GYAN BHARATI SCHOOL

Preboard (2022-2023)

Applied Mathematics (241)

SS2

Set-B (Marking Scheme)

1. (c) 2. (c) 3. (b) xplanation:

Let the number of girls be x and number of boys is 1000 – x.

70100 (1000 – x)+ 80100 × x = 76100 × 1000

70(1000 – x) + 80x = 76 × 100

70 × 1000 – 70x + 80x = 76 × 1000

10x = 1000 (76 – 70)

10x = 6000

x = 600

Hence the number of girls = 600. 4. (b) 5. (c)

6. A : B = 160 : 150, A : C =160 : 130 B/C = (B/A) x (A/C) = (150/160) x (160/130) = 15/13 = 60 : 52 ∴ In a game of 60, B can give C = (60 - 52)= **8 points**

7.(b) 8 (c) 9 (a) 10 (a) 11 (a) Here, np = 4 and npq = 2

So, q = 12 and hence p = 12 and n = 8

So, P(X = 2) = $^8C_2$ $(12)^2$ $(12)^6$ = $28(12)^8$

= 28256

12. (a)

13. Explanation:

Here, Z = X−305 Thus, for

X = 32,

Z = 32−305 = 0.4

So, (d) is the correct option

14. (b) 33

n = 34

⇒ v = 34 – 1 = 33

15. (d) 136000

Let the face value of the bond = x

Then, 10200 × =1800 ⇒ x = 36000

16. (d) If a LPP admits two optimal solutions, then it has infinite optimal solutions.

17. (b) Sampling distribution

Explanation: When a collected sample from populates is represented in the form of a frequency distribution, it is called sampling distribution.

18. (d) sample

19. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
P(Win in one game) = P(Lose in one game)
= 1/2
⇒ P(Beena to win in 3 out of 4 games) = ${}^4C_3$
(12)4 = 14 = 25%
Assertion is correct and Reason is the correct explanation for it.

20. (d) (A)is false but (R) is true.

Explanation:
Given: y = log x,
dydx = 1x
d2ydx2 = −1x2
and d3ydx3 = 2x3

21. Let the speed of the stream be y km/h. Let the distance be ‘d’ km. Then,
d/(6 – y) = 2x d/(6 + y)
y = 2
So, the speed of the stream is 2 km/h

22.
Probability of defective bucket = 0.03
n = 100
m = np = 100 × 0.03 = 3
Let X = number of defective buckets in a sample of 100
P(X = r) = $\frac{m^r e^{-m}}{r!}$, r = 0, 1, 2, 3,.....
= $\frac{3^0 e^{-m}}{0!}$ = 0.049

X = scores of students, μ = 45, σ = 5
∴ Z = $\frac{X-\mu}{\sigma}$ = $\frac{X-45}{5}$

(A) When X = 45, Z = 0
P (X > 45) = P (Z > 0) = 0.5
⇒ 50% students scored more than the mean score

Or

23. Here.

H_0: μ = 15, s = 5.5, n = 30 and $\overline{X}$ = 17

To test H_0, the statistic t is

t = X̄−μs/n−1√=17−155.5/29√ = 1.96

The table value of t at α = 1105 and 29 d.f. is 2.05.

Conclusion: Since |t| < t_α, the null hypothes is accepted.

OR

Here, we have i = 008 and $P_{\square}$ = 60,000

Using the formula $P_{\square}$ = (R + Ri) we need to determine the vaLue of R.

Using the given values in 60.000

= ₹ (R + R0.008)

we have

R = 60,000 × 8108 = 4444.44

Thus, the prize amount is ₹ 4444.44.

24. Answer:

Purchase = ₹ 40,00,000

Down payment = x

Balance = 40,00,000 – x

i = 91200 = 0.0075

n = 25 × 12 = 300

E = ₹ 30,000

⇒ 30000 = (4000000−x)×0.00751−(1.0075)−300

25. Here,

$H_0 : \mu = 12$, $s = 0.15$, $n = 10$ and $\bar{X} = 11.8$

To test H_0, the statistic t is

$t = \frac{\bar{X}-\mu}{s/\sqrt{n-1}} = \frac{11.8-12}{0.15/\sqrt{9}} = -4$

The table value of t at $\alpha = 0.05$ and 9 d.f. is 2.26

Conclusion: Since $|t| > t_\alpha$, the null hypothesis is rejected and we conclude that the sample mean differs significantly from the intended mean of 12 kg

OR

Option 1: 6%, ₹ 100 share at ₹ 120

Here, income of an investment of ₹ 120 = ₹ 6

So, income on an investment of ₹120

= ₹ $\{\frac{6}{120} \times 100\}$ = ₹ 5

Option 2 :4% , ₹10 share at ₹ 8

Here, income of an investment of ₹ 8 = ₹ 0.4

So, income on an investment of ₹ 100

= ₹ $\{\frac{0.4}{8} \times 100\}$ = ₹ 5

Hence, both the investments yield the same return.

