Solution

ALL KERALA COMMON MODEL EXAMINATION 2023-24 (DUPLICATE)

Class 12 - Mathematics

Section A

1\.

(b) x = 2, y = 3

Explanation: We have, $\begin{bmatrix}2x+y & 4x\\5x-7 & 4x\end{bmatrix}=\begin{bmatrix}7 & 7y-13\\y & x+6\end{bmatrix}$

$\Rightarrow$ 4x = x + 6 $\Rightarrow$ x = 2

and 4x = 7y - 13

$\Rightarrow$ 8 = 7y - 13

$\Rightarrow$ y = 3

2\. **(a)** 0

Explanation: Δ = $\begin{vmatrix}0 & a & -b\\-a & 0 & -c\\b & c & 0\end{vmatrix}$ = 0 + a(0 + bc) + b(-ac - 0)

= abc - abc = 0

3\.

(b) A null matrix

Explanation: Given $|A|=0$

$\therefore$ We know that adj (A) = $|A|I$

$\therefore$ adj A = 0.I = 0

$\Rightarrow$ adj A = 0

$\Rightarrow$ A(adj A) = A.0

$\Rightarrow$ A(adj A) = 0

4\.

(c) $\frac{y}{x}\cdot\left(\frac{x\log y-y}{y\log x-x}\right)$

Explanation: $x^y = y^x \Rightarrow$ y log x = x log y

$\frac{y}{x}$ + log x $\frac{dy}{dx}=\frac{x}{y}\frac{dy}{dx}$ + log y

$\Rightarrow\left(\log x-\frac{x}{y}\right)\frac{dy}{dx}$ = $\left(\log y-\frac{y}{x}\right)=\frac{x\log y-y}{x}$

$\Rightarrow\frac{dy}{dx}=\frac{x\log y-y}{y\log x-x}\times\frac{y}{x}$ = $\frac{y}{x}\times\frac{x\log y-y}{y\log x-x}$

5\. **(a)** perpendicular to z-axis

Explanation: We have,

$\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b}=3\hat{i}+\hat{j}+0\hat{k}$

Let $x\hat{i}+y\hat{j}+z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

6\.

(b) $2y-1=(\sin x-\cos x)e^x$

Explanation: $\frac{dy}{dx}=e^x\sin x$

$\int dy=\int e^x\sin x dx$

$y=\frac{1}{2}(\sin x-\cos x)e^x+C$

When x = y = 0 , we get

$0=\frac{1}{2}(\sin 0-\cos 0)e^0+C$

$C=\frac{1}{2}$

Hence, $y=\frac{1}{2}(\sin x-\cos x)e^{x}+\frac{1}{2}$
$2y-1=(\sin x-\cos x)e^{x}$

7.

(c) 41

Explanation:

Corner Point	Z = 0.7x + y
(0, 0)	0.7 $\times$ 0 + 0 = 0
(40, 0)	0.7 $\times$ 40 + 0 = 28
(30, 20)	0.7 $\times$ 30 + 20 = 41 $\longleftarrow$ Maximum
(0, 40)	0.7 $\times$ 0 + 40 = 40

8. **(a)** $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

Explanation: $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

Let:
$\vec{a}=\hat{i}+\hat{j}+0\hat{k}$
$\vec{b}=0\hat{i}+\hat{j}+\hat{k}$

$$\therefore \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 0 & 1 & 1\end{vmatrix}$$

$=\hat{i}-\hat{j}+\hat{k}$
$\Rightarrow|\vec{a}\times\vec{b}|=\sqrt{1+1+1}$
$=\sqrt{3}$

Unit vector perpendicular to $\vec{a}$ and $\vec{b}=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

9. **(a)** 0

Explanation: $\int_{-1}^{1}\ln(x+\sqrt{x^{2}+1})dx$

$$=\left[x\ln(x+\sqrt{x^{2}+1})-\int\frac{x}{\sqrt{x^{2}+1}}dx\right]_{-1}^{1}$$

$$=\left[x\ln(x+\sqrt{x^{2}+1})-\sqrt{x^{2}+1}\right]_{-1}^{1}$$

= 0

10.

(d) AB and BA both are defined

Explanation: In given matrix

order of A = 2×3

order of B $=3\times2$

AB will be defined if the number of column in A is equal to the number of rows in B

so, $(A_{2\times3})(B_{3\times2})=AB_{2\times2}$

Similarly $(B_{3\times2})(A_{2\times3})=BA_{3\times3}$

Thus, Both AB and BA are defined.

11.

(d) (40,15)

Explanation: We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by x + 2y $\leq$ 70 :

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

Region represented by 2x + y $\leq$ 95 :

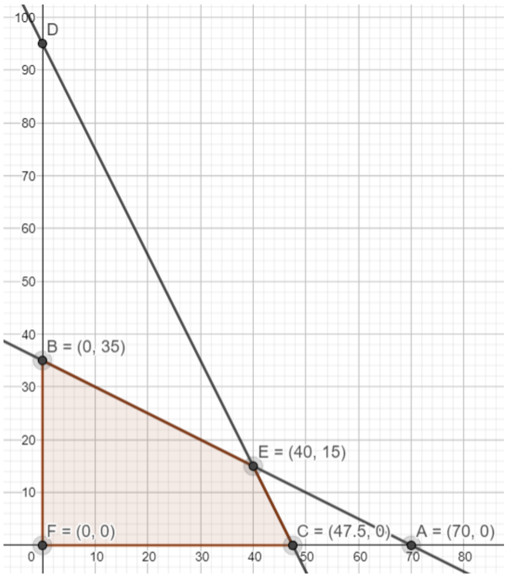
The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right) : \frac{95}{2} + 0 = \frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

12.

