Solution

PRACTICE PAPER APPLIED MATHEMATICS

Class 12 - Applied Mathematics

Section A

1.

(d) d^{n-1}

Explanation: |A| = d

$|adjA| = |A|^{n-1}$

$|adjA| = d^{n-1}$

2.

(c) 5

Explanation: 5

3.

(c) ₹ 15750

Explanation: ₹ 15750

4. **(a)** {x : |x| = 5}

Explanation: |x| = 5 is not a convex set as any two points from negative and positive x-axis if are joined will not lie in set.

5.

(b) 8

Explanation: 8

6.

(c) $\frac{105}{512}$

Explanation: n = 10, X = 6, p = q = $\frac{1}{2}$

P(X = 6) $= {}^{10}C_6\left(\frac{1}{2}\right)^{10} = \frac{105}{512}$

7.

(b) $\frac{2}{e^2}$

Explanation: $\frac{2}{e^2}$

8.

(b) $2y - x^3 = cx$

Explanation: We have,

$x\frac{dy}{dx} - y = x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x$

Comparing with $\frac{dy}{dx}$ - Py = Q

$\Rightarrow P = \frac{-1}{x}$, Q = x

I.F. = $e^{\int Pdx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$

Multiplying $\frac{1}{x}$ on both sides,

$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 1$

$\frac{d}{dx}\frac{y}{x} = 1$

$\int \frac{d}{dx}\frac{y}{x} = \int xdx$

$\frac{y}{x} = \frac{x^2}{2} + c$

$2y = x^3 + cx$

$2y - x^3 - cx = 0$

9.

(b) 3

Explanation: Let the speed of the stream be x km/hr

Then speed downstream = (10 + x) km/hr
Speed upstream
= (10 - x)km/hr
$\therefore \frac{26}{(10+x)} = \frac{14}{(10-x)}$
$\Rightarrow$ 260 - 26x = 140 + 14x
$\Rightarrow$ 40x = 120
$\Rightarrow$ x = 3 km/hr

10.
(c) K^2 |A|
Explanation: K^2 |A|

11. **(a)** 2 : 10
Explanation: 2 : 10

12.
(b) x $\in$ (-5, 2)
Explanation: x $\in$ (-5, 2)

13. **(a)** 8 m/sec, 7 m/sec
Explanation: Suppose A covers 400 m in t seconds
Then, B covers 385 m in (t + 5) seconds
$\therefore$ B covers 400 m = $\left\{\frac{(t+5)}{385} \times 400\right\}$sec
= $\frac{80(t+5)}{77}$ sec
Also, B covers 400 m = $\left(t + 7\frac{1}{7}\right)$sec
= $\frac{(7t+50)}{7}$sec
$\therefore \frac{80(t+5)}{77} = \frac{7t+50}{7}$
$\therefore$ 80(t + 5) = 11(7t + 50)
$\Rightarrow$ (80t - 77t) = (550 - 400)
$\Rightarrow$ 3t = 150
$\Rightarrow$ t = 50
$\therefore$ A's speed
= $\frac{400}{50}$ m/sec
= 8 m/sec
$\therefore$ B's speed
= $\frac{385}{55}$ m/sec
= 7 m/sec

14.
(d) The constraints are short in number
Explanation: The constraints are short in number

15.
(b) q = 3p
Explanation: Given the vertices of the feasible region are:
Q(0, 0)
A(5, 0)
B(3, 4)
C(0, 5)
Also given the objective function is Z = px + qy
Now substituting O, A, B and C in Z

Z at O(0, 0)	Z = P(0) + q(0) = 0
Z at A(5, 0)	Z = p(5) + q(0) = 5p + 0 = 5p
Z at B(3, 4)	Z = p(3) + q(4) = 3p + 4q

Z at C(0, 5)	Z = p(0) + q(5) = 0 + 5q

As per the condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5)

Then we can equate Z values at B and C, this gives

3p + 4q = 5q

3p = 5q - 4q

3p = q

16.

(c) estimating a statistic

Explanation: estimating a statistic

17.

(c) e^x f(x) + C

Explanation: $\int e^x\{f(x) + f'(x)\}\,dx = e^x f(x) + C$

t = e^xf(x)

$\frac{dt}{dx} = e^x \cdot \frac{d}{dx}(f(x)) + f(x)\frac{d}{dx}(e^x)$

= e^xf'(x) + f(x).e^x

dt = e^x(f'(x) + f(x)) dx

$\int e^x\{f(x) + f'(x)\}\,dx = \int dt = t + C$

= e^x f(x) + C

18.

(c) Method of least squares

Explanation: Method of least squares

19.

(c) A is true but R is false.

