d = $\left\| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{\|\overrightarrow{b_1} \times \overrightarrow{b_2}\|} \right\|$ Shortest distance d = $\frac{1}{\sqrt{6}}$ The lines do not intersect **OR** Eq. of line $\vec{r} = \vec{a} + \lambda\vec{b}$ Line passes through (1,2,-4) and let (a,b,c) be the D,Ratio of line then	1 ½ 1 ½ ½ ½ ½ 1/2 ½

	Eq of line is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$ Line is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ *and* $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ $\vec{a} \times \vec{b}$ *is perpendicular to* $\vec{a}$ *and* $\vec{b}$ *both* $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ $= 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ Hence D' Ratio of line is (24,36,72) Eq. of line $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$ $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$ $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$	1/2 $2\frac{1}{2}$ ½ ½ ½
34	 Drawing figure and shading the region Getting point of intersection =(√2,√2) $Area = \int_0^{\sqrt{2}} x\,dx + \int_{\sqrt{2}}^{2} \sqrt{4-x^2}\,dx$ $= \left[\frac{x^2}{2}\right]_0^{\sqrt{2}} + \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{2}}^{2}$ $=1+2(\pi/2)-1-2(\pi/4) = \frac{\pi}{2}\, sq.units$	1 $\frac{1}{2}$ 1 1 1½

35	1.a+b=b+a is true So (a,b)R(a,b). R is reflexive 2.(a,b)R(c,d) ⇒ a+d=b+c ⇒ b+c=a+d ⇒ c+b=d+a ⇒ (c,d)R(a,b) ∴R is symmetric 3. (a,b)R(c,d) and (c,d)R(e,f) (c,d)R(e,f) ⇒ a+d=b+c and e+d=f+c Adding(a+d)+ (f+c)=(b+c)+(e+d) a+f=b+e gives (a,b)R(e,f) ∴R is transitive Since R is reflexive , symmetric and transitive it is an equivalence relation OR 1.|a-a|=0 is divisible by 4 ⇒ R is reflexive 2.(a,b) ϵ R |a-b| is divisible by 4 ⇒ |b-a| is divisible by 4 ⇒ (b,a) ϵ R R is symmetric 3. (a,b) ϵ R and (c,d) ϵ R ⇒ |a-b| is divisible by 4 and |c-d| is divisible by 4 ⇒a-b=±4m,c-d=±4n ⇒ a-c=±4(m+n) ⇒|a-c| is divisible by 4 ⇒$(a,c)\epsilon$ R R is transitive ∴R is an equivalence relation Set of elements related to 1 is{1,5,9}	1 1 2 1 1 1 2 1
36	i.$y_1 = x_1^2+7$ ii.D= $\sqrt{(x_1-3)^2+(y_1-7)^2} = \sqrt{(x_1-3)^2+(x_1^2)^2}$ iii.the minimum distance=√5 units OR the nearest position of helicopter from the soldier=(1,8)	1 1 2 2

37	Let A be the event of committing an error. E_1, E_2 and E_3 be the events that Vinay,Sonia and Iqbal processed the form i.P(A)=50%of0.06+20%0.04+30%0.03=0.047 $P(E_1/A)= \frac{0.5x0.06}{0.047} = \frac{30}{47}$	2 2
38	(i)we have $\overrightarrow{OA} = 8\hat{\imath}\ km\ \overrightarrow{AB} = 6\hat{\jmath}$ vector distance from Gitika's house to school =$8\hat{\imath} + 6\hat{\jmath}$ (ii)vector distance from school to Aloke's house = $6\cos 30^0\hat{\imath} + 6sin30^0\hat{\jmath}$ = $3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$ (iii)vector distance from Gitika's house to Aloke's house= $8\hat{\imath} + 6\hat{\jmath} + 3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$ =$(8 + 3\sqrt{3})$+9$\hat{\jmath}$ **OR** The total distance travel by Gitika from her house to Aloke's house= 8 + 6 + 6 = 20 km	1 1 2 2