(c) $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

Explanation: Let

$\overrightarrow{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

then,

$8\hat{a} = 8\frac{\vec{a}}{|\vec{a}|}$

$=8.\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{5^2+(-1)^2+2^2}}$

$= \frac{8(5\hat{i}-\hat{j}+2\hat{k})}{\sqrt{30}}$

$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

13.

(d) q = 0, s = - 4

Explanation: We have

$\Delta = \begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix} = px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$

$\Rightarrow (x^2 + x) \{(4x + 2x^3) - (x^5 + 4x^3 - 3x^2 -12)\} - (3x + 1) \{(4x^2 - 2x) - (x^3 + 3x^2 + 4x + 12)) + (x - 3) \{(2x^4 - x^3 - 6x + 3) - (x^3 + 3x^2 + 2x + 6)\}$

$= px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$

$\Rightarrow -x^7 - x^6 + 0x^5 - 4x^4 + 8x^3 + 34x^2 + 75x + 21 = px^7 + qx^6 + rx^5 + sx^4 + fx^3 + ux^2 + vx + w$

p = - 1, q = - 1, r = 0, s = - 4, t - 8, u - 34, v = 75, w = 21

14.

(d) 0

Explanation: Given that, X = Set of odd numbers from the set A.

Y = Set of even numbers from the set A.

Let set A = {1, 2, 3, 4, 5, 6, 7} and Z = X + Y

We know that, sum of even and odd numbers can never be an even number.

$\therefore$ P(Z = 10) = 0

15. **(a)** $\frac{dy}{dx} + Py = Q$

Explanation: Here the degree and order of the equation is 1 and also is of the form $\frac{dy}{dx} + Py = Q$ hence it is linear differential equation in first order

16. **(a)** -1

Explanation: We know that,

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\overrightarrow{a}^2| + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a}$ ... (i)

Since,

$\vec{a}$ is perpendicular to $\vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

And according to question

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

We can rewrite equation (i) as

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 2\cos\beta + 2\cos\alpha$

1 = 1 + 1 + 1 + 0 + 2 (cos α + cos β)

1=3+2$(cos\alpha + cos\beta)$

-2=2$(cos\alpha + cos\beta)$

$\Rightarrow$cos α + cosβ = -1

17. **(a)** e^2

Explanation: $\lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} \left|\tan\left(\frac{\pi}{4} + x\right)\right|^{\frac{1}{x}}$ = $\lim\limits_{x\to0} \left|\frac{1+\tan x}{1-\tan x}\right|^{\frac{1}{x}}$

= $\lim\limits_{x\to0} \left[(1 + \tan x)^{\frac{1}{\tan x}} {}^{\tan x}\sqrt{x} \times \lim\limits_{x\to0} \left[(1 - \tan x)^{-\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}\right.$

= e $\times$ e = e^2 $\left|\because \lim\limits_{x\to0}[1 + x]^{\frac{1}{x}} = e\right|$

$\because$ f(x) is continuous at x = 0

$\therefore \lim\limits_{x\to0}$ f(x) = f(0)

$\Rightarrow e^2 = k \Rightarrow k = e^2$

18.

(c) $\frac{8}{\sqrt{29}}$

Explanation: The given equations can be reduced as: $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t\left(-\hat{i} + \hat{j} - 2\hat{k}.\right)$ and

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s\left(\hat{i} + 2\hat{j} - 2\hat{k}.\right)$

On comparing them with:

$\overrightarrow{r} = \overrightarrow{a_1} + t\overrightarrow{b_1}$, and $\overrightarrow{r} = \overrightarrow{a_2} + t\overrightarrow{b_2}$,

We get:

$\overrightarrow{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

and $\overrightarrow{a_2} = \hat{i} - \hat{j} - \hat{k}, \overrightarrow{b_2} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$\therefore S.D = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_2})}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right|$$

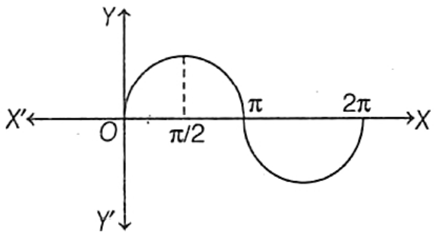
$$= \left|\frac{(2\hat{i} - 4\hat{j} - 3\hat{k}).(\hat{j} - 4\hat{k})}{\sqrt{29}}\right|$$

$$= \left|\frac{-4+12}{\sqrt{29}}\right| = \left|\frac{8}{\sqrt{29}}\right|$$

19.

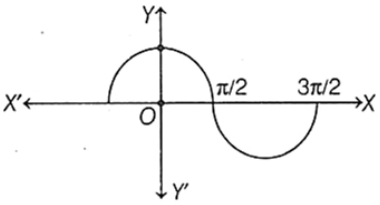
(d) A is false but R is true.

Explanation: Assertion: Given, function f(x) = sin x



From the graph of sin x, we observe that f(x) increases on the interval (0, $\frac{\pi}{2}$).

Reason: Given function is f(x) = cos x.



From the graph of cos x, we observe that, f(x) decreases on the interval (0, $\frac{\pi}{2}$).

Hence, Assertion is false and Reason is true.

20.

(c) A is true but R is false.