Explanation: Assertion: In general, the matrix A of order 2 × 2 is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now, $a_{ij} = i \times j$, i = 1, 2 and j = 1, 2

∴ a_{11} = 1, a_{12} = 2, a_{21} = 2 and a_{22} = 4

Thus, matrix A is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Reason: If A is a 4 × 2 matrix, then A has 4 × 2 = 8 elements.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: The equation of the given curve is y^2 = 9x

⇒ y^2 = 9x

Differentiating w.r.t. x, we get

2y $\frac{dy}{dx}$ = 9 ⇒ $\frac{dy}{dx} = \frac{9}{2y}$

∴ Slope of tangent at (1, 1) is

$\left(\frac{dy}{dx}\right)_{1,1} = \frac{9}{2\times1} = \frac{9}{2}$

⇒ m = $\frac{9}{2}$

∴ Equation of tangent at (1, 1) is y - 1 = $\frac{9}{2}$(x - 1)

⇒ 2(y - 1) = 9(x - 1)

⇒ 2y - 2 = 9x - 9

⇒ 0 = 9x - 9 - 2y + 2

⇒ 9x - 2y - 7 = 0

⇒ 9x - 2y = 7

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Section B

21. Construction of 3-yearly moving average

Year	Imported cotton consumption	3-yearly moving totals	3-yearly moving averages

	in India (in '000 bales)		
2010	129	-	-
2011	131	366	122.00
2012	106	328	109.33
2013	91	292	97.33
2014	95	270	90.00
2015	84	272	90.66
2016	93	-	-

22. To calculate the lump sum amount required to provide an annual scholarship of ₹ 3,000, we can use the formula for the present value of a perpetuity. A perpetuity is a series of payments that continues indefinitely.
    The formula for the present value of a perpetuity is:
    PV = $\frac{Payment}{Interest\ Rate}$
    In this case, the annual payment (scholarship) is ₹ 3,000, and the annual interest rate is 5%(0.05 as a decimal). Plug these values into the formula:
    PV = $\frac{₹\ 3000}{0.05}$
    PV = ₹ 60,000
    So, the lump sum amount required to provide an annual scholarship of ₹ 3,000, starting at the end of this year and continuing forever, is ₹ 60,000.

    OR

    Cost of laptop = ₹80,000
    Down payment = ₹20,000
    $\therefore$ Balance = ₹60,000 So, P = ₹60,000,
    i = $\frac{9}{12\times100}$
    = 0.0075
    n = 2 $\times$ 12 = 24
    EMI = $\frac{P\times i\times(1+i)^n}{(1+i)^n-1}$
    $=\frac{60{,}000\times0.0075\times(1.0075)^{24}}{(1.0075)^{24}-1}$
    $=\frac{60{,}000\times0.0075\times1.1964}{1.1964-1}$
    $=\frac{450\times1.1964}{0.1964}$
    = ₹2741.24
23. We have,
    $\int_0^1 x(1-x)^5\,dx$
    Expanding $(1 - x)^5$ by Binomial theorem, we get
    $\therefore (1 - x)^5 = 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5$
    $= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$
    So, $\int_0^1 x(1-x)^5\,dx = \int_0^1 x\,(1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5)dx$
    $=\left[\frac{x^2}{2}-\frac{5x^3}{3}+\frac{10x^4}{4}-\frac{10x^5}{5}+\frac{5x^6}{6}-\frac{x^7}{7}\right]_0^1$
    $=\frac{1}{2}-\frac{5}{3}+\frac{10}{4}-\frac{10}{5}+\frac{5}{6}-\frac{1}{7}$
    $=\frac{1}{42}$
    $\therefore \int_0^1 x(1-x)^5dx=\frac{1}{42}$
24. Given A is a square matrix of order 3 and |A| = 7, so $|A| \neq 0$ i.e. A is non-singular.
    $\therefore$ |adj A| = $|A|^2$ - 7^2 = 49.

OR

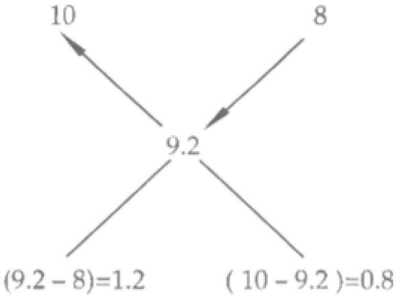
We know that A (adj A) = |A| I.

Given A (adj A) = $\begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$ $\Rightarrow$ |A| I = $\begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$

$\Rightarrow$ |A| $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix} \Rightarrow \begin{bmatrix}|A| & 0\\0 & |A|\end{bmatrix} = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$

$\Rightarrow$ |A| = 8.

