MARKING SCHEME
PRE-BOARD EXAMINATION (2023-24)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

Instructions:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks be absorbed to them.

2. If a student has attempted an extra question answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale of marks (0-80) has to be used. Please do not hesitate toward full marks if the answer deserves it.

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SECTION-A

1.	(d)	12	1
2.	(d)	–4	1
3.	(c)	4	1
4.	(c)	2	1
5.	(d)	4	1

6. (c) $\tan x - x + c$ 1

7. (d) (1, 3) 1

8. (c) 855 1

9. (d) 3 1

10. (a) $xy = c$ 1

11. (b) continuous but not differentiable 1

12. (c) 1 1

13. (d) 0 1

14. (b) –1 1

15. (d) $\frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$ 1

16. (c) 5 units 1

17. (d) $\frac{1}{x}$ 1

18. (a) $-3 \cos 3x$ 1

19. (c) Assertion (A) is true, but Reason (R) is false. 1

20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). 1

SECTION-B

21. As, $f(x) = \sin^{-1} x$, so. ½

(i) $\sin^{-1} x = \frac{\pi}{6} \Rightarrow x = \sin\frac{\pi}{6} = \frac{1}{2}$ ½

(ii) $f\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ 1

22. $y = x^x \Rightarrow \log y = x \log x$ ½

on differentiating w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \left(x\frac{1}{x} + \log x\right)$ 1

$\boxed{\frac{dy}{dx} = y(1 + \log x) = x^x(1 + \log x)}$ ½

OR

Given function is discontinuous when $x^3 - 4x = 0$ ½

$\Rightarrow x(x^2 - 4) = 0$

$\Rightarrow x(x-2)(x+2) = 0$

$\Rightarrow x = 0, -2, 2$

$\therefore$ Hence f(x) will be discontinuous at exactly 3 points (x = 0, x = –2, x = 2) 1½

23. Given $\frac{dy}{dx}=1$ ½

As, $y^2=8x,\Rightarrow 2y\frac{dy}{dx}=8$

So, $2y(1)=8\Rightarrow \boxed{y=4}$ 1

When y = 4, $y^2 = 8x \Rightarrow 8x \Rightarrow 16=8x\Rightarrow \boxed{x=2}$

Hence, the required point is P(2,4). ½

OR

Given, $\frac{dr}{dt}=3cm/sec,\ r=10cm$ ½

As, $A=\pi r^2\Rightarrow \frac{dA}{dt}=2\pi r\frac{dr}{dt}$ 1

$\frac{dA}{dt}=2\pi(10)(3)=60\pi\ cm^2/sec$

Area is increasing at the rate of 60π cm^2/sec ½

24. $I=\int_{-2}^{2}|x|\,dx=2\int_{0}^{2}|x|\,dx=2\int_{0}^{2}x\ dx$ 1

$I=2\left(\frac{x^2}{2}\right)_0^2=4-0=4$ 1

25. $I=\int\frac{dx}{(x^2+1)(x^2+4)}=\frac{1}{3}\int\frac{3}{(x^2+1)(x^2+4)}dx=\frac{1}{3}\int\frac{(x^2+4)-(x^2+1)}{(x^2+1)(x^2+4)}dx=\frac{1}{3}\int\left(\frac{1}{x^2+1}-\frac{1}{x^2+4}\right)dx$ 1

$I=\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+c$

SECTION-C

26. Let X : number of defective bulbs ½

Possible values of X : 0, 1, 2,3

P(X = 0) = P (defective = 0) = 1 (Good×Good×Good) = $\frac{15}{20}\times\frac{15}{20}\times\frac{15}{20}=\frac{27}{64}$

P(X=1) = P (defective = 1) = 3 (Good×Good×Defective) = $3\left(\frac{15}{20}\times\frac{15}{20}\times\frac{5}{20}=\frac{27}{64}\right)=\frac{27}{64}$

P(X=2) = P (defective = 2) = 3 (Good× Defective× Defective) = $3\left(\frac{15}{20}\times\frac{5}{20}\times\frac{5}{20}=\frac{27}{64}\right)=\frac{9}{64}$

P(X=3) = P (defective = 3) = 1 (Defective× Defective× Defective) = $\frac{5}{20}\times\frac{5}{20}\times\frac{5}{20}=\frac{1}{64}$

Required possibility distribution of X is

X	0	1	2	3
P(x)	27/64	27/64	9/64	1/64

2

Mean $=\Sigma xP(X)=\frac{0+27+18+3}{64}=\frac{48}{64}=\frac{3}{4}$ ½

OR

Sample space = {MF, MM, FF, FM, MM} (F=elder, M=younger)