Explanation: Assertion is true because distinct elements in Z (domain) has distinct images in Z (codomain).

Reason is false because of: A $\rightarrow$ B is said to be surjective if every element of B has at least one pre-Image in A.

Section B

21. $\sin^{-1}(\sin(-600^\circ)) = \sin^{-1}\left\{\sin\left(-600 \times \frac{\pi}{180}\right)\right\}$

$= \sin^{-1}\left\{\sin\left(-\frac{10\pi}{3}\right)\right\} = \sin^{-1}\left(-\sin\frac{10\pi}{3}\right)$

$= \sin^{-1}\left\{-\sin\left(3\pi + \frac{\pi}{3}\right)\right\} = \sin^{-1}\left\{-\left(-\sin\frac{\pi}{3}\right)\right\}$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

OR

We know that $\tan^{-1} 1 = \frac{\pi}{4}$.

$\therefore \quad \cot\left[\sin^{-1}\left\{\cos\left(\tan^{-1} 1\right)\right\}\right]$

$= \cot\left\{\sin^{-1}\left(\cos\frac{\pi}{4}\right)\right\} = \cot\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cot\frac{\pi}{4} = 1$

22. Given:- f(x) = $x^3 - 6x^2 + 12x - 18$

$\Rightarrow f'(x) = \frac{d}{dx}\left(x^3 - 6x^2 + 12x - 18\right)$

$\Rightarrow$ f'(x) = $3x^2 - 12x + 12$

$\Rightarrow$ f'(x) = $3(x^2 - 4x + 4)$

$\Rightarrow$ f'(x) = $3(x - 2)^2$

as given $x \in R$

$\Rightarrow (x - 2)^2 > 0$

$\Rightarrow 3(x - 2)^2 > 0$

$\Rightarrow$ f'(x) > 0

Hence, condition for f(x) to be increasing

Thus the given function f(x) is increasing on interval $x \in R$.

23. Given: $f(x) = x + \frac{a^2}{x}$

$\because f'(x) = 1 - \frac{a^2}{x^2}$

$f''(x) = \frac{2a^2}{x^3}$

For maxima and minima, we must have

$f'(x) = 0$

$\Rightarrow 1 - \frac{a^2}{x^2} = 0$

$\Rightarrow x^2 - a^2 = 0$

$\Rightarrow x = \pm a$

Now,

$f''(a) = \frac{2}{a} > 0$ as $a > 0$

$\because x = a$ is point of minima

$f''(-a) = \frac{-2}{a} < 0$ as $a > 0$

$\because x = -a$ is point of maxima

Hence,

Local max value = f(-a) = -2a

Local min value = f(a) = 2a.

OR

Here, curve is

$y = x^2 + 2x$

And $\frac{dy}{dt} = \frac{dx}{dt}$ ....(i)

$y = x^2 + 2x$

$\Rightarrow \frac{dy}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{dx}{dt}(2x + 2)$

using equation(i)

2x + 2 = 1

2x = -1

$x = -\frac{1}{2}$

so,

$y = x^2 + 2x$

$= \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$

$= \frac{1}{4} - 1$

$y = -\frac{3}{4}$

So, required point is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

24. $\int \frac{e^{\log \sqrt{x}}}{x} dx = \int \frac{\sqrt{x}}{x} dx$

$= \int x^{\frac{1}{2}} \times x^{-1} dx$

$= \int x^{\frac{1}{2} - 1} dx$

$= \int x^{\frac{-1}{2}} dx$

$= \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$

$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$

$= 2\sqrt{x} + c$

25. We have Maximum value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and minimum. value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$

$f'(x) = -1 + 2\cos x = 0$

$\Rightarrow \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}$

By finding the general solution, we get $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

Now, by finding the second derivative, we get that $f''\left(\frac{\pi}{3}\right) < 0$ and $f''\left(\frac{5\pi}{3}\right) > 0$

Therefore, max. value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and min. value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$

Section C

26. Let the given integral be, I= $\int \frac{1}{\sqrt{(x-1)(2-x)}} dx$

Put x = $\cos^2\theta + 2\sin^2\theta$

$\therefore$ dx = $2\cos\theta(-\sin\theta)d\theta + 4\sin\theta\cos\theta d\theta = 2\sin\theta\cos\theta d\theta$

x = $\cos^2\theta + 2\sin^2\theta$

$\Rightarrow \sin\theta = \sqrt{x-1}$

when x $\rightarrow$ 1, $\sin\theta \rightarrow 0$ or $\theta \rightarrow 0$

when x $\rightarrow$ 2, $\sin\theta \rightarrow 1$ or $\theta \rightarrow \frac{\pi}{2}$

$\Rightarrow I = \int_0^{\frac{1}{3}} \frac{2\sin\theta\cos\theta d\theta}{\sqrt{(\cos^2\theta + 2\sin^2\theta - 1)(2 - \cos^2\theta - 2\sin^2\theta)}}$

$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{2\sin\theta\cos\theta d\theta}{\sqrt{\sin^2\theta\cos^2\theta}}$ ($\sin^2\theta + \cos^2\theta = 1$)

$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{2\sin\theta\cos\theta d\theta}{\sin\theta\cos\theta}$

$\Rightarrow I = 2\int_0^{\frac{\pi}{3}} d\theta$

$\Rightarrow I = 2\theta|_0^{\frac{\pi}{2}}$

$\Rightarrow I = 2\left(\frac{\pi}{2} - 0\right) = \pi$

27. E_1 : lost card is diamond

E_2 : lost card is not diamond

let A: two cards drawn from the remaining pack are diamonds.