25. The allegation grid is as given below:



Using allegation law, we obtain

$\frac{\text{Amount lent at 10\%}}{\text{Amount lent at 8\%}} = \frac{1.2}{0.8} = \frac{3}{2}$

Thus, ₹ 10,000 are divided in two parts in the ratio 3 : 2

$\therefore$ Amount lent at 10% = ₹ $\left(\frac{3}{3+2} \times 10,000\right)$ = ₹ 6,000

Amount lent at 8% = ₹ $\left(\frac{2}{3+2} \times 10,000\right)$ = ₹ 4,000

Section C

26. Let A be the amount of bacteria present at time t and A_0 be the initial amount of bacteria. Therefore,we have,

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = \lambda A$

$\frac{dA}{A} = \lambda dt$

Integrating both sides,we get,

log A = λt + c ...(i)

when t = 0, A = A_0

log A_0 = 0 + c

c = log A_0

Using equation (i),

log A = λt + log A_0

$\log\left(\frac{A}{A_0}\right) = \lambda t$ ..(ii)

Given, bacteria triples is 5 hours, so A = 3 A_0, when t = 5,

therefore from (ii),we have,

$\log\left(\frac{3A_0}{A_0}\right) = 5\lambda$

log 3 = 5λ

$\lambda = \frac{\log 3}{5}$

Put the value of λ in equation (ii), we have,

$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5}t$

Case I: let A_1 be the number of bacteria present in 10 hours, then, we have,

$\log\left(\frac{A_1}{A_0}\right) = \frac{\log 3}{5} \times 10$

$\log\left(\frac{A_1}{A_0}\right)$ = 2 log 3

$\log\left(\frac{A_1}{A_0}\right)$ = 2(1.0986)

$\log\left(\frac{A_1}{A_0}\right)$ = 2.1972

$A_1 = A_0e^{2.1972}$

$A_1 = A_09$

Hence, there will be 9 times the bacteria present is 10 hours.

Case II: Let t_1 be the time necessary for the bacteria to be 10 times, then, we have,

$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} \times t$

$\log\left(\frac{10A_0}{A_0}\right) = \frac{\log 3}{5} \times t_1$

5 log 10 = log 3 t_1

$5\frac{\log 10}{\log 3} = t_1$

Required time is $\frac{5\log 10}{\log 3}$hours.

OR

Let A be the quantity of bacteria present in culture at any time t and initial quantity of bacteria is A_0

$\frac{dA}{dA} \propto A$

$\frac{dA}{dt} = \lambda A$

$\frac{dA}{A} = \lambda dt$

$\int \frac{dA}{A} = \int \lambda dt$

log A = λt + c ...(i)

Initially, A = A_0, t = 0

log A_0 = 0 + c

log A_0 = c

Now equation (i) becomes,

log A = λt + log A_0

$\log\left(\frac{A}{A_0}\right) = \lambda t$ ...(ii)

Given A = 2 A_0 when t = 6 hours

$\log\left(\frac{A}{A_0}\right) = 6\lambda$

$\frac{\log 2}{6} = \lambda$

Now equation (ii) becomes,

$\log\left(\frac{A}{A_0}\right) = \frac{\log 2}{6}t$

Now, A = 8 A_0

so, $\log\left(\frac{8A_0}{A_0}\right) = \frac{\log 2}{6}t$

log 2^3 = $\frac{\log 2}{6}t$

3 log 2 = $\frac{\log 2}{6}t$

18 = t

Hence, Bacteria becomes 8 times in 18 hours.

27. We are given that

C = 30,000; n = 4; S = 4000

Annual depreciation = $\frac{C-S}{n}$

$= \frac{30000-4000}{4}$

= 6500

Depreciation schedule

Year	**Annual depreciation**	**Accumulated depreciation**	**Book Value**
0	0	0	30,000
1	6500	6500	23,500
2	6500	13000	17,000
3	6500	19,500	10,500
4	6500	26,000	4000

28. i. MC = 30 + 2x.
As MC = $\frac{dC}{dx}$,
C(x) = $\int$ (MC) dx = $\int$ (30 + 2x) dx
= 30x + x^2 + k, where k is constant of integration.
Given fixed cost (in ₹) = 120 i.e. when x = 0, C(x) = 120
$\Rightarrow$ 30 $\times$ 0 + 0^2 + k = 120 $\Rightarrow$ k = 120.
$\therefore$ C(x) = 120 + 30x + x^2
$\therefore$ Total cost of producing 100 units = 120 + 30 $\times$ 100 + 100^2 = 13120 (in ₹).
ii. Cost of increasing output from 100 to 200 = C(200) - C(100)
= (120 + 30 $\times$ 200 + 200^2) - 13120 = 33000 (in ₹).
Alternatively, we can obtain it as
$\int_{100}^{200}$ (MC)dx = $\int_{100}^{200}$ (30 + 2x)dx = $\left[30x + x^2\right]_{100}^{200}$
= (30 $\times$ 200 + 200^2) - (30 $\times$ 100 + 100^2) = 33000 (in ₹).