26. Let the speed of the boat in still water be x km/h; and the speed of the stream be y km/h.

Then,

$45/(x + y) + 45/(x - y)$

$= 20$ and $12/(x + y)$

$= 4/(x - y)$

Solving these equations, we get

$x = 6$ and $y = 3$

So, the speed of the boat in still water be 6 km/h; and the speed of the stream be 3 km/h.

Here, $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$

Further, $|A| = 9 \neq 0$

A is non-singular and hence A^{-1} exists.

Here, $A^{-1} = \frac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$

Now, $X = A^{-1} B$ implies

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1$, $y = 2$, $z = 3$ is the required solution of the given system of equations.

27.

28. Here, the graph of the given constraints and the feasible region, so obtained, is shaded in the graph given below:

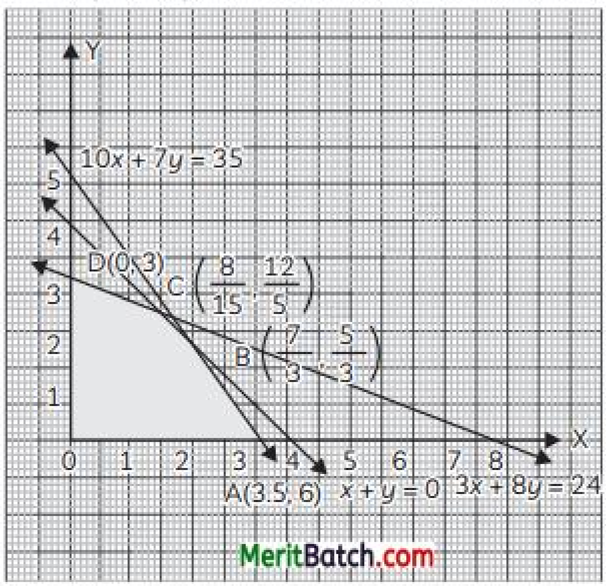
Corner point	Corresponding value of Z
O (0, 0)	5(0) + 7(0) = 0

A (3, 5, 0)	5(3.5) + 7(0) = 17.5
B (73, 53)	5(73) + 7(53) = 703 = 2313
C(85, 122	5(85)+7(125)=1245=2445
D(0, 3)	5(0) + 7(3) = 21

Thus, maximum value of Z is 2445 which occurs at C(85, 125)

OR

Here, the graph of the given constraints and the feasible region, so obtained, is shaded in the graph given below:



Corner point	Corresponding value of Z
A (3,52)	3 – 5(52) + 20 = 212
B (6, 4)	6 – 5 (4) + 20 = 6
C (4, 4)	4 – 5 (4) + 20 = 4
D (3, 3)	3 – 5 (3) + 20 = 8

Thus, minimum value of Z is 4 which occurs at C(4, 4).

29. Under pure competition, $p_d = p_s$

⇒ 8x+1 – 2 = x+32

⇒ $x^2 + 8x - 9 = 0$

⇒ x = -9, 1
⇒ x = 1
When x_0 = 1 ⇒ p_0 = 2
∴ Produce surplus = 2 – ∫10x+32dx
= 2[x24+3x2]10 = 14

OR

p = 274 – x^2
⇒ R = px = 274x – x^3
dRdx = 274 – $3x^2$.
Given MR = 4 + 3x
In profit monopolist market,
MR = dRdx
⇒ 4 + 3x = 274 – $3x^2$
⇒ x^2 + x – 90 = 0
⇒ x = -10, 9
∴ x = 9
When x_0 = 9 ⇒ p_0 = 193
∴ Consumer surplus
= ∫90(274 – 0) – 193 × 9
= [274x−x30]90
= 486

