SET 2

MOCK EXAMINATION I 2023 - 2024
APPLIED MATHEMATICS

Name : _____
Std / Sec : XII
Date :

Marks : 80
Time : 3hrs.

General Instructions:

1. This question paper contains five sections A, B, C, D and E and total of **38 questions**. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.

Section – A:

3. It comprises of questions no **1-18 MCQs and 19 & 20 are Assertion-Reason based questions of 1 mark** each.

Section – B:

4. It comprises of questions no **21-25 VSA types questions of 2 marks** each.

Section – C:

5. It comprises of questions no **26-31 SA types of questions of 3 marks** each.

Section – D:

6. It comprises of questions no **32-35 LA types of questions of 5 marks** each.

Section – E:

7. It has **3 case studies** questions no **36-38**. Each case study comprises of 3 case-based questions, where **2 VSA type questions are of 1 mark** each and **1 SA type question is of 2 marks**. Internal choice is provided in **2 marks** question in each case-study.
8. Internal choice is provided in **2 questions in Section - B**, **2 questions in Section – C,2 questions in Section - D**. You have to attempt only one of the alternatives in all such questions.
9. **Use of Calculator not permitted.**

Section –A

(Each question (1-20) carries 1 mark each, all questions are compulsory. No internal choice in this section)

1. $(-6\times 5)(\text{mod}\,7)$ is
   (a) -2 (b) 5 (c) -5 (d) 2
2. If x is a negative number, then the solution set of $-12x > 30$ is
   (a) $\{-2,-1\}$ (c) $\{\ldots,-5,-4,-3,-2\}$
   (b) $\{\ldots,-5,-4,-3\}$ (d) $\{-2,-1,0,1,2,\ldots\}$
3. If α = probability of Type-I error, then $1-\alpha$ is
   (a) Probability of rejecting H_0 when H_0 true.
   (b) Probability of not rejecting H_0 when H_0 is true.
   (c) Probability of not rejecting H_0 when H_a is true.
   (d) Probability of rejecting H_0 when H_a is true.
4. Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300. Find sample mean and margin of error.
   (a) 200,100 (b) 220,100 (c) 200,110 (d) 100,300
5. Speed of river is 6 km/hr. Speed of a motorboat in still water is 30 km/hr. How much distance can it cover downstream in 24 minutes?
   (a) 9.8 km (b) 12 km (c) 12.8 km (d) 14.4 km

6. In a binomial distribution, the probability of getting a success is $\frac{1}{4}$ and the standard deviation is 3, then its mean is

(a) 6 (b) 8 (c) 12 (d) 18

7. What is the least integer (mod 11) to which 283 is congruent?

(a) 2 (b) 4 (c) 8 (d) -8

8. The present value of a perpetuity of ₹R payable at the end of each payment period, when the money is worth *i* per period, is given by

(a) R*i* (b) $R+\frac{R}{i}$ (c) $\frac{R}{i}$ (d) $R-Ri$

9. Evaluate: $\int x(5x^2-7)^6 dx$

(a) $\frac{1}{70}\left(5x^2-7\right)^7+C$ (b) $\frac{1}{60}\left(5x^2-7\right)^7+C$ (c) $\left(5x^2-7\right)^7+C$ (d) $\frac{x^2}{2}+C$

10. The general pattern of increases or decreases in economics or social phenomena is known by

(a) Irregular trend (c) seasonal trend

(b) Secular trend (d) cyclical trend

11. A company buys a microscope at a cost of ₹25000. The company decides on a salvage value of ₹8000 and a useful life of 6 years, then annual depreciation cost is

(a) ₹2383 (b) ₹2523 (c) ₹2783 (d) ₹2833

12. The declared rate of return compounded semi-annually which is equivalent to 10.25% effective rate of return, is

(a) 10.13% (b) 10.05% (c) 10% (d) 9.89%

13. The beginning value of the investment is ₹10000 and the ending value of the investment is ₹14000 over a period of 2 years, then its CAGR is

(a) 1.83% (b) 18.23% (c) 18.32% (d) 1.4%

14. Integrating factor of $x\frac{dy}{dx}-y=x^4-3x$ is

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$

15. The point at which the maximum value of $3x+2y$ subject to the constraints $x+y\le 2, x\ge 0, y\ge 0$ is obtained, is

(a) (0,0) (b) (1.5,1.5) (c) (2,0) (d) (0,2)

16. For a random variable X, E(X)=3 and E(X²)=11. Then, variance of X is

(a) 8 (b) 5 (c) 2 (d) 1

17. Random sampling is useful as it is

(a) reasonably more accurate as compared to other methods

(b) economical in nature

(c) free from personal biases of the investigator

(d) All of the above

18. Given that mean of a normal variable X is 12 and standard deviation is 4, then find the data point if its Z-score is 5.