$P(E_1) = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{39}{52} = \frac{3}{4}$

$P\left(\frac{A}{E_1}\right) = \frac{12C_2}{51C_2} = \frac{12\times 11}{51\times 50}$

$P\left(\frac{A}{E_2}\right) = \frac{13C_2}{51C_2} = \frac{13\times 12}{51\times 50}$

$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$

$= \frac{\frac{13}{52}\times\frac{12\times 11}{51\times 50}}{\frac{13}{52}\times\frac{12\times 11}{51\times 50} + \frac{3}{4}\times\frac{13\times 12}{51\times 50}}$

$= \frac{11}{50}$

28. Re- writing the given integral as:

$I = \int \frac{x^2-2x+1}{x^4+x^2+1} dx$

$= \int \frac{1-\frac{2}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}} dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+3} dx$ - $\int \frac{2x}{x^4+x^2+1} dx$

By using Substitution Let t = x - $\frac{1}{x}$ and z = x^2

(1 + $\frac{1}{x^2}$)dx = dt and 2xdx = dz

Therefore we have,

$I = \int \frac{dt}{(t)^2+3}$ - $\frac{3}{2}$ $\int \frac{dz}{z^2+z+1}$

$I = \int \frac{dt}{(t)^2+3}$ - $\int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$

Using identity $\int \frac{1}{x^2+1}$ dx = $\tan^{-1}$ (x)

$I = \frac{1}{\sqrt{3}}$ $\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)$ - $\frac{2}{\sqrt{3}}$ $\tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right)$ + c

Substituting t as x -$\frac{1}{x}$ and z as x^2, we get

$I = \frac{1}{\sqrt{3}}$ arctan $\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)$ - $\frac{2}{\sqrt{3}}$ arctan $\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$

OR

Let the given integral be,

I = $\int_0^{\frac{\pi}{2}} \frac{1}{\cos x+2\sin x}dx$

Using sinx = $\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

And

cosx = $\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

we get

$\Rightarrow$ I = $\int_0^{\frac{\pi}{2}} \frac{1}{\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}dx$

= $\int_0^{\frac{\pi}{2}} \frac{\sec^2\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)}dx$

Let $\tan\left(\frac{x}{2}\right)$ = t

$\Rightarrow$ $\frac{1}{2}\sec^2\left(\frac{x}{2}\right)$dx = dt,

when x = 0, t = 0

and when x = $\frac{\pi}{2'}$ t = 1.

Hence,

I = $\int_0^1 \frac{2}{1-t^2+4t}dt$

= -2 $\int_0^1 \frac{1}{t^2-4t+4-5}dt$

= -2$\int_0^1 \frac{1}{(t-2)^2-5}dt$

Let t - 2 = u

$\Rightarrow$ dt = du.

Also, when t=0, u = -2.

and when t = 1, u = -1.

$\Rightarrow$ i = -2 $\int_{-2}^{-1} \frac{1}{u^2-5}dt$

= -2 $\frac{1}{2\sqrt{5}}\log_e\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|\Big|_{-2}^{-1}$

(Using $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log_e\left|\frac{x-a}{x+a}\right|$)

Hence,

I = $-\frac{1}{\sqrt{5}}\left(\log_e\left|\frac{-1-\sqrt{5}}{-1+\sqrt{5}}\right| - \log_e\left|\frac{-2-\sqrt{5}}{-2+\sqrt{5}}\right|\right)$

= $\frac{-1}{\sqrt{5}}\left(\log_e\left|\frac{\sqrt{5}+1}{\sqrt{5}-1}\right| \times \left|\frac{\sqrt{5}-2}{2+\sqrt{5}}\right|\right)$

(Using $\log_e a - \log_e b = \log_e \frac{a}{b}$)

$\Rightarrow$ I = $\frac{-1}{\sqrt{5}}\left(\log_e\left|\frac{3-\sqrt{5}}{3+\sqrt{5}}\right|\right)$

= $\frac{-2}{\sqrt{5}}\left(\log_e\left(\frac{3-\sqrt{5}}{2}\right)\right)$

(Using $\log_e a^b = b \log_e a$)

29. The given differential equation is,

(x + y) $\frac{dy}{dx}$ = 1

$\Rightarrow$ $\frac{dy}{dx}$ = $\frac{1}{x+y}$

$\Rightarrow$ $\frac{dx}{dy}$ = x + y

$\Rightarrow$ $\frac{dx}{dy}$ - x = y

This is a linear differential equation of the form:

$\frac{dx}{dy}$ + px = Q (where P = -1 and Q = y)

Now, I.F. = $e^{\int pdy} = e^{\int -dy}$ = e^{-y}.

