Applied Mathematics
Class 12
Formulas Sheet

Integrals

Basic Integration

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$
- $\int dx = x + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int e^x \, dx = e^x + C$
- $\int a^x \, dx = \frac{a^x}{loga} + C$

Substitution Method

- If $\int$ f(x) dx = F(x) + C then $\int$ f(ax+b) = $\frac{F(ax+b)}{a}$ + C
- $\int (f(x))^n$ f'(x) dx = $\frac{(f(x))^{n+1}}{n+1}$ + C
- $\int \frac{f'(x)}{f(x)}$ dx = log|f(x)| + C

- Selection of proper substitution
- 1. Try to identify the function f(x) whose derivative is given in question , put f(x) = t and then integrate
- 2. If exponent of f(x) is in fraction then put f(x) = t and then integrate
- 3. is x is replaced by some other function in logx or e^x then put that function t and integrate

Integration By Partial Fraction

Proper rational function	Partial fraction
$\frac{px+q}{(x-a)(x-b)}$, a≠b	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$, a≠b≠c	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$, a≠b	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$, x^2+bx+c can't be factorised	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

Some Special Integrals

- $\int \frac{1}{x^2-a^2}$ dx = $\frac{1}{2a}$ log|$\frac{x-a}{x+a}$|+ C
- $\int \frac{1}{a^2-x^2}$ dx = $\frac{1}{2a}$ log|$\frac{a+x}{a-x}$|+ C
- $\int \frac{1}{\sqrt{x^2+a^2}}$ dx = log| x + $\sqrt{x^2 + a^2}$ | + C
- $\int \frac{1}{\sqrt{x^2-a^2}}$ dx = log| x + $\sqrt{x^2 - a^2}$ | + C
- $\int \sqrt{x^2 + a^2}$ dx = $\frac{x}{2}\sqrt{x^2 + a^2}$ + ½ a^2log|x + $\sqrt{x^2 + a^2}$ |+C
- $\int \sqrt{x^2 - a^2}$ dx = $\frac{x}{2}\sqrt{x^2 - a^2}$ + ½ a^2log|x + $\sqrt{x^2 - a^2}$ |+C

Integration by Parts

- $\int f(x)g(x)dx = f(x)\int$ g(x)dx–{f'(x)$\int$ g(x)dx}dx +C, Where f(x) is first and g(x) is second function
- How to choose first and second function
- I – Inverse trigonometric function
- L– Logarithmic function
- A– Algebraic function
- T– Trigonometric function
- E– Exponential function
- $\int$ (f(x)+f'(x))e^x dx = f(x) e^x + C

Properties of Definite Integrals

- 1. $\int_a^b f(x)dx = \int_a^b f(t)dt$
- 2. $\int_a^b f(x)dx = - \int_b^a f(x)dx$
- 3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- 4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- 5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Marginal Functions

- C(x) = $\int MC\,dx + C$
- AC = $\frac{c(x)}{x}$
- R(x) = $\int MR\,dx + C$
- R(x) = px
- P = $\frac{R(x)}{x}$, where p is demand

Consumer Surplus & Producer Surplus

- C. S = $\int_{0}^{x0} D(x)dx$ $-$ Xo.Po
- P. S = Xo.Po $-$ $\int_{0}^{x0} S(x)dx$
- At Equillibrium C. S = P. S

Differential Equation

- Order and Degree of a Differential Equation The order of a differential equation is the order of the highest order derivative of the dependent variable with respect to independent variable appearing in the equation. If each term involving derivatives of a differential equation is a polynomial (or can be expressed as polynomial), then the highest exponent of the highest order derivative is called the degree of the differential equation.
- If any term of a differential equation cannot be expressed as a polynomial in the derivative (or derivatives), then the degree of the differential equation is not defined.

Growth and Decay Model

- Most common growth and decay models is exponential growth and decay. Exponential growth or decay is a process that occurs when the instantaneous rate of change (i.e. derivative) of a quantity with respect to time is proportional to the quantity itself. Exponential growth and decay can be seen in number of natural applications such as population growth, compound interest, radioactive decay, carbon dating or Newton's law of cooling etc

Inferential Statistics

Difference between Population & Sample	
Population	Sample
1. It focuses on the identification of the characteristic. **2. A survey done of an entire population is accurate and more precise with no margin of error.** **3. It is collection of all elements having same characteristic.** **4. The measurable characteristic of population is called parameter.**	1. It makes inference about the population. 1. A survey done using a sample of the population has a margin of error. 1. It is a subgroup of members of the population. 2. The measurable characteristic of sample is called statistic.