29. Let X be a random variable denoting the number of defective bolts in a sample of 4 bolts drawn from a bag containing 5 defective bolts and 20 good bolts. Then, X can take the values 0, 1, 2, 3 and 4.
Now,we have,
P(X = 0) = P(no defective bolts)
= $\frac{^{20}C_4}{^{25}C_4}$ = $\frac{4845}{12650}$ = $\frac{969}{2530}$
P(X = 1) = P(1 defective bolt)
= $\frac{^5C_1 \times ^{20}C_3}{^{25}C_4}$ = $\frac{5700}{12650}$ = $\frac{114}{253}$
P(X = 2) = P(2 defective bolts)
= $\frac{^5C_2 \times ^{20}C_2}{^{25}C_4}$ = $\frac{1900}{12650}$ = $\frac{38}{253}$
P(X = 3) = P(3 defective bolts)
= $\frac{^5C_3 \times ^{20}C_1}{^{25}C_4}$ = $\frac{200}{12650}$ = $\frac{4}{253}$
P(X = 4) = P(4 defective bolts)
= $\frac{^5C_4}{^{25}C_4}$ = $\frac{5}{12650}$ = $\frac{1}{2530}$
Thus, the probability distribution of X is as follows:

X	0	1	2	3	4
P(X)	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

OR

X	1	2	3	4	5	6
P(X)	k	4k	9k	8k	10k	12k

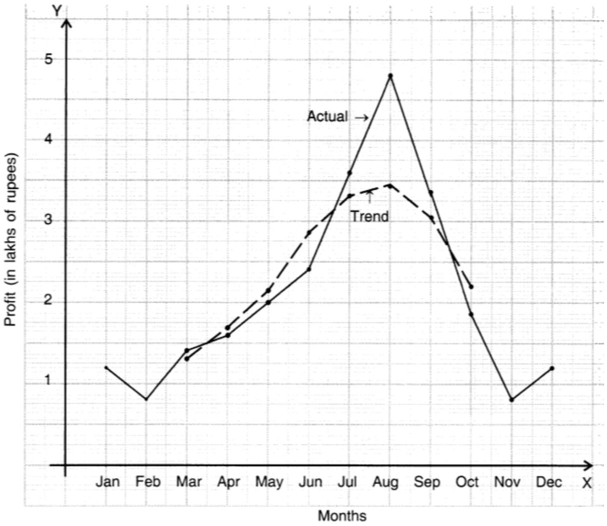
We know that, $\sum P_i = 1$
$\Rightarrow 44k = 1 \Rightarrow k = \frac{1}{44}$
$\therefore \sum XP(X)$ = k + 8k + 27k + 32k + 50k + 72k + 0
$= 190k = 190 \times \frac{1}{44} = \frac{95}{22}$
i. $P(X \geq 4)$ = P(X = 4) + P(X = 5) + P(X = 6)
= 8k + 10k + 12k = 30k $= 30 \cdot \frac{1}{44} = \frac{15}{22}$
ii. So, $E(X) = \sum XP(X) = \frac{95}{22}$ = 4.32
iii. Also, $E(X^2) = \sum X^2P(X)$ = k + 16k + 81k + 128k + 250k + 432k
= 908k = 908 $\times \frac{1}{44}$ $\left[\because k = \frac{1}{44}\right]$
= 20.636 = 20.64 (approx)
$\therefore E(3X^2) = 3E(X^2) = 3\times$ 20.64 = 61.9

30. Since we are to calculate four monthly moving averages, so the period is even, therefore, we have to calculate centred moving averages.

Calculation of 4-monthly centred moving averages:

Months	*Profit (in lakh of rupees)*	*four monthly moving total*	*four monthly moving average*	*four monthly centre moving average*
Jan	1.2			
Feb	0.8			
		5.0	1.25	
Mar	1.4			1.35
		5.8	1.45	
Apr	1.6			1.65
		7.4	1.85	
May	2.0			2.125
		9.6	2.4	
Jun	2.4			2.8
		12.8	3.2	
Jul	3.6			3.375
		14.2	3.55	
Aug	4.8			3.475
		13.6	3.4	
Sept	3.4			3.05
		10.8	2.7	
Oct	1.8			2.25
		7.2	1.8	
Nov	0.8			
Dec	1.2			

We get the following graph from the above data:



The dotted curve shows four monthly moving averages.