30. Let p denote the probability of drawing a white ball from an urn containing 5 white, 7 red and
8 black balls, Then, p = 55+7+8 = 0.25
Therefore, q = 1 – p = 1 – 0.25 = 0.75
Let X denote the number of white balls in 4 draws with replacement. Then, X is a binomial variate with parameters n = 4 and p = 0.25
such that P(X = r) = ^4C_r $(0.25)^r(0.75)^{4-r}$; r = 0, 1, 2, 3, 4
Now, Probability that all are white balls = P(X) = $^4C_4$ $(0.25)^4(0.75)^0$
= 0.25^4, or 1256
Probability that only 3 are white balls = P(X = $^4C_3(0.25)^3(0.75)^1$ = $4(0.25)^3$ $(0.75)^1$, or 364
Probability that none is white balls = P(X = 0) = $^4C_0$ $(0.25)^0(0.75)^4$ = $(0.75)^4$, or 81256
Probability that at least 3 are white balls = P(X ≥ 3)
= P(X = 3) + P(X = 4)
= 364 + 1256 { From (i) and (ii)}
= 13256

OR

According to the problem,

$Z = X-65.512$

(A) $P(46 < X < 68)$

$= P(-1.63 < Z < 0.21)$

$= P(0 < Z < 1.63) + P(0 < Z < 0.21) = 0.4484 + 0.0832 = 0.5316$ Thus, the number of students having weight between 46 and 68 kg is $700 \times 0.5316 = 372$ (B) $P(X > 68) = P(Z > 0.21)$

$= 0.5 - P(0 < Z < 0.21)$

$= 0.5 - 0.0832$

$= 0.4168$

Thus, the number of students having weight more than 68 kg is $700 \times 0.4168 = 292$

Answer:

$n = 10 \times 2 = 20,$

$S = 10, 21, 760,$

$i = \frac{5}{200} = 0.025$, R = ?

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow \quad 1021760 = R\left[\frac{(1+0.025)^{20} - 1}{0.025}\right]$$

$$\Rightarrow \quad 1021760 = R\left[\frac{1.6368 - 1}{0.025}\right]$$

$$\Rightarrow \quad R = \left[\frac{1021760 \times 0.025}{0.6386}\right]$$

$$\Rightarrow \quad R = ₹\ 40{,}000$$

Mr Mehra set aside an amount of ₹ 40,000 at the end of every six months.

31.

32.

Answer:

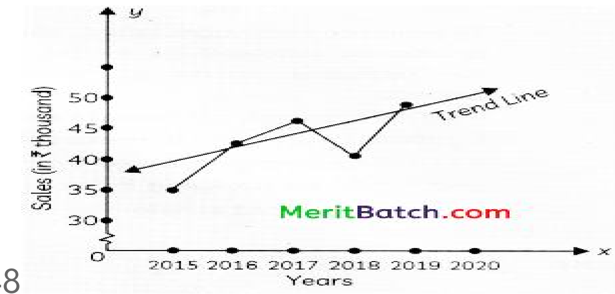
Year	Y	X	X^2	XY
2015	35	−2	4	−70
2016	42	−1	1	−42
2017	46	0	0	0
2018	41	1	1	41
2019	48	2	4	96
	212		10	25

$$a = \frac{\Sigma Y}{n} = \frac{212}{5} = 42.4$$

and $$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{25}{10} = 2.5$$

$$Y_C = 42.4 + 2.5X$$

33.

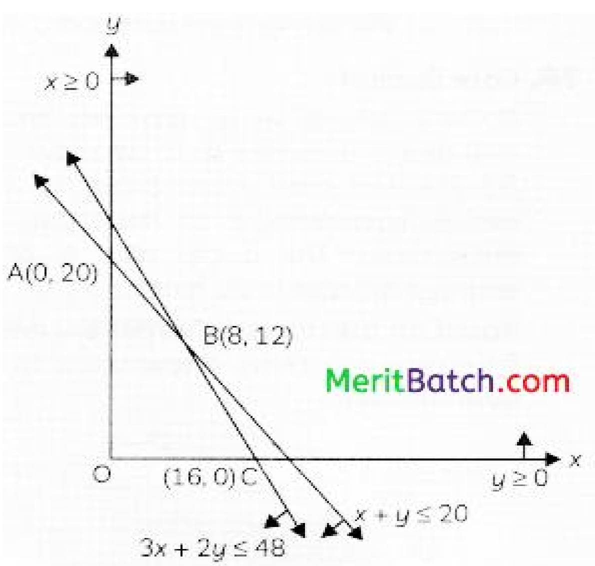
Year	Y	3-year moving average
2015	35	–
2016	42	41
2017	46	43
2018	41	45
2019	48 	–

OR

33.

