MOCK EXAMINATION   I   2022-2023
APPLIED MATHEMATICS (241)

Exam No: _____   **Max. Marks: 80**

Std / Sec: XII-D   **Time: 3 hrs.**

Date   :

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
   Section – A:
3. It comprises of **20 MCQs of 1 mark** each.
   Section – B:
4. It comprises of **5 VSA types questions of 2 marks** each.
   Section – C:
5. It comprises of **6 SA types of questions of 3 marks** each.
   Section – D:
6. It comprises of **4 LA types of questions of 5 marks** each.
   Section – E:
7. It has **3 case studies**. Each case study comprises of 3 case-based questions, where **2 VSA type questions are of 1 mark** each and **1 SA type question is of 2 marks**. Internal choice is provided in **2 marks** question in each case-study.
8. Internal choice is provided in **2 questions in Section - B**, **2 questions in Section – C**, **2 questions in Section - D**. You have to attempt only one of the alternatives in all such questions.
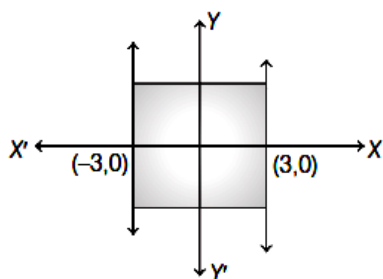
🔸 **Use of Calculator not permitted.**

Section –A

(Each question(1-20) carries 1 mark each, all questions are compulsory. No internal choice in this section)

1. If $7x = 3 \pmod 5$, then the value of x is

   (a) 1   (b) 2   (c) 3   (d) 4

2. The inequality representing the following graph is

   (a) $|x| < 3$ (b) $|x| \le 3$ (c) $|x| > 3$ (d) $|x| \ge 3$

3. The t-distribution depends on a parameter known as

   (a) Critical value   (b) p-value   (c) degrees of freedom   (d) Hypothesis

4. Consider the following hypothesis test:

   $H_o : \mu \leq 12$
   $H_a : \mu > 12$

   A sample of 25 provided a sample mean as 14 and a sample standard deviation as 4.32, and then the value of the test statistic is

   (a) 2.11   (b) 2.21   (c) 2.31   (d) 2.41

5. A person can row a boat upstream at 10 km/h and downstream at 18 km/h, what is the speed of boat in still water?

   (a) 14 km/h   (b) 12 km/h   (c) 4 km/h   (d) 16 km/h

6. _____ states that the sampling distribution of the sample mean approaches a normal distribution as the sample size gets larger, no matter what is the shape of the population distribution.

   (a) Normal Distribution   (c) Population

   (b) Statistical Inference   (d) Central Limit Theorem

7. A can run 22.5 m while B runs 25 m. In a kilometer race B beats A by:

   (a) 100 m   (b) $100\dfrac{1}{9}$ m   (c) 25 m   (d) 50 m

8. What is the face value of sinking fund that yields a dividend of ₹2400 at 5% semi-annually?

   (a) ₹48000   (b) ₹96000   (c) ₹58000   (d) ₹92000

9. Find x_0, if the demand function for a commodity is given by $p = 100 - 8x$ at equilibrium price $p_0 = 4$

   (a) 11   (b) 10   (c) 14   (d) 12

10. The sum of the squares of the deviations of the values of y from their corresponding trend values is

    (a) least   (b) larger   (c) zero   (d) best fit

11. A company buys a machine at a cost of ₹5000. The company decides on a salvage value of ₹1000 and a useful life of 5 years. What is its annual depreciation as per linear method?

    (a) ₹80   (b) ₹800   (c) ₹880   (d) ₹1200

12. The general solution of the differential equation $ydx - xdy = 0$ is

    (a) $xy = C$ (b) $x = Cy^2$ (c) $y = Cx$ (d) $y = Cx^2$

13. An investment of ₹20000 becomes ₹32000 in 5years, then the CAGR is given by

    (a) $\dfrac{\sqrt[5]{1.6} - 1}{100}$ (b) $\dfrac{\sqrt[5]{1.6} + 1}{100}$ (c) $\sqrt[5]{1.6} - 1 \times 100$ (d) $\sqrt[5]{1.6} + 1 \times 100$

14. The corner points of the shaded unbounded feasible region of an LPP are (0,4), (0.6,1.6) and (3,0) as shown in figure. The minimum value of the objective function occurs $Z = 4x + 6y$ at
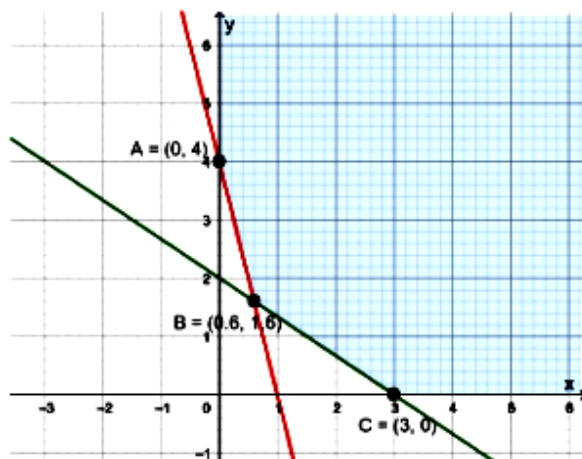
(a) (0.6, 1.6) only

(b) (3,0) only

(c) (0.6, 1.6) only and (3,0)only

(d) at every point of the line segment joining points (0.6, 1.6) and (3,0)

15. Zinc and Copper are in the ratio of 5:3 in 400g of an alloy. How much of copper (in gm) should be added to make the ratio 5:4?

