No. of pages - 24

(M)

PRE-BOARD EXAMINATION (2023-24)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours | **Maximum Marks : 80**

समय : 3 घंटे | **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड क में प्रश्न संख्या 1 से 18 तक बहु विकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड ख में प्रश्न संख्या 21 से 25 तक अति लघु उत्तरीय(VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड ग में प्रश्न संख्या 26 से 31 तक लघु उत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच- पाँच अंकों के प्रश्न हैं।
7. खंड ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में, खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, one internal choice has been provided in 2 questions in section B, 3 questions in section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\begin{bmatrix} 3 & 2 \\ 1 & y \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$, then $(2x + y)$ is : 1

(a) 4 (b) 8

(c) 10 (d) 12

2. If the matrix $\begin{bmatrix} 0 & 3 & 2 \\ K+1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of K is : 1

(a) 0 (b) 3

(c) –3 (d) –4

3. If A is a square matrix of order 3 and $|A| = 2$, then the value of $|\text{adj } A|$ is : 1

(a) 1 (b) 2

(c) 4 (d) 8

4. If $A = \begin{bmatrix} 2 & 2023 \\ 0 & 1 \end{bmatrix}$, then $|A|$ is equal to : 1

(a) 0 (b) 1

(c) 2 (d) 2023

5. If matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k^K is : 1

(a) 1 (b) 0

(c) 2 (d) 4

6. $\int \tan^2 x \, dx$ equals : 1

(a) $\sec x + c$ (b) $\tan x + c$

(c) $\tan x - x + c$ (d) $\tan x + x + c$

7. The point which lies in the half-plane $x + y - 6 \leq 0$ is : 1

(a) (5, 2) (b) (2, 5)

(c) (8, 1) (d) (1, 3)

8. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15).

If $Z = 18x + 9y$ be the objective function, then maximum value of Z is : 1

(a) 450 (b) 684

(c) 855 (d) 875

9. The value of $(\hat{i} \times \hat{j}).\hat{k} + (\hat{j} \times \hat{k}).\hat{i} + (\hat{k} \times \hat{i}).\hat{j}$ is : 1

(a) 0 (b) 1

(c) 2 (d) 3

10. The general solution of the differential equation $x\,dy + y\,dx = 0$ is : 1

(a) $xy = c$ (b) $y = cx$

(c) $x + y = c$ (d) $x - y = c$

11. The function $f(x) = |x|, x = 0$ is : 1

(a) continuous and differentiable (b) continuous but not differentiable

(c) differentiable but not continuous (d) neither differentiable nor continuous

12. If $(\hat{i} + 2\hat{j} + 2\hat{k}) \times (p\hat{i} + q\hat{j} + 2024\hat{k}) = \vec{0}$, then $(2p - q + 1)$ is equal to : 1

(a) -1 (b) 0

(c) 1 (d) 2

13. $\int_0^{\pi/2}(\sin^{2023}x-\cos^{2023}x)dx$ equals : 1

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) 2023 (d) 0

14. The value of 'p' for which the lines $\frac{x-1}{2}=\frac{1-y}{2}=\frac{z}{1}$ and $\frac{x-6}{1}=\frac{y}{p}=\frac{3-z}{4}$ are at right angles is : 1

(a) 0 (b) –1

(c) 1 (d) –3

15. A unit vector along the vector $2\hat{i}-2\hat{j}+\hat{k}$ is : 1

(a) $2\hat{i}-2\hat{j}+\hat{k}$ (b) $\frac{2\hat{i}-2\hat{j}+\hat{k}}{5}$

(c) $\frac{2\hat{i}-2\hat{j}+\hat{k}}{\sqrt{3}}$ (d) $\frac{2\hat{i}-2\hat{j}+\hat{k}}{3}$

16. Distance of the point (–3, –4, –5) from z-axis is : 1

(a) 0 (b) –5 units

(c) 5 units (d) 10 units

17. The integrating factor for solving the differential equation $x\frac{dy}{dx}-y=x^5$ is : 1

(a) e^{-x} (b) e^x

(c) x (d) $\frac{1}{x}$

18. If $y = 4\sin^3 x - 3\sin x$, then $\frac{dy}{dx}$ is : 1

(a) $-3\cos 3x$ (b) $-\cos 3x$

(c) $3\cos x$ (d) $-\frac{\cos 3x}{3}$

Question number 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below :

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : The principle value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$. 1

Reason (R) : Domain of $\cos^{-1} x$ is $\mathbb{R}$.

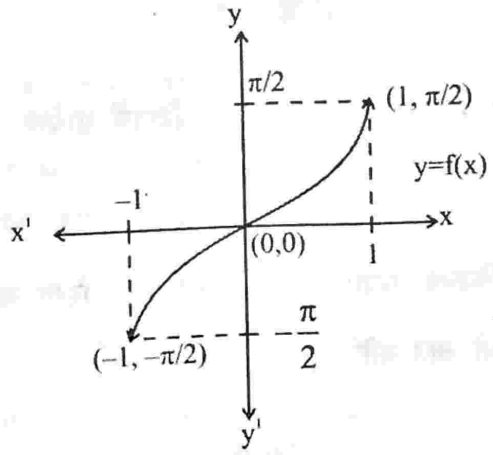
20. Assertion (A) : Given two independent events A and B such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then P (A and not B) $= \frac{1}{3}$. 1

Reason (R) : For two independent events A and B, P(A and B) = P(A) . P(B).