(i) Let A : Both children are male

B : atleast one of the children is male {MF, FM, MM}

$$P(A) = \frac{1}{4}, P(B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$$

Thus $P\left(\frac{A}{B}\right) = \frac{1}{3}$ 1½

(ii) Let C : Both children are female {FF}

D : Elder child is female {FM, FF}

$$P(C) = \frac{1}{4}$$

$$P(D) = \frac{2}{4}$$

$$P(C \cap D) = \frac{1}{4}$$

Thus $P\left(\frac{C}{D}\right) = \frac{1}{2}$ 1½

27. $(x^2 - y^2)dx = -2xy\,dy$

$$\boxed{\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}} = \frac{1}{2}\left(\frac{y}{x} - \frac{x}{y}\right) \text{ (Homogenous diff. equation)}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

Thus, $v + x\frac{dv}{dx} = \frac{1}{2}\left(v - \frac{1}{v}\right)$

$$\Rightarrow x\frac{dv}{dx} = \frac{-(v^2+1)}{2v}$$

$\Rightarrow \int \frac{2v}{v^2+1}dv = -\int \frac{1}{x}\,dx$ 1

$$\Rightarrow \log|v^2+1| = \log\left(\frac{c}{x}\right)$$

$\Rightarrow \boxed{y^2 + x^2 = cx}$ required solution of given differential equation. 1

OR

$\boxed{\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{1}{(1+x^2)^2}}$ Linear differential equation ½

Integrating factor = $e^{\int \frac{2x}{1+x^2}dx} = (1+x^2)$ 1

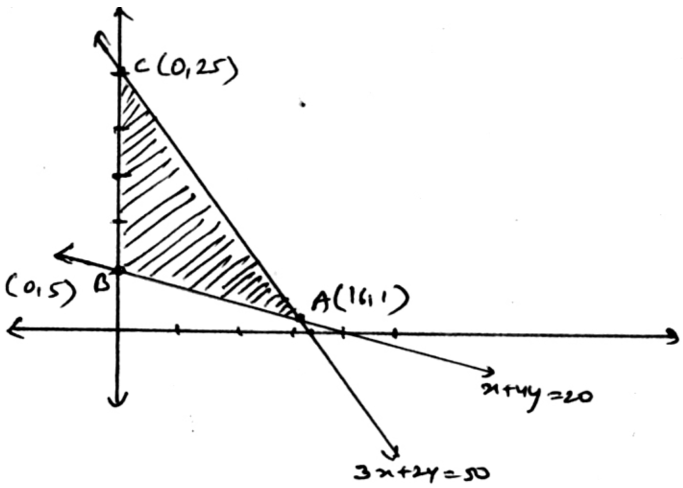
Thus, equation of given difference is $y.(1+x^2) = \int \frac{1}{1+x^2}\,dx$

$\boxed{y = \frac{\tan^{-1}x}{1+x^2} + \frac{c}{1+x^2}}$ or $\boxed{y(1+x^2) = \tan^{-1}x + c}$ 1

When $x = 1,\ y = 0 \Rightarrow c = -\frac{\pi}{4}$

$\therefore \boxed{y(1+x^2) = \tan^{-1} x - \frac{\pi}{4}}$ required particular solution ½

28. On plotting the graph of $3x + 2y \le 50$, $x + y \ge 20$, $x \ge 0$, $y \ge 0$, we get



1½

Shaded region ABC is the solution set of given system of inequalities.

Corner points are A(16,1), B (0,5), C (0,25).

$Z_A = 10(16) + 15(1) = 160 + 15 = 175$

$Z_B = 10(0) + 15(3) = 0 + 75 = 75$

$Z_C = 10(0) + 15(25) = 0 + 375 = 375$

Thus, maximum value of Z is 375 when x = 0, y = 25. 1½

29. f(x) = sin 3x

$f'(x) = 3\cos 3x = 0 \Rightarrow 3x = \frac{\pi}{2}, \frac{3\pi}{2}, ...$

$\therefore x = \frac{\pi}{6} \in \left[0, \frac{\pi}{2}\right]$ and $x = \frac{\pi}{2} \in \left[0, \frac{\pi}{2}\right]$

Possible intervals are $\left[0, \frac{\pi}{6}\right]$ and $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ 1½