31. Given n = 25, $\bar{x}$ = 14, S = 4.32, μ_0 = 12

i. t = $\frac{\bar{x}-\mu_0}{\frac{S}{\sqrt{n}}} = \frac{14-12}{\frac{4.32}{\sqrt{25}}}$

= $\frac{10}{4.32}$ = 2.31

$\therefore$ t = 2.31

and degrees of freedom = 25 - 1 = 24

ii. $\because$ t = 2.31 > 0

So, p-value of 2.31 = Area under the t-distribution curve to the right of t

From the t-distribution table, we find that t = 2.31 lies between 2.064 and 2.492 for which area lies between 0.01 and 0.025, so p-value lies between 0.01 and 0.025

$\therefore$ 0.01 < p-value < 0.025

iii. Given, α = 0.05

Since p-value < 0.05

So, reject H_0.

iv. Reject H_0, if t $\leq t_\alpha$

t_α = $t_{0.05}$

From the table, $t_{0.05}$ = 1.711 with df = 24

$\because$ 2.31 > 1.711

$\therefore$ Reject H_0

Section D

32. The above information can be expressed in the form of the following table:

	P	**Q**	**Minimum requirement**
Vitamin A	3	4	8
Vitamin B	5	2	11
Price	₹ 60 per kg	₹ 80 per kg	

Let the mixture contain 'x' kgs and 'y' kgs of food P and Q respectively.

Cost of food P = 60x

Cost of food Q = 80y

Cost of mixture = 60x + 80y

Now,

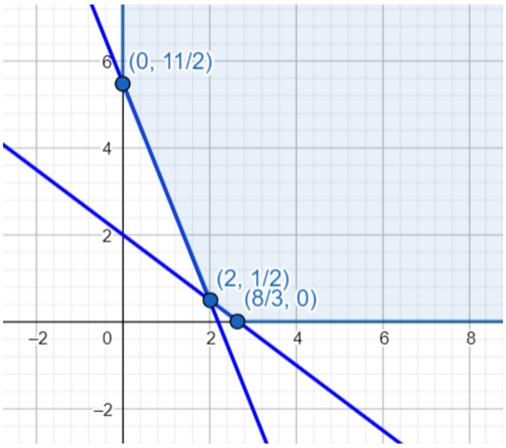
$\Rightarrow 3x + 4y \geq 8$

i.e. the minimum requirement of vitamin A from the mixture of P and Q is 8 units, each of which contains 3units and 4units respectively.

$\Rightarrow 5x + 2y \geq 11$

i.e. the minimum requirement of vitamin B from the mixture of P and Q is 11 units, each of which contains 5units and 2units respectively.

Hence, the mathematical formulation of the LPP is as follows:



The feasible region is Unbounded.

The corner points of the feasible region are as follows:

Point	**Value of Z = 60x + 80y**
A(0, 5.5)	440
B(2, 0.5)	160
$C\left(\frac{8}{3}, 0\right)$	160

Z is minimised on the line joining points B(2, 0.5) and C$\left(\frac{8}{3}, 0\right)$.

The minimum cost of mixture is ₹160

OR

Let the company manufacture x souvenirs of Type A and y souvenirs of Type B

Therefore, $x \geq 0$, $y \geq 0$

The given information can be compiled in a table as follows:

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on Type A souvenirs is 50 paisa and on Type B souvenirs is 60 paisa. Therefore, profit gained on x souvenirs of Type A and y

souvenirs of Type B is ₹0.50x and ₹0.60y respectively

Total Profit, Z = 0.5x + 0.6y

The mathematical formulation of the given problem is,

Max Z = 0.5x + 0.6y

Subject to constraints,

$5x + 8y \leq 200$

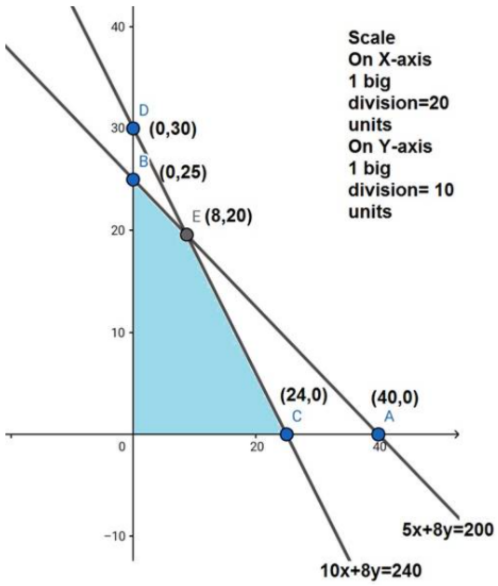
$10x + 8y \leq 240$

$x \geq 0, y \geq 0$

Region $5x + 8y \leq 200$: line 5x + 8y = 200 meets axes at A(40, 0), B(0, 25) respectively. Region containing origin represents the solution of the inequation $5x + 8y \leq 200$ as (0, 0) satisfies $5x + 8y \leq 200$