34. Answer:
Let x be the number of guests for the booking
Clearly, x >100 to avail discount
Profit, P = [4800 – 20010(x – 100)]x = 6800
x – 20x^2
⇒ dPdx = 6800 – 40x ⇒ x = 170
As d2Pdx2 = -40 < 0, ∀ x
∴ Manufacturing loo dolls will maximise the profit of the company and, profit = ₹ 1,50,000.

35. Let the number of tables and chairs be x and y respectively
(Max profit) Z = 22x+ 18y
Subject to constraints:
$x + y \leq 20$
$3x + 2y \leq 48$
$x, y \geq 0$



The feasible region OABCA is closed (bounded)

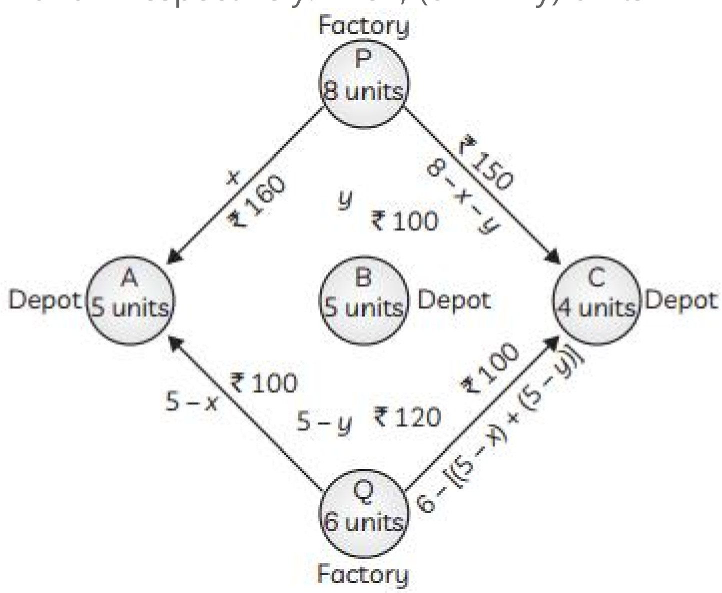
Corner points	Z = 22x + 18y
O(0, 0)	0
A(0, 20)	360

B(8, 12)	392
B(8, 12)	352

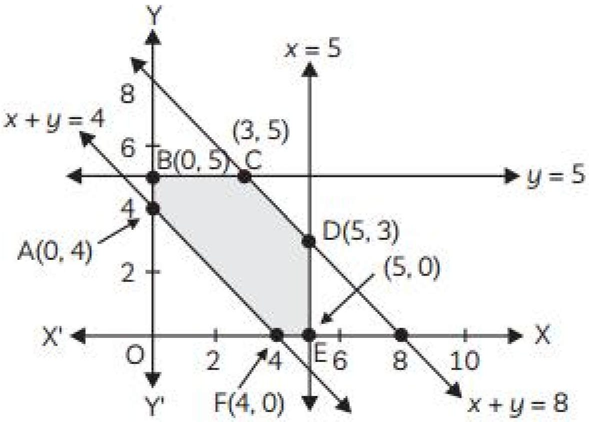
Buying 8 tables and 12 chairs will, maximise the profit

Or

Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively. Then, $(8 - x - y)$ units will be transported to the depot at C.



Hence, we have $x, y \geq 0$ and $8 - x - y \geq 0$
i. e., $x + y \leq 8$ and $x, y \geq 0$
Now, the weekly requirement of the depot at A is 5 units of the commodity. So, $(5 - x)$ units need to be transported to the depot A from the factory at Q. Obviously, $5 - x > 0$.
Similarly, $(5 - y)$ units and $\{6 -(5 - x + 5 - y)\}$
$= (x + y - 4)$ units are to be transported from the factory at Q to the depots at B and C respectively. Thus,
$5 - y \geq 0, x + y - 4 \geq 0$
or $y < 5, x + y > 4$
Let Z be the total transportation cost. Then,
$Z = 160x + 100y + 150(8 - x - y) + 100(5 - x) + 120 (5 - y) + 100 (x + y - 4)$
$= 10(x - 7y + 190)$
Thus, the required LPP is
Minimise $Z = 10(x -7y + 190)$
subject to constraints
$x + y \leq 8, x + y \geq 4, x \leq 5, y \leq 5, x, y \geq 0$
Let us draw the graph for the system of inequalities representing constraints.



The feasible region is ABCDEF shown (shaded) in Fig. 6, which is bounded.
The coordinates of the corner points of the feasible region ABCDEF are A(0, 4), B(0, 5),C (3, 5), D (5, 3), E (5, 0) and F (4, 0)
Let us evaluate the objective function Z =10(x -7y + 190) at the corner points.