SECTION-B

This section comprises of very short answer (VSA) type questions of 2 marks each.

21. The graph of an inverse trigonometric function f(x) is given below. Observe the graph and answer the following questions : ½



(i) If $f(x)=\frac{\pi}{6}$, then find the value of x. ½

(ii) What is the value of $f\left(-\frac{1}{\sqrt{2}}\right)$? 1

22. If $y = x^x$, then prove that $\frac{dy}{dx} = x^x(1+\log x)$ 2

OR

Find the points at which the function $f(x)=\frac{x^2+1}{x^3-4x}$ is discontinuous.

23. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate. 2

OR

The radius of a circle is increasing uniformly at the rate of 3 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

24. Evaluate : $\int_{-2}^{2} |x|\, dx$ 2

25. Evaluate : $\int \frac{dx}{(x^2+1)(x^2+4)}$ 2

SECTION-C

This section comprises short answer (SA) type questions of 3 marks each.

26. From a lot of 20 bulbs which include 5 defectives, a sample of 3 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution. 3

OR

A couple has two children :

(i) Find the probability that both children are males, if it is known that atleast one of the children is male. 1½

(ii) Find the probability that both children are females, if it is known that elder child is a female. 1½

27. Solve the differential equation $(x^2 - y^2)dx + 2xy\ dy = 0$. 3

OR

Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that $y = 0$, when $x = 1$.

28. Solve the following linear programming problem graphically: 3

Maximize $Z = 10x + 15$

Subject to the constraints :

$3x + 2y \leq 50$

$x + 4y \geq 20$

$x \geq 0,\ y \geq 0$

29. Find the intervals in which the function given by $f(x) = \sin 3x,\ x \in \left[0, \frac{\pi}{2}\right]$ is : 3

(a) increasing

(b) decreasing

30. If $x = a \sin^2\theta$ and $y = a \cos^2\theta$, then find $\frac{d^2y}{dx^2}$. 3

OR

If $y = 3 \cos (\log x) + 4 \sin (\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

31. Evaluate : $\int_0^{\pi/4} e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) dx$ 3

SECTION-D

This section comprises long type answer (LA) type questions of 5 marks each.

32. If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations : 5

$-3x - 2y - 4z = -1$

$2x + y + +2z = 5$

$2x + y + 3z = 6$

OR

If $A=\begin{pmatrix}3 & 2 & 0\\ 1 & 4 & 0\\ 0 & 0 & 5\end{pmatrix}$, shows that $A^2 - 7A + 10\, I_3 = 0$. Hence find A^{-1}.

33. Show that the relation S in set $\mathbb{R}$ of real numbers defined by : 5

$$S=\{(a,b): a \le b^2,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

Is neither reflexive, nor symmetric, nor transitive,

OR

Check whether a function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$ is one-one and onto or not.

34. Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1. 5

35. For the vectors $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$, find the value of $|\vec{a}.\vec{b}|$ and $|\vec{a}\times\vec{b}|$. Hence verify that : 5

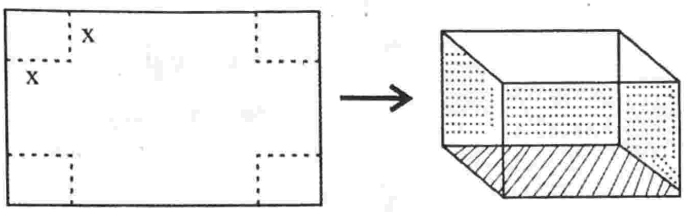
$$|\vec{a}.\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$$

SECTION-E

This section comprises 3 case based questions of 4 marks each.

Case Study-1

36. A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flags.



Based on the above information, answer the following questions, if x cm is the length of each square cut from corners.

(i) Write volume of box, V(x) as a function in terms of x. 1

(ii) What should be the side of the square to be cut-off so that the volume is maximum? 1

(iii) (a) Find the maximum volume of the open box. 2

OR

(b) Find the total area of the removed squares.

Case Study-2

37. A building contractor undertake a job to construct 5 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many workers are not present and still the work gets completed on time is 0.35.

The probability that work will be completed on time when all workers are present is 0.80.

Let E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present;

and

A : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job? 1

(ii) What is the probability that construction will be completed on time? 1

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time. 2

OR

(b) What is the probability that all workers were present given that the construction work was completed on time?

38. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

Two such wires lie along the following lines :

$$\ell_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$$

$$\ell_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$$

On the basis of the above information, answer the following questions :

(i) Find the angle between lines ℓ_1 and ℓ_2. 2

(ii) Find the point of intersection of the lines ℓ_1 and ℓ_2. 2