Region $10x + 8y \leq 240$: line 10x + 8y = 240 meets axes at C(24, 0), D(0, 30) respectively. Region containing origin represents the solution of the inequation $10x + 8y \leq 240$ as (0, 0) satisfies $10x + 8y \leq 240$

Region $x, y \geq 0$: it represents first quadrant



The corner points of the feasible region are O(0, 0), B(0, 25), E(8, 20), C(24, 0)

The values of Z at these corner points are as follows:

Corner Points	Z = 0.5x + 0.6y
O	0
B	15
E	16
C	12

The maximum value of Z is attained at E(8, 20)

Thus, 8 souvenirs of Type A and 20 souvenirs of Type B should be produced each day to get the maximum profit of ₹16.

33. The given inequalities are

$3y - 2x < 4$ ...(i)

$x + 3y > 3$ ...(ii)

and $x + y \leq 5$ ...(iii)

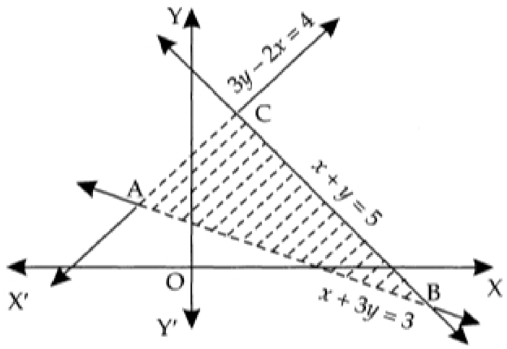
To draw the graph of $3y - 2x < 4$:

We draw the straight line 3y - 2x = 4 which passes through the points (-2, 0) and $\left(0, \frac{4}{3}\right)$. The line divides the plane into two parts.

Further, as O(0, 0) satisfies the inequality $3y - 2x < 4$.

($\because 3 \times 0 - 2 \times 0 = 0 < 4$), therefore, the graph consists of that part of the plane divided by the line 3y - 2x = 4 which contains the origin.

Similarly, draw the graphs of other two inequalities $x + 3y > 3$ and $x + y \leq 5$.



Shade the common part of the graphs of all the three given inequalities (i), (ii) and (iii).

The solution set consists of all the points in the shaded part of the coordinate plane shown in fig. The points on the line segment BC are included in the solution.

34. We have,

p = probability that a blade is defective = $\frac{1}{500}$, n = 10

$\therefore$ m = np = $\frac{1}{500} \times 10 = \frac{1}{50}$ = 0.02

Let X denote the number of defective blades in a packet of 10 blades

Then,

P(X = r) = $\frac{m^r e^{-m}}{r!}$

$= \frac{(0.02)^r e^{-0.02}}{r!}$, r = 0, 1, 2, ...

i. We have, P(X = 0) = $e^{-0.02}$ = 0.98019

$\therefore$ Number of packets containing no defective blade in a consignment of 10,000 packets

= 10,000 $\times$ P (X = 0)

= 10,000 $\times$ 0.98019 = 9801.9 = 9802, approximately

ii. We have,

P(X = 1) = 0.02 $\times$ $e^{-0.02}$ = 0.02 $\times$ 0.98019 = 0.0196038

$\therefore$ Number of packets containing one defective blade in a consignment of 10,000 packets

= 10000 $\times$ P(X = 1)

= 10000 $\times$ 0.0196038

= 196.038

= 196, approximately

iii. We have,

P(X = 2) = $\frac{(0.02)^2 e^{-0.02}}{2!} = \frac{0.0004 \times 0.98019}{2}$ = 0.000196

$\therefore$ Required number of packets = 10000 $\times$ P(X = 2)

= 10000 $\times$ 0.000196 = 1.96 = 2, approximately

iv. We have,

P(X = 3) = $\frac{(0.02)^3 e^{-0.002}}{3!}$ = 0.0000013

So, required number of packets = 10000 $\times$ P(X = 3) = 0.013 = 0, approximately.

OR

Let A_i(i = 1, 2) denote the event of getting an ace in ith draw. Since the cards are drawn with replacement. Therefore,

P(A_i) = Probability of getting an ace in i^{th} draw = $\frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

and $P\left(\overline{A_i}\right)$ = 1 - P(A_i) = $1 - \frac{1}{13} = \frac{12}{13}$, i = 1, 2

Let X denote the number of aces in two draws. Then, X can take values 0, 1, 2.