(a) 72g   (b) 200g   (c) 50g   (d) 76g

16. A measurable characteristic of a sample is called

(a) Population   (b) Statistic   (c) Sampling   (d) Estimation

17. Irregular variations in a time series are caused by

(a) Lockouts and strikes   (c) Epidemics

(b) Floods   (d) All of these

18. In a government school, a random sample of 150 students is taken to check whether a student's average calorie intake is 1600 or not. The collected data of average calories intake of sample students is presented in a frequency distribution, which is called a

(a) Statistic   (c) Parameter

(b) Sampling distribution   (d) Population Sampling

Directions (19-20)the below given questions are of the type Assertion and Reason. Each question contains Assertion and Reason. Each question has 4 choices (a), (b), (c) and (d)out of which ONLY ONE is correct. So select the correct choice.

(a) Both the Assertion and Reason is true, and the reason is correct explanation for assertion.

(b) Both the Assertion and Reason is true, and the reason is not correct explanation for assertion.

(c) Assertion is true, reason is false

(d) Assertion is false, reason is true

3

19. **Assertion:** India and Pakistan are two equally capable Cricket teams with star performers. Probability that India beat Pakistan in world cup is 4 matches out of 5 is 31.25%

    Reason: The probability of r success in n trails, denoted by $P(X = r)$ is given by $P(X = r) = {}^nC_r p^r q^{n-r}, r = 0,1,2,.....n$ where p denotes success and q denotes failure in each trail.

20. **Assertion:** The Nominal rate of return is the total amount of money earned by an investment without considering various expenses.

    Reason: The various expenses before factoring nominal rates are such as taxes, legal/investment fees, and inflation.

Section – B

(Each question (21-25) carries 2mark each, all questions are compulsory. In case of internal choice attempt only one question)

21. What is the sum of money needed now, so as to get ₹6000 at the beginning of every month forever, when the money is worth 6% per annum compounded monthly?

22. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the value of $2a - b$.

    (OR)

    Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. Find the possible values of x and y if x, y are natural numbers.

23. A person can row a boat 5 km an hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

    (OR)

    Two runners A and B complete a 100 metres race in 36seconds and 48 seconds respectively, by how many metres will A defeat B?

24. A diet is to contain at least 80 unit of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available costing ₹5 per unit and ₹6 per unit respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problem to minimize the cost of diet that consists of mixture of these two foods.

25. Find the nominal rate (compounded quarterly) which is equivalent to 5% effective rate of interest. [Given $(1.05)^{1/4} = 1.01227$]

(Each question (26-31) carries 3 marks each, all questions are compulsory. In case of internal choice attempt only one question)

26. Find the intervals in which the function given by $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is

    (i) strictly increasing   (ii) strictly decreasing

27. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap books and pastel sheets made by them using recycled paper at the rate of ₹20, ₹15, and ₹10 per unit respectively. School A sold 25 paper bags, 10 scrap books and 30 pastel-sheets. School B sold 20 paper bags, 15 scrap books and 30 pastel-sheets while School C sold 25 paper bags, 18 scrap books and 35 pastel-sheets. Using matrices, find the total amount raised by each school.

28. Evaluate: $\int \frac{e^{2x}}{2 + e^x} dx$

    (OR)

    Evaluate: $\int (1 + x) \log x \, dx$

29. The marginal cost function of a product is given by $MC = \frac{x}{\sqrt{x^2 + 400}}$. Find the total cost and average cost if the fixed cost is ₹1200.

    (OR)

    For a particular commodity the demand function $D(x) = 26 - \frac{x^2}{100}$ and the supply is $S(x) = \frac{x^2}{400} + 6$. Determine to the nearest rupee the consumer's surplus if market attains equilibrium.

30. A couple wishes to purchase a house for ₹1500000 with a down payment of ₹400000. If they can amortize the balance at an interest rate of 9% per annum compounded monthly for 10 years, then find the monthly installment (EMI). [Given $(1.0075)^{-120} = 0.4079$]

31. A machine costing ₹50000 is to be replaced at the end of 10 years, when it will have a salvage value of ₹5000. In order to provide money at the time for a machine costing the same amount, a sinking fund is set up. If equal payments are placed in the fund at the end quarter and the fun earns at the rate of 8% compounded quarterly, then what should each payment be?

