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CODE NUMBER	241/1/1
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**INDIAN SCHOOL MUSCAT
FIRST PRE-BOARD EXAMINATION 2023
APPLIED MATHEMATICS - 241**



CLASS : XII
DATE: 04-12-2023

TIME ALLOTTED : 3 HRS.
MAXIMUM MARKS: 80

GENERAL INSTRUCTIONS:

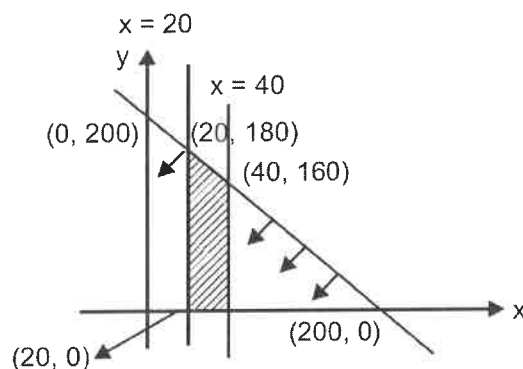
- (a) This question paper consists of 38 questions divided into five sections A, B, C, D and E.
- (b) Section-A has 18 MCQs and 2 Assertion-Reason based questions of **one mark** each (Q1 to Q 20).
- (c) Section-B has 5 questions of **two marks** each (Q 21 to Q 25).
- (d) Section-C has 6 questions of **three marks** each (Q 26 to Q 31).
- (e) Section-D has 4 questions of **five marks** each (Q 32 to Q 35).
- (f) Section- E comprises of 3 Case-study questions of **four marks** each (Q 36 to Q 38).
- (g) There is no overall choice. However, internal choice has been provided in some questions.
 1. A man can row 7.5 km/h in still water. if the stream is flowing at the rate of 1.5 km/hr, it takes him 50mins to row to a place and return. How far is the place?
a. 3km b. 2km c. 4km d. 2.5km
 2. A matrix A of order 3x3 has determinant 5 what is the value of |3A| ?
a. 625 b. 135 c. 45 d. 125
 3. While determining the intervals where a function f(x) having domain (-1, ∞) is increasing, a student found that $f'(x) = \frac{(x)^2}{(2+x)^2(x+1)}$. Which of the following statement is correct?
a. f is always increasing in (-1, ∞) b. f is always decreasing in (-1, ∞)
c. f is always increasing in its domain except at x = 0 d. f is increasing only in (0, ∞)
 4. The expected value of a discrete random variable 'x' is given by _____
a) P(x) b) $\sum P(x)$ c) $\sum x P(x)$ d) 1
 5. The central limit theorem states that if the sample size _____
a) increases sampling distribution must approach normal distribution
b) decreases then the sample distribution must approach normal distribution
c) increases then the sampling distribution much approach an exponential distribution
d) decreases then the sampling distribution much approach an exponential distribution
 6. Increase in number of patients in the hospitals due to heat stroke is a
a. cyclical trend b. secular trend c. irregular trend d. seasonal trend.
 7. If the cash equivalent of a perpetuity of ₹1200 payable at the end of each quarter is ₹ 96,000, then the rate of interest convertible quarterly.
a. 4% b. 5% c. 5.5% d. 6.5%

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8. For an L.P.P. the objective function is $Z = 400x + 300y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.

Find the coordinates at which the objective function is maximum.

- (a) (20, 0) (b) (40, 0)
(c) (40, 160) (d) (20, 180)