Now, P(X = 0) = Probability of getting no ace in two draws

$\Rightarrow$ P(X = 0) = $P\left(\overline{A_1} \cap \overline{A_2}\right) = P\left(\overline{A_1}\right) P\left(\overline{A_2}\right) = \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

$\Rightarrow$ P(X = 1) = Probability of getting an ace in either of the two draws

$\Rightarrow$ P(X = 1) = $P\left(\left(A_1 \cap \overline{A_2}\right) \cup \left(\overline{A_1} \cap A_2\right)\right)$

$\Rightarrow$ P (X = 1) = $P\left(A_1 \cap \overline{A_2}\right) + P\left(\overline{A_1} \cap A_2\right)$

$\Rightarrow$ P (X = 1) = $P(A_1)P\left(\overline{A_2}\right)+P\left(\overline{A_1}\right)P(A_2)=\frac{1}{13}\times\frac{1}{13}=\frac{1}{169}$

Thus, the probability distribution of X is given by:

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$\therefore \Sigma p_i x_i = 0\times\frac{144}{169}+1\times\frac{24}{169}+2\times\frac{1}{169}=\frac{26}{169}$

and, $\Sigma p_i {x_i}^2 = 0\times\frac{144}{169}+1\times\frac{24}{169}+4\times\frac{1}{169}=\frac{28}{169}$

Hence, $\bar{X}$ = Mean = $\Sigma p_i x_i = \frac{26}{169}=\frac{2}{13}$

and, Var(X) = $\sum p_i x_i^2-(\Sigma p_i x_i)^2=\frac{28}{169}-\left(\frac{2}{13}\right)^2=\frac{24}{169}$

$\therefore$ S.D. = $\sqrt{\operatorname{Var}(X)}=\sqrt{\frac{24}{169}}=\frac{2\sqrt{6}}{13}$

Hence, Mean = $\frac{2}{13}$ and S.D. = $\frac{2\sqrt{6}}{13}$

35. It is given that C = ₹50,000 and r = 0.08

The depreciation charge for the 8th year is obtained by subtracting the book value at the end of the 8th year from the book value at the end of the 7th year.

The book value at the end of the 7th year.

= C(1 - r)^7 = 50,000 (1 - 0.08)^7 = 50,000 (0.92)^7

= 50,000 (0.5578466)

= ₹27892.33

The book value at the end of the 8th year

= C(1 - r)^8 = 50,000 (1 - 0.08)^8

= 50000 (0.92)^8

= 50,000 (0.5132188)

= ₹25660.94

Hence depreciation charge for the 8th year

= ₹ 27892.33 - ₹ 25660.94

= ₹ 2231.39

The scrap value of the machine is given by (book value at the end of 10th year)

S = C(1 - r)^{10} = 50,000 (1 - 0.08)^{10}

= 50,000 (0.92)^{10}

= 50,000 (0.4343884)

= ₹21719.42

Section E

36. **Read the text carefully and answer the questions:**

Rohit's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 ft of wire fencing.

(i) To create a garden using 200 ft fencing, we need to maximise its area.

(ii) Required relation is given by 2x + y = 200

(iii) Area of the garden as a function of x can be represented as

A(x) = x · y - x(200 - 2x) = 200x - 2x^2

OR

$A(x) = 200x - 2x^2$

$\Rightarrow A'(x) = 200 - 4x$

For the area to be maximum $A'(x) = 0$

$\Rightarrow 200 - 4x = 0 \Rightarrow x = 50$ ft

37. **Read the text carefully and answer the questions:**

EMI or equated monthly installment, as the name suggests, is one part of the equally divided monthly outgoes to clear off an outstanding loan within a stipulated time frame. The EMI is dependent on multiple factors, such as:

- Principal borrowed
- Rate of interest
- Tenure of the loan
- Monthly/annual resting period

For a fixed interest rate loan, the EMI remains fixed for the entire tenure of the loan, provided there is no default or part-payment in between. The EMI is used to pay off both the principal and interest components of an outstanding loan.

Example:

A person amortizes a loan of ₹1500000 for renovation of his house by 8 years mortgage at the rate of 12% p.a. compounded monthly.

(Given $(1.01)^{96} = 2.5993$, $(1.01)^{57} = 1.7633$)

(i) ₹ 24379.10

(ii) ₹ 1055326.20

(iii) ₹ 10553.26

OR

₹ 13825.84

38. Let x, y and z ₹ be the investments at the rates of interest of 6%, 7% and 8% per annum respectively. Then,

Total investment = ₹5000

$\Rightarrow x + y + z = 5000$

Now, Income from first investment of ₹ x = ₹$\frac{6x}{100}$

Income from second investment of ₹ y = ₹$\frac{7y}{100}$

Income from third investment of ₹ z = ₹$\frac{8z}{100}$

$\therefore$ Total annual income = ₹$\left(\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}\right)$

$\Rightarrow \frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100} = 358$ [$\because$ Total annual income = ₹358]