As $f'\left(\frac{\pi}{2}\right) = +ve \Rightarrow f(x)$ is increasing function

As $f'\left(\frac{\pi}{4}\right) = -ve \Rightarrow f(x)$ is decreasing function

Thus, f(x) is increasing for $x \in \left[0, \frac{\pi}{6}\right]$ and

f(x) is decreasing for $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ 1½

30. As $x = a\sin^2\theta,\ y = a\cos^2\theta$

Thus, $x + y = a \Rightarrow \boxed{\frac{dy}{dx} = -1}$ 2

On differentiating w.r.t. x

$\boxed{\frac{d^2y}{dx^2} = 0}$ 1

OR

$y = 3\cos(\log x) + 4\sin(\log x)$

$y' = \dfrac{-3\sin(\log x) + 4\cos(\log x)}{x}$

$\Rightarrow xy' = -3\sin(\log x) + 4\cos(\log x)$ 1½

Again differentiating w.r.t. x,

$x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \dfrac{-3\cos(\log x) - 4\sin(\log x)}{x} = \dfrac{-y}{x}$

$\boxed{x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0}$ 1½

31. $I = \int_0^{\pi/4} e^x\left(\dfrac{2+\sin 2x}{1+\cos 2x}\right)dx$

$= \int_0^{\pi/4} e^x\left(\dfrac{2+2\sin x\cos x}{2\cos^2 x}\right)dx = \int_0^{\pi/4} e^x(\sec^2 x + \tan x)dx$ 1

$= \left(e^x \tan x\right)_0^{\pi/4}$ 1

$= e^{\pi/4} - 0$

$\boxed{I = e^{\pi/4}}$ 1

SECTION-D

32 $AB = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ 2

The given equation can be written as

$$\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix}$$

$A\,X = C$

$X = A^{-1}C \quad (\because AB = I \Rightarrow A^{-1} = B)$

$X = BC$ 1

$$= \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 9 \\ -15 \\ 1 \end{bmatrix}$$

Thus,

$$\left.\begin{array}{r} x = 9 \\ y = -15 \\ z = 1 \end{array}\right]$$ 2

OR

32. $A^2 = \begin{pmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{pmatrix}$

Now,

$$A^2 - 7A + 10I = \begin{pmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{pmatrix} + \begin{pmatrix} -21 & -14 & 0 \\ -7 & -28 & 0 \\ 0 & 0 & -35 \end{pmatrix} + \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ 3

Thus, $A^2 - 7A + 10I = 0$

On pre-multiplying by A^{-1},

$A - 7I = 10A^{-1} = 0$

$$A^{-1} = \frac{1}{10}[7I - A] = \frac{1}{10}\left[\begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} - \begin{pmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}\right]$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ 2

33. $S = \{(a, b) : a \leq b^2\}$

(Any example which reject the inequality awarded the marks)

For reference, one such example

- As, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin S$, since $\frac{1}{2} \nleq \frac{1}{4}$, So R is not Reflexive. 1½
- As $(1,2) \in S$ but $(2, 1) \notin S$.
- Since $1 < 2^2$ but $2 \nless 1^2$. So R is not Symmetric. 1½

- As $(9,4) \in S$ and $(4,\sqrt{5}) \in S$, but $(9,\sqrt{5}) \notin S$. Since $9<4^2$, $4<5$ but $9 \nless 5$. So, <u>R is not Transitive</u>. 2

OR

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$

$\Rightarrow x_1 + x_1x_2^2 = x_2 + x_1^2x_2$

$\Rightarrow x_1 - x_2 = x_1x_2(x_1 - x_2)$

$\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$

Either $\boxed{x_1 = x_2}$ or $\boxed{x_1 = \frac{1}{x_2}}$ 2½

So, f(x) is not one-one function.

Let y = f(x) $\Rightarrow y = \frac{x}{1+x^2} \Rightarrow \boxed{x = \frac{1 \pm \sqrt{1-4y^2}}{2y}}$

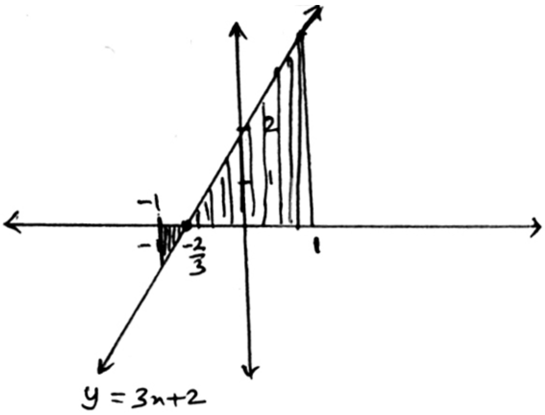
It is defined when $1 - 4y^2 \geq 0 \Rightarrow \left(y - \frac{1}{2}\right)\left(y + \frac{1}{2}\right) \leq 0$

Range = $\left[-\frac{1}{2}, \frac{1}{2}\right]$ = co-domain.