9. In a 50m race A can give a start of 5m to B and a start of 14 m to C. In the same race how much start can B give to C?
a. 9 m b. 10 m c. 12 m d. 11 m
10. If $A = \begin{pmatrix} 5 & x \\ y & 0 \end{pmatrix}$ and $A = A'$ then
(a) $x=0, y=5$ (b) $x=y$ (c) $x+y=5$ (d) $x-y=5$
11. It is given that at $x = 1$, the function $f(x) = x^3 - 12x^2 + kx + 7$ attains a maximum value, then the value of k is
a. 10 b. 12 c. 21 d. 13
12. If the probability that a bomb dropped from a plane will strike the target is 60% and if 10 bombs are dropped, then the respective values of mean and variance is
a) 0.6, 0.24 b) 6, 2.4 c) 0.4, 0.16 d) 4, 1.6
13. For a Poisson Distribution, if mean = 1, then $P(X = 1)$ is ?
a) $\frac{1}{e}$ b) e c) $\frac{1}{e}$ d) can be determined
14. The interest rate per year is 6% and the compounding occurs every month then interest rate per compounding period is
a. 0.06 b. 0.005 c. 0.006 d. 0.05
15. Consider the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$
If the corner points of the feasible region are (0, 0), (5, 0), (3, 4) and (0, 5).
Let $Z = px + qy$, where $p, q > 0$. Condition on p and q, so that the maximum value of Z occurs at both (3, 4) and (0, 5) is:
a. $p = q$ b. $p = 2q$ c. $p = 3q$ d. $q = 3p$
16. Tea worth Rs.126 per kg and Rs135perKg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs 153 per Kg, then the price of the third variety in per Kg is
a. Rs. 170 b. Rs. 175.50 c. Rs. 145 d. Rs. 165.50
17. Given that $x = at^2$ and $y = 2at$, then $\frac{d^2y}{dx^2}$ is
a. $2at^3$ b. $-\frac{1}{2at}$ c. $-\frac{1}{2at^3}$ d. $-\frac{1}{t^2}$
18. A machine costing ₹ 50,000 has a useful life of four years the estimated scrap value is ₹ 10,000 then the annual depreciation is
a. ₹ 20000 b. ₹ 12000 c. ₹ 10000 d. ₹ 5000
19. **Assertion A:** Selection of football team for FIFA world cup is an example of Purposive sampling.
Reason R: A purposive sampling is done based on the characteristics of the population and is not random so we can select a better team not by chance but by characteristics.

20. **Assertion A:** If a is any positive real number, then $a + \frac{1}{a} \geq 2$
Reason R: Let a and b be distinct positive real numbers then $\frac{a+b}{2} > \sqrt{ab}$
21. Evaluate: $\int \frac{2}{(x+1)x^2} dx$ **OR** $\int_{-1}^3 (|x-1| + |x-2|) dx$
22. Find the present value of a perpetuity of ₹ 3120 payable at the beginning of each year, if money is worth 6% effective.
23. Find the particular solution of the differential equation,
 $(\log x) \sqrt{4+y^2} dx + \frac{1}{x} dy = 0$, given that, at $x = 1$, $y = 0$.
24. A money lender charges interest at the rate of ₹10 per ₹100 per half year, payable in advance. What effective rate of interest does he charge per annum?

OR

M/s JBC Earthmovers was founded on April 1, 2018 by a RK Sharma. The revenue numbers for firm are shown in the adjacent table. Find the compound annual growth rate of the company. Given: $(1.833)^{\frac{1}{3}} = 1.2239$

2018-2019	2019-2020	2020-2021
₹300000	₹ 250000	₹ 550000

25. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. Formulate the linear programming to maximize the amount of vitamin A in the diet, stating the objective function and the constraints.
26. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively, while a third pipe C can empty the full tank in 20 minutes. All the three pipes are opened in the beginning, but pipe C is closed 6 minutes before the tank is filled in what time will the tank be full.
27. Consider two families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a child and that of protein is 45 grams for a man, 55 grams for a woman and 33 grams for children.

Based on the above information, answer the following questions.

- a) Represent the requirement of calories and proteins for each person in matrix form.
b) Evaluate the requirement of calories of family A using matrix algebra.

OR

Given the following equations for two related markets A and B, find the equilibrium conditions for each market and the price for each by Cramer's rule:

$$x_d(A) = 82 - 3 p_A + p_B$$

$$x_s(A) = -5 + 15 p_A$$

$$x_d(B) = 92 + 2 p_A - 4 p_B$$

$$x_s(A) = -6 + 32 p_B$$

where x_d and x_s denotes the quantity demanded and quantity supplied respectively and p_A and p_B represents the price for each market.

28. The population of a city increases at a rate proportional to the number of inhabitants present at any time t , if the population of city was 200000 in 1990 and 250000 in 2000.
i) Show that the growth constant is $\frac{1}{10} \log \frac{4}{5}$
ii) Find the population in 2010.
29. Assuming that half the population are consumers of chocolate so that the chance of an individual being a consumer is $\frac{1}{2}$ and if 100 investigators each take 10 individuals to see whether they are consumers. How many investigators would you expect to report that 3 people or less were consumers?

OR

The probability that a man aged 50 years will die within a year is 0.01125. What is the probability that out of 12 such men at least eleven will reach their 51st birthday?

