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PREBOARD EXAM 2023-24

CLASS – XII

MATHEMATICS (041)

Time Allowed : 3 Hours Maximum Marks : 80

General Instructions: This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

1. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
2. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
3. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
4. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
5. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark

1. If A is any square matrix of order 3 × 3 such that $|A| = 3$, Then the value of $|adjA|$ is

(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) 27

2. If A = [a_{ij}] is a symmetric matrix of order n, then

(a) $a_{ij} = 1/a_{ij}$ for all i, j (b) $a_{ij} \neq 0$ for all i,j

(c) $a_{ij} = a_{ji}$ for all i,j (d) $a_{ij} = 0$ for all i, j

3. Write the element a_{23} of a 3 x 3 matrix A = (a_{ij}) whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.

(a) 2 × 3 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) None of these

4. If for any square matrix A, A(adjA) = [6 0 0 6] then value of $|A|$

(a) 3 (b) 6 (c) 8 (d) 1

5. If a matrix A is both symmetric and skew symmetric then matrix A is

(a) scaler matrix (b) a diagonal matrix (c) a zero matrix (d) rectangular matrix

6. If $x = t^2$ and $y = t^3$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2t}{3}$ (b) $2t$ (c)$3t$ (d) $\frac{3t}{2}$

7. If the function f(x) is continuous at x= 0, then the value of k is

$$f(x) = \begin{cases} \frac{\sin(4x)}{9x}, & x \neq 0 \\ k^2 & x = 0 \end{cases}$$

(a) $\frac{-3}{2}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\frac{4}{9}$

8. The value of $\int_{-a}^{a} sin^3 x\, dx$ is

(a) a (b) a/3 (c) 1 (d) 0

9. Evaluate $\int \frac{2}{1+\cos cos\, 2x} dx$

(a) tan $tan\, x$ +c (b) log tan $tan\, x$ +c (c) tan(x/2) +c (d) log(1+cos2x)+c

10. The integrating factor of differential equation cos $cos\, x \frac{dy}{dx}$ + y sin x = 1 is

(a) cos x (b) tan x (c) sec x (d) sin x

11. If m is the order and n is the degree of given differential equation,

$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^5 + x^4 = 0$ then what is the value of m + n

(a) 5 (b) 9 (c) 4 (d) 7

12. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

(a) 5 (b) 10 (c) 14 (d) 16

13. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(a) 2/3 (b). 3/2 (c) 5/2 (d) 2/5

14. The scalar projection of the vector $2\hat{i} + 3\hat{j} - 5\hat{k}$ on the vector $5\hat{i} + 5\hat{j} + 5\hat{k}$ is

(a) $\frac{2}{5\sqrt{3}}$ (b) 0 (c) 25 (d) None of these

15. The value of i. $(j \times k) + j.(k \times i) + k.(j \times i)$ is

(a) 1 (b) 3 (c) 0 (d) -1

16. The corner points of the feasible region determined by the following system of linear inequalities: 2x + y ≤ 10, x + 3y ≤ 15, x, y ≥ 0 are (0,0), (5,0), (3,4), (0,5). Let Z= px + qy, where p,q > 0. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

(a) p = q (b) p = 2q (c) p = 3q (d) q = 3p

17. The Solution set of system $3x + 6y \geq 80$, $4x + 3y \geq 100$, $x, y \geq 0$ is

(a) Lies in I quadrant (b) Lies in II quadrant

(c) Lies in III quadrant (d) Lies in IV quadrant

18. Three balls are drawn from a bag containing 2 red and 5 black balls, if the random variable X represents the number of red balls drawn, then X can take values

(a) 0, 1, 2 (b) 0, 1, 2, 3 (c) 0 (d) 1, 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both *A* and *R* are true and *R* is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) *A* is true but *R* is false.

(d) A is false but *R* is true.

19. Assertion (A) : f: $N \to N$ given by $f(x) = 5x$ is injective but not surjective

Reason (R) : If co-domain ≠ range , then the function is not surjective.

20. Assertion (A) : The direction-cosines of the line joining the points $(1, 0, 0)$ and $(0, 1, 1)$ is $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Reason (R) : direction ratios = direction cosines.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Find the value of $cos^{-1}\left(cos\left(\frac{7\pi}{6}\right)\right)$.

22. Find the rate of change of the area of a circle with respect to its radius 'r' when r = 6 cm

OR

Show that the function f given by $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is increasing on R

23. Find dy/dx if $2x + 3y = \sin y$.

24. For what value of 'a' the vectors: $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

OR

Find the value of λ if $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal given that $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

25. Find the Vector equation of the line which passes through the point (-2,4,-5) and is parallel to the line $\frac{x+3}{2} = \frac{2-y}{5} = \frac{2z+3}{6}$

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find : $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$.

27. If $y = x^{\cos x} + (\cos x)^x$, find $\frac{dy}{dx}$.

OR

If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x(x^2+1)y_1 = 2$.

28. Solve the differential equation: $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$.

OR

Solve differential equation $cos^2 x\frac{dy}{dx} + y = tanx$ $(0 \leq x < \frac{\pi}{2})$

29. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\sin x}}{\sqrt{\sin\sin x} + \sqrt{\cos\cos x}} dx$

OR

Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

30. Solve the following Linear Programming Problem graphically:

Maximize $Z = 17.5x + 7y$

subject to the constraints,

$x + 3y \leq 12$

$3x + y \leq 12$

$x, y \geq 0$

31. A die is thrown twice and the sum of numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared at least once.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Let $A = \{x \in Z; 0 \leq x \leq 12\}$. Check whether the relation $R = \{(a, b); a, b \in A\ |a - b|$ is divisible by 4$\}$ in the set A is reflexive , symmetric or transitive.

OR

Let L be the set of lines in XY plane and R be the relation in L defined as R={ (l_1, l_2): l_1 *is parallel to* l_2}. Prove that the relation R is an equivalence relation. Find the set of all the lines which are related to the line $y = 2x + 4$.

33. Find the product of A=$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ *and* **B**= $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence solve the system of linear equations :

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

34. Using integration, find the area bounded by the curves $y = x^2, and\ y = |x|$.

OR

Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1.

35. Find the coordinates of the image of the point **(1, 6, 3)** with respect to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$; where 'λ' is a scalar. Also, find the distance of the image from the y axis.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each)

36.**Case study 1:** Read the following passage and answer the questions given below:

You want to make two gardens in the shape of square and circle in front of your house. If you purchase a wire of length 28m to fence these gardens and you have used *x* meters of wire to fence circular garden.

(i) What is the Radius of the circular garden and side of square garden?

(ii) If you want to minimize the combined area of both gardens without wasting the wire of length 28m. Then How much length of the wire will be needed to fence the circular garden. And how much length of the wire will be needed to fence the squared garden

37. **Case study 2:** Read the following passage and answer the questions given below.

The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(i) What is the number of days it will take for the plant to grow to the maximum height?
(ii) What is the maximum height of the plant?
(iii) What will be the height of the plant after 2 days?

OR

If the height of the plant is $\frac{7}{2}$ cm, then what is the number of days it has been exposed to the sunlight ?

38. Case study 3: Read the text carefully and answer the questions:

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1, and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35, and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

i. When the doctor arrives late, what is the probability that he comes by metro?

ii. When the doctor arrives late, what is the probability that he comes by other means of transport?