So, f(x) is onto function. 2½

34. $y = 3x + 2$

It is the equation of line.



1½

$$\text{Area} = \left|\int_{-1}^{-2/3} y\ dx\right| + \int_{-2/3}^{1} y\ dx$$

$$\text{Area} = \left|\int_{-1}^{-2/3} (3x+2)\ dx\right| + \int_{-2/3}^{1} (3x+2)\ dx$$ 1

$$= \left|\left(\frac{3x^2}{2} + 2x\right)_{1}^{-2/3}\right| + \left[\frac{3x^2}{2} + 2x\right]_{-2/3}^{1}$$ 1

$$= \left|\frac{2}{3} - \frac{4}{3} - \frac{3}{2} + 2\right| + \left(\frac{3}{2} + 2 - \frac{2}{3} + \frac{4}{3}\right)$$

$$= \frac{1}{6} + \frac{25}{6} = \frac{26}{6} = \frac{13}{3}\text{sq.units}$$ 1½

35. $\vec{a}.\vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})(2\hat{i} - 3\hat{j} + 5\hat{k}) = 2 - 6 + 15 = 11$ 1

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -3 & 5 \end{vmatrix} = 19\hat{i} + \hat{j} - 7\hat{k}$$

$|\vec{a} \times \vec{b}| = \sqrt{361 + 1 + 49} = \sqrt{411}$ 1½

Now, $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = 11^2 + (\sqrt{411})^2 = 121 + 411 = 532$

RHS $|\vec{a}|^2 |\vec{b}|^2 = 14 \times 38 = 532$

Thus, $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$ 2½

SECTION-E

36. (i) V(x) = L×B×H = (18 – 2x) (18 – 2x) x = $4x^3 - 72x^2 + 324x$ 1

(ii) $\frac{dV}{dx} = 12(x-3)(x-9) = 0 \Rightarrow x = 3$ As $\boxed{x \neq 9}$ 1

$\frac{d^2V}{dx^2} = 12(2x - 12) \left.\frac{d^3V}{dx^2}\right|_{x=3} < 0$, so volume is maximum at x = 3.

(iii) $V_{max} = (12)(12)3 = 432 cm^3$

OR

Total area of the removed squares = $4x^2 = 4(9) = 36\ cm^2$ 2

37. (i) $P(E_1) = 0.65$, $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

$P(E_2) = 0.35$ (Probability that all workers are present for the job) 1

(ii) $P(A) = P\left(\frac{A}{E_1}\right)P(E_1) + P\left(\frac{A}{E_2}\right)P(E_2)$

$= 0.35 \times 0.65 + 0.80 \times -0.35$

$\boxed{P(A) = 0.5075}$ (Probability that construction will be completed on time) 1

(iii) (a) $P\left(\frac{E_1}{A}\right) = \frac{P(A/E_1) \times P(E_1)}{P(A)} = \frac{0.35 \times 0.65}{0.5075} = \frac{0.65}{1.45}$

$\boxed{P\left(\frac{E_1}{A}\right) = \frac{65}{145} = \frac{13}{29}}$

(b) $P\left(\frac{E_2}{A}\right) = \frac{P(A/E_2) \times P(E_2)}{P(A)} = \frac{0.35 \times 0.80}{0.5075} = \frac{16}{29}$ 2

38. (i) Angle between the lines

$$\cos\theta = \left|\frac{3(-1) + (-2)(3) + (-1)(-2)}{\sqrt{14}\ \sqrt{14}}\right| = \left|\frac{-3 - 6 + 2}{14}\right| = \frac{1}{2}$$

$\boxed{\theta = \frac{\pi}{3}}$

(ii) Let $\frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} = a$ and $\frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} = b$

$$\left.\begin{aligned} x &= 3a-1 \\ \therefore\ y &= -2a+3 \\ z2 &- a - 2 \end{aligned}\right] \text{and} \left.\begin{aligned} x &= -b \\ y &= 3b+7 \\ z &= -26-7 \end{aligned}\right]$$

For intersecting lines, 3a – 1 = –b and –2a + 3 = 36 + 7

on solving we get a = 1, b = –2

Thus, point of intersecting is (2, 1, –3)