(a) 5 (b) 4 (c) 16 (d) 32

Directions (19-20) the below given questions are of the type Assertion and Reason. Each question contains Assertion and Reason. Each question has 4 choices (a), (b), (c) and (d)out of which ONLY ONE is correct. So, select the correct choice.

(a) Both the Assertion and Reason is true, and the reason is correct explanation for assertion.

(b) Both the Assertion and Reason is true, and the reason is not correct explanation for assertion.

(c) Assertion is true, reason is false

(d) Assertion is false, reason is true

19. **Assertion:** The maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$ is 113.
Reason: $p'(x) = -72 - 36x \Rightarrow x = -2, p''(x) = -36 < 0, p(-2) = 113$

20. **Assertion:** Poisson distribution is applied for discrete random variable.
Reason: In a Poisson distribution, the mean and variance are equal.

Section – B

(Each question (21-25) carries 2mark each, all questions are compulsory. In case of internal choice attempt only one question)

21. The present value of a perpetual income of ₹x at the end of each six months is ₹40000. Find the value of x if money is worth 6% compounded semi-annually.

22. (a) Find $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, then find k so that $A^2 = kA - 2I$.

(OR)

(b) Find value of $2a + 3b - c$, if $A = \begin{bmatrix} 0 & -1 & 28 \\ a-8 & 0 & 3b \\ -c+2 & 2 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

23. A shopkeeper deals in two items-wall hangings and artificial plants. He has ₹15000 to invest and a space to store atmost 80 pieces. A wall hanging costs him ₹300 and an artificial plant ₹150. He can sell a wall hanging at a profit of ₹50 and an artificial plant at a profit of ₹18. Assuming that he can sell all the items that he buys, formulate a linear programming problem in order to maximize his profit.

24. (a) A boatman takes twice as long as to go upstream to a point as to return downstream to the starting point. If the speed of a boat in still water is 15 km/h, what is the speed of the stream.

(OR)

(b) In a 10 km race, A, B and C each running at uniform speed get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then find by how many metres does A beat C?

25. A machine produces washers of thickness 0.50 mm. To determine whether the machine is in proper working order, a sample of 10 washers is chosen for which the mean thickness is 0.53 mm and the standard deviation is 0.03 mm. Test the hypothesis at 5% level of significance that the machine is working in proper order.[Given $t_{0.025}$=2.262 at v=9]

SECTION – C

(Each question (26-31) carries 3 marks each, all questions are compulsory. In case of internal choice attempt only one question)

26. (a) Evaluate: $\int \frac{x^2}{1-x^6} dx$

(OR)

(b) Evaluate: $\int \left(x^2+1\right) \log x dx$

27. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

28. Find the intervals in which is $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

(i) strictly increasing (ii) strictly decreasing

29. (a) The demand and supply functions for a commodity are $p = x^2 - 6x + 16$ and $p = \frac{x^2}{3} + \frac{4}{3}x + 4$, respectively. Find the consumers surplus at the equilibrium point, when $x < 7$.

(OR)

(b) The marginal cost function of a product is given by $MC = \frac{x}{\sqrt{x^2+400}}$. Find the total cost and the average cost if the fixed cost is ₹1000.

30. Amrita bought a car worth ₹12,50,000 and makes a down payment of ₹3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

[Given $(1.0125)^{-48} = 0.5508565$]

31. A machine costing ₹2,00,000 has effective life of 7 years and its scrap value is 30,000. What amount should the company put into a sinking fund earning 5% p.a., so that it can replace the machine after its usual life? Assume that a new machine will cost ₹3,00,000 after 7 years.

[Given $(1.05)^7 = 1.407$]

SECTION – D

(Each question (32-35) carries 5 marks each, all questions are compulsory. In case of internal choice attempt only one question)

32. (a) It is known that 3% of plastic buckets manufactured in a factory are defective. Using the poisson distribution on a sample of 100 buckets, find the probability of

(i) Zero defective buckets

(ii) Atmost one bucket is defective (Use $e^{-3}=0.049$)

(OR)

(b) An experiment succeeds thrice as often as it fails. Find the probability that in the next 5 trails, there will be atleast 3 successes.

