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**ALL KERALA COMMON MODEL EXAMINATION 2023 - 24**

**MATHEMATICS [041]**

**Time Allowed :** 180 mins **Maximum Marks :** 80

**General Instructions:**

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion - Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) - type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) - type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) - type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

| | **Section A** | |
|---|---|---|
| 1 | If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x, y is<br>a) x = 3, y = 1<br>b) x = 2, y = 3<br>c) x = 2, y = 4<br>d) x = 3, y = 3 | **[1]** |
| 2 | The value of the determinant $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$ is<br>a) 0<br>b) a<br>c) - a<br>d) b | **[1]** |

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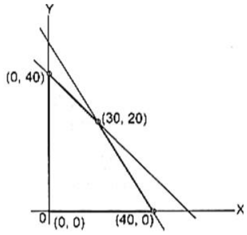
| 3 | If A is singular then A(adj A) = ?<br>a) None of these<br>b) A null matrix<br>c) A unit matrix<br>d) A symmetric matrix | **[1]** |
| 4 | If $x^y = y^x$ ,find $\frac{dy}{dx}$<br>a) x log x<br>b) 0<br>c) $\frac{y}{x} \cdot \left(\frac{x\log y - y}{y\log x - x}\right)$<br>d) None of these | **[1]** |
| 5 | The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is<br>a) perpendicular to z - axis<br>b) parallel to z - axis<br>c) parallel to y - axis<br>d) parallel to x - axis | **[1]** |
| 6 | Find the equation of a curve passing through the point (0, 0) and whose differential equation is y ' = $e^x$ sin x.<br>a) $2y + 1 = e^x(sin2x - cosx)$<br>b) $2y - 1 = (\sin x - \cos x)e^x$<br>c) $3y - 1 = e^x(sinx - cos2x)$<br>d) $4y - 1 = e^x(sinx - cos2x)$ | **[1]** |
| 7 | The maximum value of Z = 0.7x + y for feasible region given below is<br><br> | **[1]** |

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| | a) 40<br>b) 50<br>c) 41<br>d) 45 | |
| 8 | A unit vector perpendicular to both $\hat{\imath} + \hat{\jmath}$ and $\hat{\jmath} + \hat{k}$ is<br>a) $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} + \hat{k})$<br>b) $\hat{\imath} + \hat{\jmath} + \hat{k}$<br>c) $\hat{\imath} - \hat{\jmath} + \hat{k}$<br>d) $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$ | **[1]** |
| 9 | $\int_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right)dx = ?$<br>a) 0<br>b) $\log\frac{1}{2}$<br>c) log 2<br>d) $\frac{1}{2}\log 2$ | **[1]** |
| 10 | If A = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$ then<br>a) only BA is defined<br>b) only AB is defined<br>c) AB and BA both are not defined<br>d) AB and BA both are defined | **[1]** |
| 11 | The point at which the maximum value of x + y, subject to the constraints x + 2y$\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is<br>a) (20, 35)<br>b) (30, 25)<br>c) (35, 20) | **[1]** |

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| | d) (40,15) | |
| 12 | Find a vector in the direction of vector $5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has a magnitude of 8 units.<br><br>a) $\frac{40}{\sqrt{30}}\hat{\imath} + \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$<br><br>b) $-\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$<br><br>c) $\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$<br><br>d) $\frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} - \frac{16}{\sqrt{30}}\hat{k}$ | **[1]** |
| 13 | Let $\begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix}$ = px $^7$ + qx $^6$ + rx $^5$ + sx $^4$ + tx $^3$ + ux $^2$ + vx + w then which of the following is not true?<br><br>a) w = 21, v = 75<br><br>b) p = - 1, t = 8<br><br>c) p = q = - 1<br><br>d) q = 0, s = - 4 | **[1]** |
| 14 | Number X is randomly selected from the set of odd numbers and Y is randomly selected from the set of even numbers of the set {1, 2, 3, 4, 5, 6, 7}. Let Z = (X + Y).What is P (Z = 10) equal to?<br><br>a) $\frac{1}{3}$<br><br>b) $\frac{1}{2}$<br><br>c) $\frac{1}{5}$<br><br>d) 0 | **[1]** |
| 15 | A first order linear differential equation, is a differential equation of the form<br><br>a) $\frac{dy}{dx} + Py = Q$<br><br>b) $\frac{dy}{dx} + Px = Q$<br><br>c) $\frac{dy}{dx} + Py = 0$ | **[1]** |

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| | d) $\frac{dy}{dx} = Q$ | |
| 16 | Let $\vec{a}, \bar{b}, \bar{c}$ be three unit vectors such that $\left\|\vec{a} + \vec{b} + \vec{c}\right\| = 1$ and $\vec{a}$ is perpendicular to $\vec{b}$ . If $\vec{c}$ makes angle $\alpha$ and $\beta$ with $\vec{a}$ and $\vec{b}$ respectively, then cos $\alpha$ + cos $\beta$ =<br><br>a) - 1<br><br>b) $\frac{3}{2}$<br><br>c) $-\frac{3}{2}$<br><br>d) 1 | **[1]** |
| 17 | Let f(x) = $\begin{cases} \left\|\tan\left(\frac{\pi}{4} + x\right)\right\|^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$ then the value of k such that f(x) holds continuity at x = 0 is<br><br>a) $e^2$<br><br>b) $\frac{1}{e^2}$<br><br>c) e<br><br>d) None of these | **[1]** |
| 18 | Find the shortest distance between the lines $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$<br><br>a) $\frac{8}{\sqrt{31}}$<br><br>b) $\frac{8}{\sqrt{35}}$<br><br>c) $\frac{8}{\sqrt{29}}$<br><br>d) $\frac{8}{\sqrt{33}}$ | **[1]** |
| 19 | **Assertion (A):** The function f(x) = sin x decreases on the interval (0, $\frac{\pi}{2}$ ).<br>**Reason (R):** The function f(x) = cos x decreases on the interval (0, $\frac{\pi}{2}$ ).<br><br>a) Both A and R are true and R is the correct explanation of A.<br><br>b) Both A and R are true but R is not the correct explanation of A.<br><br>c) A is true but R is false. | **[1]** |