It is given that the combined income from the first two investments is ₹70 more than the income from the third

$\therefore \frac{6x}{100}+\frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow 6x + 7y - 8z = 7000$

Thus, we obtain the following system of simultaneous linear equations:

$x + y + z = 5000$

$6x + 7y + 8z = 35800$

$6x + 7y - 8z = 7000$

This system of equations can be written in matrix form as follows:

$$\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 6 & 7 & -8\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}5000\\ 35800\\ 7000\end{bmatrix}$$

or, AX = B, where A = $\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 6 & 7 & -8\end{bmatrix}$, X = $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and B = $\begin{bmatrix}5000\\ 35800\\ 7000\end{bmatrix}$

Now, |A| = $\begin{vmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 6 & 7 & -8\end{vmatrix}$ = 1 (-56 - 56) - (-48 - 48) + (42 - 42) = $-16 \neq 0$

So, A^{-1} exists and the solution of the given system of equations is given by $X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in $A = [a_{ij}]$. Then,

$C_{11} = -112$, $C_{12} = 96$, $C_{13} = 0$, $C_{21} = 15$, $C_{22} = -14$,

$C_{23} = -1$, $C_{31} = 1$, $C_{32} = -2$ and $C_{33} = 1$

$\therefore$ adj A = $\begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

So, $A^{-1} = \frac{1}{|A|}$ (adj A) = $-\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Hence, the solution is given by

$X = A^{-1}B = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -560000 & +537000 & +7000 \\ 480000 & -501200 & -14000 \\ 0 & -35800 & +7000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$

$\Rightarrow$ x = 1000, y = 2200 and z = 1800

Hence, three investments are of ₹1000, ₹2200 and ₹1800 respectively.

OR

i. The input-output table is as given below:

Output		Input	
		Industry I	Industry II
	Industry I	0.20	0.40
	Industry II	0.50	0.30

Let A be the input-output coefficient matrix or, the technology matrix. Then,

$A = \begin{bmatrix} 0.20 & 0.40 \\ 0.50 & 0.30 \end{bmatrix}$

ii. Let ₹x_1 crores and ₹x_2 crores be the gross outputs of industries I and II respectively to meet the demand of ₹180 crores and ₹270 crores respectively. Then, the gross output matrix X = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ satisfies the equation

(I - A) X = D, where D = $\begin{bmatrix} 180 \\ 270 \end{bmatrix}$ is the demand matrix

Now, A = $\begin{bmatrix} 0.20 & 0.40 \\ 0.50 & 0.30 \end{bmatrix}$

$\Rightarrow$ I - A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.20 & 0.40 \\ 0.50 & 0.30 \end{bmatrix} = \begin{bmatrix} 0.80 & -0.40 \\ -0.50 & 0.70 \end{bmatrix}$

$\Rightarrow$ |I - A| = 0.56 - 0.20 = 0.36 $\neq 0$

We find that |I - A| > 0 and diagonal elements of A are all less than 1. So, Hawkins-Simon conditions are satisfied and hence the system is viable.

adj (I - A) = $\begin{bmatrix} 0.70 & 0.40 \\ 0.50 & 0.80 \end{bmatrix}$

$\therefore$ $(I - A)^1 = \frac{1}{|1-A|}$ adj (I - A)

$\Rightarrow (I - A)^{-1} = \frac{1}{0.36}\begin{bmatrix} 0.70 & 0.40 \\ 0.50 & 0.80 \end{bmatrix}$

Now, (I - A) X = D

$\Rightarrow X = (I - A)^{-1}D$

$\Rightarrow X = \frac{1}{0.36}\begin{bmatrix} 0.70 & 0.40 \\ 0.50 & 0.80 \end{bmatrix}\begin{bmatrix} 180 \\ 270 \end{bmatrix} = \frac{1}{0.36}\begin{bmatrix} 126 + 108 \\ 90 + 216 \end{bmatrix} = \begin{bmatrix} 650 \\ 850 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 650 \\ 850 \end{bmatrix}$

$\Rightarrow x_1 = 650, x_2 = 850$

Hence, the gross outputs of industries I_1 and II_2 must be ₹650 crores and ₹850 crores respectively.

iii. Row sum has no economic meaning First column sum gives us the cost of intermediate inputs to produce a rupee worth of output by industry I. Thus, to produce an output of worth ₹1 the industry I require the input of worth ₹(0.20 + 0.50) = ₹0.70 and the balance of ₹0.30 denotes the value of addition per rupee. Likewise, second column sum gives us the cost of intermediate inputs to produce a rupee worth of output by industry II. Thus, to produce an output of worth ₹1 the industry II requires input of worth ₹(0.40 + 0.30) = ₹0.70 and the balance of ₹0.30 denotes the value addition per rupee.