33. There are two types of fertilizers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs atleast 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹10 per kg and 'B' costs ₹8 per kg, then graphically determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost.

34. (a) A firm has the cost function $C=\frac{x^3}{3}-7x^2+111x+50$ and demand function $x=100-p$.

(i) Write the total revenue function in terms of x.

(ii) Formulate the total profit function P in terms of x.

(iii) Find the profit maximizing level of output x.

(iv) What is the maximum profit?

(OR)

(b) A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m^3. If building of tank costs ₹100 per square metre for the base and 50 per square metre for the sides, find the cost of least expensive tank.

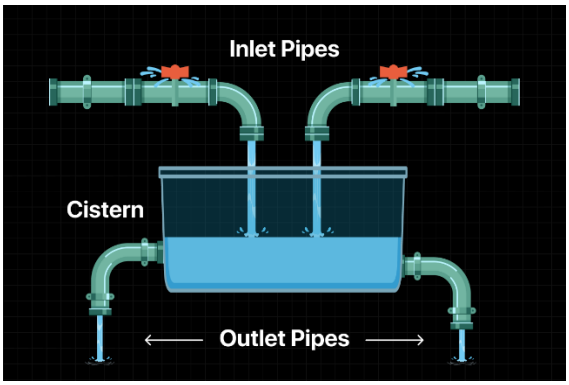
35. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, find AB. Hence, solve the system of equations:

$x-y=3, 2x+3y+4z=17, y+2z=7$

SECTION – E

(All the questions (36-38) are compulsory. In case of internal choice attempt only one question)

36. **Case Study I:** A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hours. B and C together fill the tank in 10 hours. A and C together fill the tank in $7\frac{1}{2}$ hours.



Based on the above information, answer the following questions:

(a) In how much time will A, B and C fill tank? **[1m]**

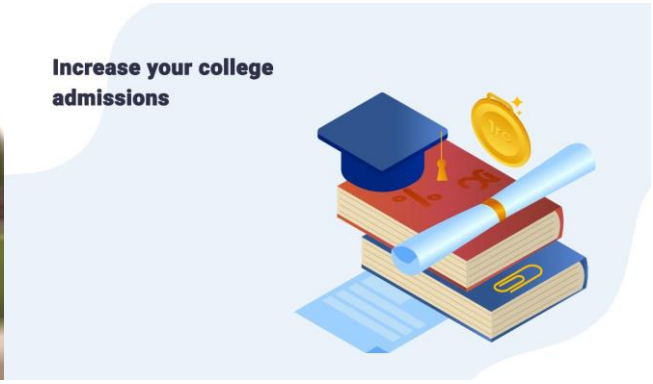
(b) In how much time will A separately fill the tank? **[1m]**

(c) In how much time will B separately fill the tank? **[2m]**

(OR)

In how much time will C separately fill the tank? **[2m]**

37. **Case Study II:** Let X denotes the number of colleges where you will apply after your results and P(X= r) denotes your probability of getting admission in x number of colleges it is given that



$$P(X=r)=\begin{cases} kx, & x=0 \text{ or } 1 \\ 3kx, & x=2 \\ 2k(5-x), & x=3 \text{ or } 4 \\ 0, & x>4 \end{cases}$$ Where k is positive constant.

Based on the above information, answer the following questions. Show steps to support your answers.

(a) Find the value of k. **[1m]**

(b) What is the probability that atleast 2 admissions? **[1m]**

(c) Calculate the mathematical expectation of number of admissions **[2m]**

(OR)

Find the variance of the distribution **[2m]**

38. **Case Study III:** When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sales details (in ₹lakh) of Steel Industry during 2011-2015. Fit a straight-line trend by the method of least squares and tabulate the trend values, estimate the trend for the year 2018. **[4m]**

Years	2011	2012	2013	2014	2015
Sales(₹lakhs)	76	80	130	144	138

(OR)

Strikes is a very powerful weapon used by trade unions and other labour associations to get their demand accepted generally involves quitting of work by a group of workers for the purpose of bringing the pressure on their employer so that their demands get accepted. When workers collectively cease to work in a particular industry, they are said to be on strike.

The table give below shows the average number in lakhs, of working days lost in strikes during each year of period 2009-2018 was

Years	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Sales	1.5	1.8	1.9	2.2	2.6	3.7	6.2	6.4	3.6	5.4

Calculate three yearly moving average. Draw the graphs for moving average **[4m]**

**********All the best**********