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FIRST PREBOARD EXAMINATION 2019-2020
CLASS XII : MATHEMATICS-041

Time: 3 hours Max Marks: 80

General Instructions

- All questions are compulsory
- The question paper consists of **36 questions** divided into 4 sections A, B, C and D
- **Section A** contains **20 questions of 1 mark each**, **Section B** is of **6 questions of 2 marks** each, **Section C** comprises of **6 questions of 4 marks each** and **Section D** comprises of **4 questions of 6 marks each**
- There is no overall choice. However, **internal choice** has been provided in **3 questions of 1 mark each** and **2 questions of 2 marks** each, **2 questions of 4 marks** each and **2 questions of 6 marks** each . You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted

SECTION A

Q1-Q10 are multiple choice questions. Select the correct option

1. Let A be a square matrix of order 3 and $|A|$ = 5, then find $|4A|$.

 (a) 125 (b) 320 (c) 60 (d) 80

2. If A=[a_{ij}]=$\begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and B=[b_{ij}]=$\begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, which among the following is $a_{21}+b_{12}$?

 (a) 0 (b) 4 (c) -2 (d) 2

3. What is the value of (i x j). k +(j x k). i

 (a) 0 (b) -1 (c) 2 (d) -2

4. If the events E and F are independent, find P(F) if P(E)=2/5 and P(EUF)=3/5

(a) 1/3 (b) 2/3 (c) 3/5 (d) cannot find

5. The point which does not lie in the half plane $2x+3y-12<0$ is

(a) (1,2) (b) (2,1) (c) (2,3) (d) (-3,2)

6. . If $Tan^{-1}x= \frac{\pi}{10}$, for some $x\in R$, then the value of $cot^{-1}x$ is ……

(a) $\frac{2\pi}{5}$ (b) $\frac{\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{10}{\pi}$

7. There are 4 rotten eggs in a lot containing 20 eggs. Find the probability of getting a rotten egg if 2 eggs are chosen at random

(a) 1/5 (b) 32/95 (c) 16/95 (d) 1/2

8. $\int\frac{dx}{\sqrt{16-49x^2}} =$

(a) $\sin^{-1}\frac{7x}{4} + c$ (b) $\frac{1}{8}\log\left|\frac{4+7x}{4-7x}\right| + c$

(c) $\frac{1}{8}\log\left|\frac{4-7x}{4+7x}\right| + c$ (d) $\frac{1}{7}\sin^{-1}\frac{7x}{4} + c$

9. The sum of intercepts cut off by the plane $2x+y-z=5$ on the coordinate axes is

(a) 2/5 (b) 5/2 (c) 2 (d) 1/2

10. The direction ratios of the line $x=-3, \frac{y-4}{3} = \frac{2-z}{1}$ is

(a) (-3,4,-2) (b) (3,-4,-2) (c) (0,3,-1) (d) (1,3,-1)

(Q11-Q15) Fill in the blanks

11. If $f(x)=[x]$, greatest integer function of x, and $g(x)=|x|$, modulus function of x, then the value of $fog(\frac{-5}{2})$ is __________________

12. If $x=2at^2$, $y=at^4$, then dy/dx = __________________________

13. A square matrix is such that $(A-I)^3+(A+I)^3-7A=$____________

14. At some particular time the area and perimeter of a variable square are increasing at the rate of $12cm^2$/sec and 4cm/sec respectively. The length of the side of the square at that time is ___________

OR

For the curve $y=5x-2x^3$ if x increases at the rate of 2units/sec then at x=3, the slope of the curve is changing at ___________.

15. Unit vector perpendicular to both $\vec{a} = i - 7j + 7k$ and $\vec{b} = 3i - 2j + 2k$ is _________

OR

If projection of $\vec{a} = \lambda i + j + 4k$ and $\vec{b} = 2i + 6j + 3k$ is 4 units, then λ = _____________

(Q16-Q20) Answer the following questions

16. Form the differential equation of the family of concentric circles centered at origin having different radii

OR

If m denotes the order and n denotes the degree of the differential equation

$\frac{d^3y}{dx^3} - (\frac{dy}{dx})^{\frac{2}{3}}=0$ find m+n?

17. If $\int_0^a \frac{dx}{4+x^2} = \frac{\pi}{8}$, find the value of a?

18. If the matrix A=$\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -a \end{bmatrix}$ is given to be symmetric matrix, find the values of a and b?

19. Evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x dx$ using properties of definite integrals

20. Evaluate $\int (\frac{x-1}{x^2}) e^x dx$

SECTION B

21. A coin is tossed 3 times. Find P(A/B) where A: atmost 2 tails, B: at least one tail

22. Find the acute angle between the lines $\vec{r} = (4i-3j-k) + \gamma(3i+4j+5k)$ and $\vec{r} = (i-j-10k) + \mu(4i - 3j + 5k)$

23. If $\vec{a} = 2i + 2j + 3k$, $\vec{b} = -i + 2j + k$ and $\vec{c} = 3i + j$ are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to $\vec{c}$ then find the value of λ ?

OR

Show that $\left[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

24. Using Rolle's theorem , find the points on the curve $y=x^2$ where $x\in [-2,2]$ and the tangent is parallel to X axis

25. Discuss the continuity of the function f(x) at x = ½ when $f(x)=\begin{cases} \frac{1}{2}+x & \text{if } 0\le x\le\frac{1}{2} \\ 1 & \text{if } x=\frac{1}{2} \\ \frac{3}{2}-x & \text{if } \frac{1}{2}\le x\le 1 \end{cases}$

26. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ then prove that $x+y+z=xyz$

OR

Show that the relation S in the set of real numbers defined as $S=\{(a,b): a,b\in \mathbf{R} \text{ and } a\le b^3\}$ is neither reflexive nor transitive

SECTION C

27. Let the function $f : N \rightarrow R$ be a function defined by $f(x) = 4x^2 + 12x + 15$. Prove that $f : N \rightarrow S$ is invertible, where S is the range of f, and hence find its inverse.

28. . A cooperative society of farmers has 50 hectares of land to grow two crops X and Y. The profits from crops X and Y per hectare are estimated as Rs. 10500 and Rs.9000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare respectively. Further, no more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond, which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society? Formulate the above L.P.P mathematically and then solve it graphically.

29. Find the value of a for which function defined by $f(x) = \begin{cases} \frac{1-\cos ax}{x \sin x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is continuous at x=0.

OR

If $\sqrt{1-x^2} + \sqrt{1-y^2}$ = a (x –y) , show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

30. Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$

31. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

32. Three cards are drawn successively *with replacement* from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Also find the mean of the distribution

OR

In an examination a student either guesses or copies or knows the answer of MCQ with four choices. The probability that he makes a guess is 1/3, and the probability that he copies answer is 1/6. The probability that his answer is correct , given that he copied it is 1/8. Find the probability that he copies the answer to question, given that he correctly answered it.

SECTION D

33. Using the properties of determinants prove that

$$\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

If $P = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find PA and *hence* solve the system of equations

$x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1$

34. Find the area of region bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

35. Show that the altitude of the right circular cone of maximum volume can be inscribed in a sphere of radius R is $\frac{4}{3}R$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

OR

A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of the semicircular ends is $\pi:\pi + 2$

36. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0.Also find the distance of the plane obtained above, from the origin

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