Given: $e^{-0.315} = 0.87371$

30. Hole punching machine is set to punch a hole 1.84 cm in diameter in a strip of sheet metal in a manufacturing process. The strip of metal is then creased and sent on to the next phase of production, where a metal rod is slipped through the hole. It's important that the hole be punched to the specified diameter of 1.84 cm. To test punching accuracy, technicians randomly sampled 16 punched holes and measured the diameters. The sample data (in centimeters) has a mean of 1.85 and variance 0.0064. Set up null and alternate hypothesis to test if the machine is working properly (whether the holes are being punched an average of 1.84 centimeters), at an alpha level of 0.05. Assume the punched holes are normally distributed in the population. Given: $t_{0.05,15} = 2.131$
31. A person has set up a sinking fund to have ₹100,000 after 10 years for his children's college education. How much amount should he set aside annually into an account paying 5% per annum compounded half yearly. Given: $1.025^{20} = 1.6386$
32. A company has approximated the marginal cost and marginal revenue functions for one of its products by $MC = 81 - 16x + x^2$ and $MR = 20x - 2x^2$ respectively. Determine the profit maximizing output and the total profit at the optimum output, assuming fixed cost as zero.

OR

The demand and supply functions for commodities are $p = 16 - 6x + x^2$ and $p = 4 + \frac{4}{3}x + \frac{1}{3}x^2$ respectively. Find each of the following assuming $x \leq 5$,

- i) the equilibrium point
ii) the consumer's surplus at the equilibrium point
iii) the producer's surplus at the equilibrium point
33. Shanaia takes a loan of ₹ 5,00,000 from a bank at the interest rate of 6% per annum for 10 years she wants to pay back the loan in equated monthly installments. Find her EMI by using
i) flat rate method
ii) reducing balance method given that $(1.005)^{-120} = 0.5496$

OR

Jayant amortizes a loan of ₹15,00,000 for renovation of his house by 8 years mortgage at the rate of 12% per annum compounded monthly. Find

- i) the equated monthly installment.
ii) the principal outstanding at the beginning of 40th month
iii) the interest paid in the 40th month. [Given $(1.01)^{96} = 2.5933$, $(1.01)^{57} = 1.7633$.]
34. A Company produces three products every day. Their production on a certain day is 45 tons. It is found that production of the third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of the second product. Determine the production level of each product using matrix inversion method.
35. Fit a straight-line trend by the methods of least squares to the following data and estimate the trend value for 2022. Also plot the trend on the graph sheet.

Year	2013	2014	2015	2016	2017	2018	2019
Production in thousands	80	90	92	83	94	99	92

36. A pharmaceutical company manufactures two drugs - drug A and drug B. The process involves two steps - synthesis and testing. Each lot of drug A requires 15-man hours for synthesis and 3-man hours for testing. Each lot of drug B requires 5-man hours for synthesis and 2-man hours for testing. For synthesizing and testing, the maximum man hours available per week are 390 and 24 respectively. The company makes a profit of Rs 3500 on each lot of drug A and Rs 8000 on each lot of drug B.
- Formulate the linear programming problem.
 - Represent the constraints on the graph depicting the feasible region.
 - Using corner point method find out how many lots of drug A and drug B should be manufactured each week to make a maximum profit



OR

Using Iso profit method find out how many lots of drug A and drug B should be manufactured each week to make a maximum profit

37. In a fighter jet training program entrance test the scores can be modelled using a normal distribution with a mean of 80 and standard deviation 4.
- Find the probability of candidates scoring between 72 and 84.
 - Find the minimum score (rounded off to integer) necessary to get on the program if it takes only the top 2.5% of candidates on a test.



Given: $P(z > -1.96) = 0.975$

38. In mathematics modular arithmetic is a system of arithmetic for integers where numbers "wraparound" when reaching a certain value called modulus. A familiar use of modular arithmetic is in the 12-hour clock in which the day is divided into two 12-hour periods. If the time is 7:00 now, then 8 hours later it will be 3:00. Simple addition would result in $7+8=15$, but clocks "wrap around" every 12 hours. Because the hour number starts over after it reaches 12, this is arithmetic modulo 12. In terms of the definition, 15 is congruent to 3 modulo 12. So 15:00 on a 24-hour clock is displayed 3:00 on a 12-hour clock. Further to the above information answer the following questions.
- Evaluate $(123 \times 217 \times 365) \text{ mod } 11$
 - What is the least positive of x for which $100 \equiv x \pmod{7}$
 - Evaluate $(7^{291} \times 6^{500}) \text{ mod } 10$

OR

Find the last digit of $2^{100} + 100!$

******END OF THE QUESTION PAPER******