The general solution of the given differential equation is given by the relation,

x(I.F) = $\int$(Q $\times$ I.F) dy + C

$\Rightarrow xe^{-y} = \int (y.e^{-y})dy + C$

$\Rightarrow xe^{-y} = y.\int e^{-y}\, dy - \int \left[\frac{d}{dy}(y)\int e^{-y}dy\right] dy + C$

$\Rightarrow xe^{-y} = y(-e^{-y}) - \int (-e^{-y})\, dy + C$

$\Rightarrow xe^{-y} = -ye^{-y} + \int e^{-y}\, dy + C$

$\Rightarrow xe^{-y} = -ye^{-y} - e^{-y} + C$

$\Rightarrow x = -y - 1 + Ce^{y}$

$\Rightarrow x + y + 1 = Ce^{y}$

OR

The given differential equation is,

$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$

$\Rightarrow \frac{dy}{dx} = \frac{2x^2+x}{x^3+x^2+x+1}$

$\Rightarrow dy = \frac{2x^2+x}{(x+1)(x^2+1)}dx$

Integrating both sides, we get

$\int dy = \int \left\{\frac{2x^2+x}{(x+1)(x^2+1)}\right\} dx$

$\Rightarrow y = \int \left\{\frac{2x^2+x}{(x+1)(x^2+1)}\right\} dx$

Let $\frac{2x^2+x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$

$\Rightarrow 2x^2 + x = (A + B)x^2 + (B + C)x + (A + C)$

Comparing the coefficients on both sides, we get

A + B = 2 ...(i)

B + C = 1 ...(ii)

A + C = 0 ...(iii)

Solving (i), (ii) and (iii), we get

$A = \frac{1}{2}, B = \frac{3}{2}, C = -\frac{1}{2}$

$\therefore y = \frac{1}{2}\int \frac{1}{(x+1)}dx + \int \frac{\frac{3}{2}x - \frac{1}{2}}{x^2+1}dx$

$= \frac{1}{2}\int \frac{1}{(x+1)}dx + \frac{1}{2}\int \frac{3x}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$

$= \frac{1}{2}\int \frac{1}{(x+1)}dx + \frac{3}{4}\int \frac{2x}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$

$= \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$

Hence, $y = \frac{1}{2}\log|x + 1| + \frac{3}{4}\log|x^2 + 1| - \frac{1}{2}\tan^{-1}x + C$ is the solution to the given differential equation.

30. We need to maximize z = 3 x + 5y

First, we will convert the given inequations into equations, we obtain the following equations:

x + 2y = 20, x + y = 15, y = 5, x = 0 and y = 0

The line x + 2y = 20 meets the coordinate axis at A(20,0) and B(0,10). Join these points to obtain the line x + 2y = 20

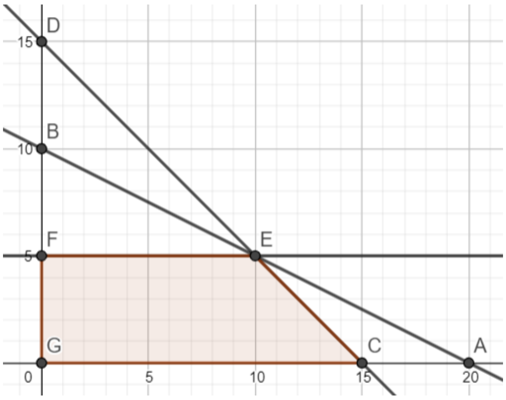
Clearly, (0, 0) satisfies the inequation $x + 2y \leq 20$. So, the region in xy -plane that contains the origin represents the solution set of the given equation.

The line x + y = 15 meets the coordinate axis at C(15,0) and D(0,15). Join these points to obtain the line x + y = 15 Clearly, (0,0) satisfies the inequation $x + y \leq 15$. So, the region in XY -plane that contains the origin represents the solution set of the given equation.

y = 5 is the line passing through (0,5) and parallel to the X-axis. The region below the liney = 5 will satisfy the given inequation.

Region represented by $x \geq 0$ and $y \geq 0$ (non -negative restrictions)

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations. These lines are drawn using a suitable scale.



The corner points of the feasible region are O(0,0), C(15,0), E(10,5) and F(0,5)

The values of objective function at the corner points are as follows:

Corner point : Z = 3x + 5y

O(0,0) : $3 \times 0 + 5 \times 0 = 0$

C(15,0) : $3 \times 15 + 5 \times 0 = 45$

E(10, 5) : $3 \times 10 + 5 \times 5 = 55$

F(0, 5) : $3 \times 0 + 5 \times 5 = 25$

We see that the maximum value of objective function Z is 55 which is at E(10,5)

Thus, the optimal value of objective function Z is 55.

OR

We have to maximize Z= 60 x + 15y First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 50, 3x + y = 90, x = 0 and y = 0

Region represented by x + y $\leq$ 50 :

The line x + y = 50 meets the coordinate axes at A(50, 0) and B(0, 50) respectively. By joining these points we obtain the line 3x + 5 y = 15 Clearly (0, 0) satisfies the inequation x + y $\leq$ 50 . Therefore, the region containing the origin represents the solution set of the inequation x + y $\leq$ 50

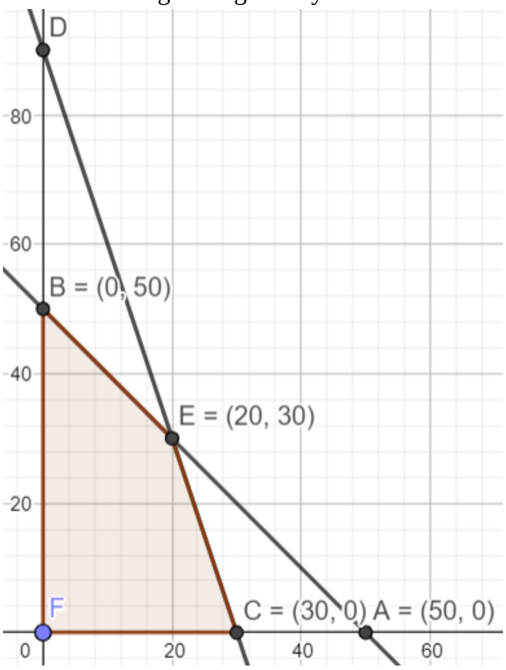
Region represented by 3x + y $\leq$ 90 :

The line 3x + y = 90 meets the coordinate axes at C(30, 0) and D(0, 90) respectively. By joining these points we obtain the line 3x + y = 90 Clearly (0, 0) satisfies the inequation 3x + y $\leq$ 90 . Therefore, the region containing the origin represents the solution set of the inequation 3x + y $\leq$ 90

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. Therefore, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0.