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| | d) A is false but R is true. | |
| 20 | **Assertion (A):** A function f: Z → Z defined as f(x) = $x^3$ is injective. **Reason (R):** A function f: A → B is said to be injective if every element of B has a pre - Image in A.<br><br>a) Both A and R are true and R is the correct explanation of A.<br><br>b) Both A and R are true but R is not the correct explanation of A.<br><br>c) A is true but R is false.<br><br>d) A is false but R is true. | **[1]** |
| | **Section B** | |
| 21 | Evaluate:$\sin^{-1}$ (sin( - 600°))<br><br>**OR**<br><br>For the principal value, evaluate$\cot[\sin^{-1}\{\cos(\tan^{-1}1)\}]$ | **[2]** |
| 22 | Prove that the function f(x) = $x^3$ - 6$x^2$ +12x - 18 is increasing on R. | **[2]** |
| 23 | Find the points of local maxima or local minima and corresponding local maximum and local minimum valuesof the function. Also, find the points of inflection, if any:f(x) = x +$\frac{a^2}{x}, a > 0, x \neq 0$ .<br><br>**OR**<br><br>A particle moves along the curve y = $x^2$ + 2x. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate? | **[2]** |
| 24 | Evaluate$\int \frac{e^{\log\sqrt{x}}}{x} dx$ | **[2]** |
| 25 | Find the maximum and minimum values of f(x) = ( - x + 2sin x) on<br><br>$[0, 2\pi]$. | **[2]** |
| | **Section C** | |
| 26 | Evaluate the definite integral:$\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}} dx$ | **[3]** |
| 27 | A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond. | **[3]** |
| 28 | Evaluate the integral:$\int \frac{(x-1)^2}{x^4+x^2+1} dx$<br><br>**OR** | **[3]** |

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| | Evaluate: $\int_0^{\pi/2} \frac{dx}{(\cos x+2\sin x)}$ | |
| 29 | Find the general solution of the differential equation: $(x+y)\frac{dy}{dx} = 1$<br><br>**OR**<br><br>Solve thedifferential equation: $(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$ | **[3]** |
| 30 | Solve the Linear Programming Problemgraphically:Maximize Z = 3x + 5y Subject to x + 2y ≤ 20 x + y ≤ 15 y ≤ 5 x, y ≥ 0<br><br>**OR**<br><br>Solved the linear programming problem graphically:Maximize Z = 60x + 15y Subject to constraints x + y ≤ 50 3x + y ≤ 90 x, y ≥ 0 | **[3]** |
| 31 | If x = a (1 - cos$\theta$ ), y = a ( $\theta$ + sin $\theta$ ) prove that $\frac{d^2y}{dx^2} = -\frac{1}{a}$ at $\theta = \frac{\pi}{2}$ | **[3]** |
| | **Section D** | |
| 32 | Sketch the graph of y = \|x + 3\| and evaluate $\int_{-6}^{0} \lvert x+3 \rvert \, dx$ | **[5]** |
| 33 | Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:<br><br>1. R = {(x, y): x and y work at the same place}<br><br>2. R = {(x, y): x and y live in the same locality}<br><br>**OR**<br><br>Let R be a relation on$N \times N$ , defined by (a, b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ $N \times N$ .Show that R is an equivalence relation. | **[5]** |
| 34 | If A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & - & 1 \end{bmatrix}$ , then find $A^{-1}$ and hence solve the system of equations<br>x+ 2y+z= 4<br>-x+y+z=0<br>and x- 3y + z = 4. | **[5]** |
| 35 | Find the perpendicular distance of the point (1, 0, 0) from the line$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ .Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.<br><br>**OR** | **[5]** |

Show that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular, if $a^2 (v + w) + b^2 (u + w) + c^2 (u + v) = 0$ and, parallel, if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$

**Section E**

36 **Read the text carefully and answer the questions:** A shopkeeper sells three types of flower seeds $A_1$ , $A_2$ , $A_3$ . They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively. **[4]**

Based on the above information:

1. Calculate the probability that a randomly chosen seed will germinate.
2. Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.
3. A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

   **OR**

4. If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

37 **Read the text carefully and answer the questions:** If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition. **[4]**

1. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$ .
2. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$ .
3. If ABCD is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$ , then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$ .

   **OR**

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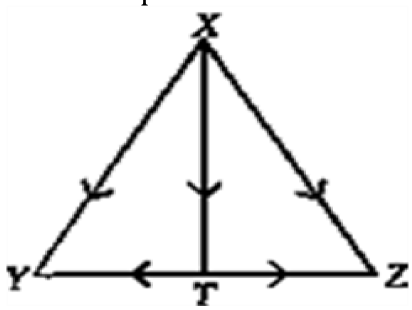
4. If T is the mid point of side YZ of $\triangle$ XYZ, then what is the value of $\overrightarrow{XY} + \overrightarrow{XZ}$.

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**Read the text carefully and answer the questions:** The temperature of a person during an intestinal illness is given by f(x) = - $0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in $^o$ F at x days.

1. Is the function differentiable in the interval (0, 12)? Justify your answer.
2. If 6 is the critical point of the function, then find the value of the constant m.

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