Types of sampling

- 1. Probability Sampling
  - Simple Random Sampling
  - Systematic Sampling
  - Startified Sampling
  - Cluster Sampling

  2. Non Probability Sampling

  --- Conveneince Sampling

  --- Voluntary Response Sampling

  --- Judgement Sampling

  --- Snowball Sampling

Difference between Parameter and Statistics	
Parameter	Statistics
1. It is a characteristic of a population. **2. It is a numeric value that is taken from the entire population such as population mean.** **1. Generally denoted by Greek alphabets.** **2. For ex.- The average weight of all males in India. The average test score of all student in a class.**	1. It is a characteristic of a sample. 2. A statistics is the numerical value taken from a sample and calculated from the sample observation alone. 3. Generally denoted by English alphabet. 4. For ex.- The average weight of 1000 males in India. The average test of 200 students in a class of 500 students.

- $sample\ mean\ \bar{x} = \frac{\Sigma x_i}{n}$
- $sample\ standard\ deviation\ (s) =$
- $\sqrt{\frac{\Sigma(x_{1'} \_ \bar{x})^2}{n-1}}\ or\ \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x_{1'}}{n}\right)^2}$
- Margin of error = $Z_{\alpha/2}\ \frac{\sigma}{\sqrt{n}}$
- Central Limit Theorem
- $\bar{x} = \mu$

  σ² = σ²/*n*
- *So* **σ(x) = σ/** √n
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How to set Null Hypothesis

- Depending on the how the problem is stated, the alternate hypothesis can be one of the following 3 cases:
  Case 1: H1 : $\bar{x} = \mu$. Used when the true sample mean is not equal to the comparison mean. Use Two Tailed T Test.
  Case 2: H1 : $\bar{x} > \mu$. Used when the true sample mean is greater than the comparison mean. Use Upper Tailed T Test.
  Case 3: H1 : $\bar{x} < \mu$. Used when the true sample mean is lesser than the comparison mean. Use Lower Tailed T Test.
  Where $\bar{x}$ is the sample mean and μ is the population mean for comparison

One Sample test

- t= $\frac{x-\mu}{s/\sqrt{n}}$ if n ≥ 30 , t= $\frac{x-\mu}{s/\sqrt{n-1}}$ if n < 30
- One Tailed test
- If $|t| > t(\alpha)$ then null hypothesis reject
- TwoTailed test
- If $|t| > t(\alpha/2)$ then null hypothesis reject
- Degree of freedom = n-1

Two sample test

- t= $\frac{(x1-x2)-D}{\sqrt{\frac{s1^2}{n1}+\frac{s2^2}{n2}}}$
- One Tailed test
- If |t|> $t(\alpha)$ then null hypothesis reject
- TwoTailed test
- If |t|> $t(\alpha/2)$ then null hypothesis reject
- Degree of freedom n1+n2-2

Financial Mathematics

- Perpetuity :
  - Type 1 - if payment made at the end of payment term $P = \frac{R}{i}$
  - Type 2 - if payment made in the beginning of payment term $P = R + \frac{R}{i}$

Sinking Fund :

$$A = R \times \frac{(i+i)^n - 1}{i} \text{ or } A = R \times S_{n/i}$$

- Bond :
  - PV (Fair value) = $\frac{C[1-(1+i)^{-n}]}{i}$ + F $(1 + i)^{-n}$
    - Where is F is maturity amount

Approx YTM = $\frac{C+\frac{F-Pv}{N}}{\frac{F+Pv}{2}}$

Depreciation :Linear or straight line method

D = $\frac{C-S}{N}$

Depriciation % = $\frac{D}{C-S}$ x 100

- Effective Rate Of Return:
  - r_e = $(1 + \frac{r}{100p})^p$ - 1
  - r = annual rate of interest
  - p = number of compounding in a year

  If compounded continuously then $r_e = e^r - 1$

- CAGR :

  CAGR =$(\frac{FV}{PV})^{1/n}$ - 1

- EMI :
  - 1. Flat rate Method-
    - EMI = $\frac{P+I}{n}$
  - 2. Reducing Balance method-
    - EMI = $\frac{P \times i \times (1+i)^{\,n}}{(1+i)^n - 1}$ = $\frac{P \; x \; i}{1-(1+i)^{-n}}$ = $\frac{P}{a_{n/i}}$
    - Total Interest Paid = EMI x n - P