The feasible region is given by



The corner points of the feasible region are O(0, 0), C(30, 0) E(20, 30) and B(0, 50)

The values of Z at these corner points are as follows given by

Corner point Z = 60 x + 15 y

O(0, 0) : $60 \times 0 + 15 \times 0 = 0$

C(30, 0) : $60 \times 30 + 15 \times 0 = 1800$

E(20, 30) : $60 \times 20 + 15 \times 30 = 1650$

B(0, 50) : $60 \times 0 + 15 \times 50 = 750$

Therefore, the maximum value of Z is 1800 at the point (30, 0) Hence, x = 30 and y = 0 is the optimal solution of the given LPP.

Thus, the optimal value of Z is 1800.This is the required solution.

31. Given,

y = a (θ + sinθ) ...(i)

x = a (1 - cosθ) ...(ii)

To prove: $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$

To prove above equation we need to find differentiation twice wrt x

As $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$\frac{\mathrm{dy}}{\mathrm{d}\theta} = \frac{\mathrm{d}}{\mathrm{d}\theta}\mathrm{a}(\theta + \sin\theta) = \mathrm{a}(1 + \cos\theta)$ ...(iii)

Similarly,

$\frac{dx}{d\theta} = \frac{d}{d\theta}a(1 - \cos\theta) = \mathrm{a}\sin\theta$ ...(iv)

$\left[\because \frac{\mathrm{d}}{\mathrm{dx}}\cos\mathrm{x} = -\sin\mathrm{x}, \frac{\mathrm{d}}{\mathrm{dx}}\sin\mathrm{x} = \cos\mathrm{x}\right]$

$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\frac{\mathrm{dy}}{\mathrm{dx}}}{\frac{\mathrm{dx}}{\mathrm{d}\theta}} = \frac{\mathrm{a}(1+\cos\theta)}{\mathrm{asin}\theta} = \frac{(1+\cos\theta)}{\sin\theta}$ ...(v)

Differentiating again w.r.t x:

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{(1+\cos\theta)}{\sin\theta}\right) = \frac{d}{dx}(1 + \cos\theta)\csc\theta$

Using product rule and chain rule of differentiation together:

$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2} = \left\{\csc\theta\frac{\mathrm{d}}{\mathrm{d}\theta}(1 + \cos\theta) + (1 + \cos\theta)\frac{\mathrm{d}}{\mathrm{d}\theta}\csc\theta\right\}\frac{\mathrm{d}\theta}{\mathrm{dx}}$

$\frac{\mathrm{d}^2y}{\mathrm{dx}^2} = \{\csc\theta(-\sin\theta) + (1 + \cos\theta)(-\csc\theta\cot\theta)\}\frac{1}{\mathrm{a}\sin\theta}$ [using equation (iv)]

$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2} = \left\{-1 - \csc\theta\cot\theta - \cot^2\theta\right\}\frac{1}{\mathrm{a}\sin\theta}$

As we have to find $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$

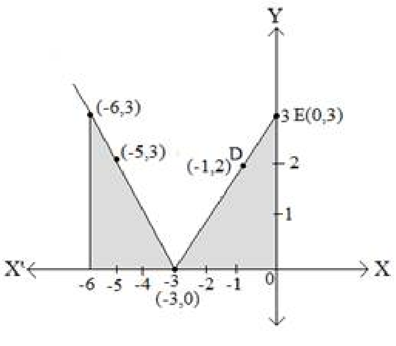
$\therefore$ put $\theta = \pi/2$ in the above equation:

$\frac{d^2y}{dx^2} = \left\{-1 - \csc\frac{\pi}{2}\cot\frac{\pi}{2} - \cot^2\frac{\pi}{2}\right\}\frac{1}{\mathrm{a}\sin\frac{\pi}{2}} = \frac{\{-1-0-0\}1}{a}$

$\frac{d^2y}{dx^2} = -\frac{1}{a}$

Hence Proved.

Section D

32. 

$y = |x + 3|$

$y = |x + 3| = \begin{cases} -(x+3) & for \quad x < -3 \\ x + 3 & for \quad x \geqslant -3 \end{cases}$

When x < - 3

y = - x - 3

when x = - 4,

y = 4 - 3 = 1

When x = - 5,

y = 5 - 3 = 2
When x = - 6,
y = 6 - 3 = 3
Required Area = Area of region ABC + Area of region OAD
$= \int_{6}^{-3} |x+3|\, dx + \int_{-3}^{0} |x+3| dx$
$= \int_{6}^{-3} (-x-3)\, dx + \int_{-3}^{0} (x+3) dx$
$= \left[\frac{-x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$
$= \left[\left(-\frac{9}{2}+9\right) - (-18+18)\right] + \left[(0+0) - (\frac{9}{2}-9)\right]$
$= \ (\frac{9}{2}+0) + (0+\frac{9}{2})$
$\Rightarrow 9$ sq. units
Hence, the required area is 9 square units.