    [Given $(1.02)^{40} = 2.208$]

(Each question (32-35) carries 5 marks each, all questions are compulsory. In case of internal choice attempt only one question)

32. For a Poisson distribution model, if arrival rate of passengers at an airport is recorded 30 per hour on a given day. Find

    (i) The probability of 4 or fewer arrivals in the first 10 minutes of an hour.

    (ii) The probability of 10 or more arrivals in an hour given that there are 8 arrivals in the first 10 minutes of that hour. [Given: $e^{-5} = 0.0067$]

    (OR)

    In a district exam scores of 300 students of class XII are recorded at the end of the session.

    (i) Sudha scored 420 marks in the same batch. What can you say about her performance as compared to the batch of 300 students?

    (ii) How much has Abhay scored if he has done better than 44.83% of his batchmates?

    [Given: F(0.56)=0.7123, F(-0.56)=0.0594, Z score for 44.83%=-13]

33. The cost of manufacturing of certain items consists of ₹1600 as overheads, ₹30 per item as the cost of material and the labour cost $\dfrac{x^2}{100}$ for x items produced. How many items must be produced to have minimum average cost?

    (OR)

    If the demand function is given by $x = \dfrac{600 - p}{8}$, where the price is ₹p per unit and the manufacturer produces x units per week at the cost of ₹$(x^2 + 78x + 2500)$, find the value of x for which the profit is maximum.

34. A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹48 per unit and product B is sold for ₹40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum revenue. Formulate the above as a linear programming problem and solve it, indicating clearly the feasible region in the graph.

35. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations: $3x + 4y + 7z = 14, 2x - y + 3z = 4, x + 2y - 3z = 0$

SECTION – E

(All the questions(36-38) are compulsory. In case of internal choice attempt only one question)

36. **CASE I:**

An overhead water tank has three pipes A, B and C attached to it (as shown in figure). The inlet pipes A and B can fill the empty tank independently in 12 hours and 8 hours respectively. The outlet pipe C alone can empty a full tank in 15 hours.
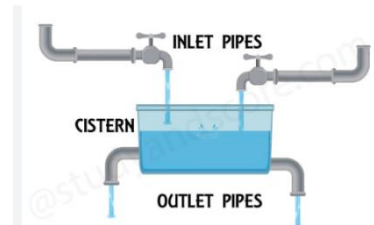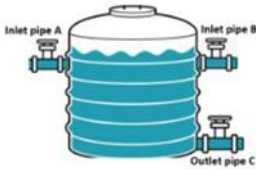
Based on the above information, answer the following questions. Showsteps to support your answers.

(a) For a routine cleaning of the tank, the tank needs to be emptied. If pipes A and B are closed at the time when the tank is filled to three-fifth of its total capacity, how long will pipe C take to empty thetank completely? **[1m]**

(b) How long will it take for the empty tank to fill completely, if all the three pipes are opened simultaneously? **[1m]**

(c) On a given day, pipes A, B and C are opened (in order) at 4 am, 7 am and 8 am respectively, to fill the empty tank. In how many hours will the tank be filled completely? **[2m]**

(OR)

Given that the tank is half-full, only pipe C is opened at 5 AM, to empty the tank. After closing thepipe C and an hour's cleaning time, tank is filled completely by pipe A and B together. What is thetotal time taken in the whole process? **[2m]**

37. **CASE II:**

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production. Mathematically, for finding a line of best-fit to represent a trend, many methods are available. Methods like moving-averages and least-squares squares are someof the techniques to predict such trends.

Mr. Anil kumar runs a rice mill factory and the record of his productions (in metric tonnes) for the period of 2012- 2016 is as follows:

Year	2012	2013	2014	2015	2016
productions (m tonnes)	80	90	92	83	94

Based on the above information, answer the following questions. Show steps to support your answers.

(a) By taking year 2014 as origin, use method of least-squares to find the best-fit trend line equation for Mr. Anil kumar's rice mill factory. Show the steps of your working. **[2m]**

(OR)

Demonstrate the technique to fit the best-suited straight-line trend by the method of 3-years moving averages. Also draw the trend line. **[2m]**

(b) Estimate the likely production of the rice mill factory during 2018? **[1m]**

(c) Mr. Anil kumar's wishes to grow his business to yearly production of 103tonnes. In which year will he be able to reach her target? **[1m]**

38. **CASE III:**

A factory manufactures Bulbs for household and industry purpose. A team wanted to check the quality of the bulbs manufactured by the factory. So that they planned to take sample of bulbs to check. From a lot of 15 bulbs which include 5 defective bulbs, a sample of 4 bulbs is drawn one by one with replacement.

Based on the above information, answer the following questions. Show steps to support your answers. Find,

(a) The probability of success and failure, p and q. **[1m]**

(b) find the mean of the distribution **[1m]**

(c) The probability distribution of defective bulbs **[2m]**

(OR)

The probability of atleast 2 defective bulbs **[2m]**

All the best