Corner point	Corresponding value of Z
A(0, 4)	1620
B (0, 5)	1550 (minimum)
C(3, 5)	1580
D (5, 3)	1740
E (5, 0)	1950
F (4, 0)	1940

Z is minimum at B(0, 5) and minimum cost is ₹ 1500

So, the minimum transportation cost is ₹ 1500 when 0,5 and 3 units are transported from factory at P and 5, 0 and 1 units are transported from factory at Q to the depots A, B and C respectively.

36. (A) If a gives B, a start of 40m, it means that in the same time A runs 1000 metres while B runs (1000 – 40) m = 960 cm
∴B runs 1000 m in y seconds
∵ B runs 960 m in (y1000 × 960) sec
= 2425 sec.

(B) A had given a start of 30 seconds to B, then A had run for (y – 30) seconds.

(C) From Q(A) B wins by 19 sec
∴2425 y – x = 19
25x – 24 y = – 475 .... (i)
In second condition, B wins by 40 m

$\therefore$ 1000 – $\frac{1000}{x}$ (y – 30) = 40
$\Rightarrow$ $\frac{1000}{x}$ (y – 30) = 960
$\Rightarrow$ $\frac{25}{x}$ (y – 30) = 24
24x – 25y = – 750 .... (ii)
On solving (i) & (ii), we get
x = 125 seconds
So, A complete, race in 125 sec.

OR

B beats A by (25 – 22.5) m = 2.5 m in running 25 m Thus, in running 25 m, B beats A by 2.5 m .
$\therefore$ In running y km i.e, 1000 m B beats A by ($\frac{2.5}{25}$ × 1000) m = 100 m.

37. (A) Area of trapezium
= $\frac{1}{2}$ (sum of parallel sides) × distance between parallel sides
= $\frac{1}{2}$ × (16 + 22) × 20 = 380 m^2

(B) PQ^2 = 62 + 202
= 36 + 400
= 436

PQ = $\sqrt{436}$ = 20.8 cm

(C) We have s = RP^2 + RQ^2
Since, RP^2 = 16^2 + x^2
= 256 + x^2
and RQ^2 = 22^2 + $(20 – x)^2$
= 484 + 400 + x^2 – 40x
$\therefore$ s = $2x^2$ – 40x + 1140
$\frac{ds}{dx}$ = 4x – 40
Foul minimum value,
put $\frac{ds}{dx}$ = 0
4x – 40 = 0
$\Rightarrow$ x = 10
$\frac{d^2x}{dx^2}$ = 4> 0
$\therefore$ Minimum value of s
= $2(4)^2$ – 40 (4) + 1140
= 32 – 160 + 1140
= 1172 – 160
= 1012

OR

We have f(x) = log (x + 3)
there, f(x) is defined for x > – 3
on differentiating w.r.t x,
we get f(x) = $\frac{1}{x+3}$
and x > -3
For f(x) to be decreasing function f(x) < 0 and x > – 3.
$\frac{1}{x+3}$ < 0 and x > – 3
⇒ x < – 5 and x > – 3
Hence, for no value of x, f(x) is decreasing.

38. (A) Amount of money is needed to buy the new machine at the end of 15 years
₹ 2,30,000 – ₹ 10, 000 = ₹ 2, 20,000

(B) Amount of money that is contributed to the sinking fund at the end of every year
= ₹ 10,000.

(C) Here, amount (A) accumulated in the sinking fund over 15 years is given by
A = P$\left[\frac{(1+i)^n-1}{i}\right]$
where P = ₹ 10,000; i = 0.06 and n = 15 years
So, A = 10,000 $\left[\frac{(1+0.06)^{15}-1}{0.06}\right]$
= ₹ 2, 32,759
Thus, amount of interest in the sinking fund = ₹ 2, 32, 759 – (₹ 10,000 × 15) = ₹ 82, 759

OR

Cost of new machine after discount
= ₹ 2,30,000 – 5 % of ₹ 2,30,000
= ₹ 2,18,5000
New cost of new machine after adding VAT
= ₹ 2,18,500 + 18% of ₹ 2,18,500
= ₹ 2,57,8300.

On 1st April, 2015, Dreams Ltd. purchased an AC for and incurred towards freight, towards carriage and towards installation charges. It has been estimated that the machinery will have a scrap value of at the end of the useful life which is four years. What will be the annual depreciation and the value of machinery after four years according to linear method?