33. i. A be the set of human beings.
R = {(x, y) : x and y work at the same place}
Reflexive:
$\therefore$ x and x works together R
$\therefore$ (x, x) $\in$ R
$\Rightarrow$ R is reflexive
Symmetric: If x and y work at the same place, which implies, y and x work at the same place
$\therefore$ (y, x) $\in$ R
$\Rightarrow$ R is symmetric
Transitive: If x and y work at the same place then x and y work at the same place and y and z work at the same place,
$\Rightarrow$ (x, z) $\in$ R and
Hence,
$\Rightarrow$ R is transitive

ii. A be the set of human beings.
R = {(x, y) : x and y lives in the same locality}
Reflexive: Since x and x lives in the same locality,
$\Rightarrow$ (x, x) $\in$ R
$\Rightarrow$ R is reflexive
Symmetric: Let (x, y) $\in$ R
$\Rightarrow$ x and y lives in the same locality
$\Rightarrow$ y and x lives in the same locality
$\Rightarrow$ (y, x) $\in$ R
$\Rightarrow$ R is symmetric.
Transitive: Leet (x, y) $\in$ R and (y, z) $\in$ R
(x, y) $\in$ R
$\Rightarrow$ x and y lives in the same locality and (y, z) $\in$ R
$\Rightarrow$ y and z lives in the same locality
$\Rightarrow$ x and z lives in the same locality
$\Rightarrow$ (x, z) $\in$ R
$\Rightarrow$ R is transitive

OR

Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$
We shall show that R satisfies the following properties

i. Reflexivity:
We know that a + b = b + a for all a, b $\in$ N.
$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$
So, R is reflexive.

ii. Symmetry:
Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c
$\Rightarrow$ c + b = d + a
$\Rightarrow$ (c, d) R (a, b).
$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$
This shows that R is symmetric.

iii. Transitivity:
Let (a, b) R (c, d) and (c, d) R (e, f). Then,
(a, b) R (c, d) and (c, d) R (e, f)
$\Rightarrow$ a + d = b + c and c + f = d + e
$\Rightarrow$ a + d + c + f = b + c + d + e
$\Rightarrow$ a + f = b + e
$\Rightarrow$ (a, b) R (e, f).
Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)
This shows that R is transitive.
$\therefore$ R is reflexive, symmetric and transitive
Hence, R is an equivalence relation on $N \times N$

34. Given, A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$
Now, |A| = 1 (1 + 3) - 2 (- 1 -1) + 1 (3-1)
= 4 + 4 + 2=10
$\Rightarrow \quad |A| \neq 0$, hence A^{-1} exists.
Now, cofactors of elements of |A| are,
$A_{11} = (-1)^2 \begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix} = (1+3) = 4$
$A_{12} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = 2$
$A_{13} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = (3-1) = 2$
$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix} = -(2+3) = -5$
$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1-1) = 0$
$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} = -1(-3-2) = 5$
$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2-1) = 1$
$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -(1+1) = -2$
$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = (1+2) = 3$
$\therefore \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$
= $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$
and $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$
$\Rightarrow A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$
Given system of equations can be written in matrix from as
$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$

i.e. AX = B

where,

$A=\begin{bmatrix}1&2&1\\-1&1&1\\1&-3&1\end{bmatrix} B=\begin{bmatrix}4\\0\\4\end{bmatrix}$ and $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

Clearly, $X=A^{-1}B=\frac{1}{10}\begin{bmatrix}4&-5&1\\2&0&-2\\2&5&3\end{bmatrix}\begin{bmatrix}4\\0\\4\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\8+0+(-8)\\8+0+12\end{bmatrix}=\frac{1}{10}\begin{bmatrix}20\\0\\20\end{bmatrix}=\begin{bmatrix}2\\0\\2\end{bmatrix}$

Therefore,on comparing corresponding elements, we get x = 2, y = 0 and z = 0

35. Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector $\vec{b}=2\hat{i}-3\hat{j}+8\hat{k}$

Now,

$\overrightarrow{PQ}=0\hat{i}-\hat{j}-10\hat{k}$

$\therefore \vec{b}\times\overrightarrow{PQ}$ = $\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-3&8\\0&-1&-10\end{vmatrix}$

= $38\hat{i}+20\hat{j}-2\hat{k}$

$\Rightarrow|\vec{b}\times\overrightarrow{PQ}|=\sqrt{38^2+20^2+2^2}$

= $\sqrt{1444+400+4}$

= $\sqrt{1848}$

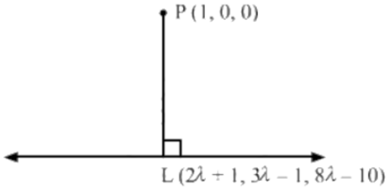
d = $\frac{|\vec{b}\times\overrightarrow{PQ}|}{|\vec{b}|}$

= $\frac{\sqrt{1848}}{\sqrt{77}}$

= $\sqrt{24}$

= $2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ are given by

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda$

$\Rightarrow x=2\lambda+1$

$y=-3\lambda-1$

$z=8\lambda-10$

Suppose the coordinates of L be

$(2\lambda+1,-3\lambda-1,8\lambda-10)$

Since,The direction ratios of PL are proportional to,

$2\lambda+1-1,-3\lambda-1-0,8\lambda-10-0$, i.e., $2\lambda,-3\lambda-1,8\lambda-10$

Since,The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda)-3(-3\lambda-1)+8(8\lambda-10)$ = 0

$\Rightarrow \lambda$ = 1 Substituting λ = 1 in $(2\lambda+1,-3\lambda-1,8\lambda-10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1}=\frac{y-0}{-4-0}=\frac{z-0}{-2-0}$

= $\frac{x-1}{1}=\frac{y}{-2}=\frac{z}{-1}$

$\Rightarrow\vec{r}=\hat{i}+\lambda(\hat{i}-2\hat{j}-\hat{k})$

OR

The given equations are

al + bm + cn = 0 ....(i)

and, $ul^2 + vm^2 + wn^2 = 0$ ..(ii)

From (i), we get

n = $-\left(\frac{al+bm}{c}\right)$

Substituting n = $-\left(\frac{al+bm}{c}\right)$ in (ii), we get

$ul^2 + vm^2 + w\frac{(al+bm)^2}{c^2} = 0$

$\Rightarrow (c^2u + a^2w)l^2 + 2abwlm + (c^2v + b^2w)m^2 = 0$

$\Rightarrow \left(a^2w + c^2u\right)\left(\frac{l}{m}\right)^2 + 2abw\left(\frac{l}{m}\right) + \left(b^2w + c^2v\right) = 0$ ....(iii)

This is a quadratic equation in $\frac{l}{m}$. So, it gives two values of $\frac{l}{m}$. Suppose the two values be $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$.

$\therefore \quad \frac{l_1}{m_1}, \frac{l_2}{m_2} = \frac{b^2w+c^2v}{a^2w+c^2u} \Rightarrow \frac{l_1l_2}{b^2w+c^2v} = \frac{m_1m_2}{a^2w+c^2u}$ .....(iv)

Similarly, by making a quadratic equation in $\frac{m}{n}$, we obtain

$\frac{m_1m_2}{a^2w+c^2u} = \frac{n_1n_2}{a^2v+b^2u}$ ....(v)

From (iv) and (v), we get

$\frac{l_1l_2}{b^2w+c^2v} = \frac{m_1m_2}{a^2w+c^2u} = \frac{n_1n_2}{a^2v+b^2u} = \lambda$ (say)

$\Rightarrow l_1l_2 = \lambda\left(b^2w + c^2v\right), m_1m_2 = \lambda\left(a^2w + c^2u\right), n_1n_2 = \lambda\left(a^2v + b^2u\right)$

For the given lines to be perpendicular, we must have

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow \lambda\left(b^2w + c^2v\right) + \lambda\left(a^2w + c^2u\right) + \lambda\left(a^2v + b^2u\right) = 0$

$\Rightarrow a^2 (v + w) + b^2 (u + w) + c^2 (u + v) = 0$

For the given lines to be parallel, the direction cosines must be equal and so the roots of the equation (iii) must be equal.

$\therefore 4a^2 b^2 w^2 - 4(a^2w + c^2u)(b^2w + c^2v) = 0$ [On equating discriminant to zero]

$\Rightarrow a^2 c^2 vw + b^2 c^2 uw + c^4 uv = 0$

$\Rightarrow a^2 vw + b^2 c^2 uw + c^2uv = 0$

$\Rightarrow \frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$ [Dividing throughout by uvw] Hence the required result is proved

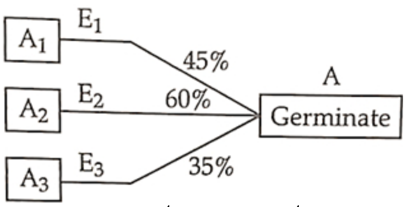
Section E

36. **Read the text carefully and answer the questions:**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i)



Here, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$, $P(E_3) = \frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

$= \frac{490}{1000} = 4.9$

(ii) Required probability = $P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10}\times\frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

(iii) Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

= $\frac{1}{2}$

and P(E_2) = $\frac{13}{52} = \frac{1}{4}$

Then, P($E_1 \cap E_2$) = P(E_1) $\cdot$ P(E_2)

= $\frac{1}{2}, \frac{1}{4}$ = $\frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A ∩ B) = P(A)

$\Rightarrow$ P(B) - P(A ∩ B) = 0

$\Rightarrow$ P(A ∩ B) = P(B)

$\therefore$ P(A|B) = $\frac{P(A\cap B)}{P(B)}$

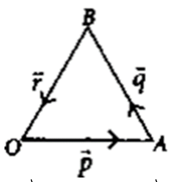
= $\frac{P(B)}{P(B)}$

= 1

37. **Read the text carefully and answer the questions:**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

(i) Let OAB be a triangle such that



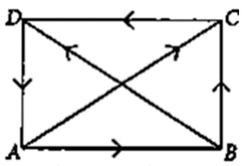
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

(ii) From triangle law of vector addition,

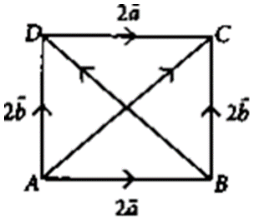
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



= $\overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

= $\overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

(iii) In $\triangle$ABC, $\overrightarrow{AC}$ = $2\vec{a}$ + $2\vec{b}$ ...(i)



and in $\triangle$ABD, $2\vec{b} = 2\vec{a} + \overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

= $2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

38. **Read the text carefully and answer the questions:**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o}F at x days.

(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

(ii) f(x) = -0.2x + m

At Critical point

0 = -0.2 $\times$ 6 + m

m = 1.2