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Pre Board-II SESSION-2023-24

Class -XII

MATHEMATICS

SET-I

Time Allowed 3 Hours **Maximum Marks-80**

General Instructions:

1. This Question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 marks each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 long Answer (LA) type questions of 5 marks each.
6. Section E has 3 Source based/ passage based/integrated units of assessment of 4 marks each with sub-parts.

Section-A

(Multiple Choice Questions)Each question carries 1 mark

1. Let A be square matrix of order 3 such that adj (4A) = k (adj A) then the value of k is

(a) 4 (b) 8 (c) 12 (d) 16

2. If $|A| = 3 \, and \, A^{-1} = \begin{bmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{bmatrix}$ *then write the AdjA.*

(a) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$

3. *If a function f by f(x)* $= \begin{cases} \frac{k\cos x}{\pi - 2x}, & if x \neq \frac{\pi}{2} \\ 3, & if x = \frac{\pi}{2} \end{cases}$ *continuous at x=* $\frac{\pi}{2}$

then value of k is.

(a) 2 (b) 3 (c) 6 (d)-6

4. If A= $\begin{vmatrix} x & -3 & 1 \\ 2 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ and xyz=7, x+y-6z=11 then A.adj A is equal to.

(a) $-5I$ (b) 5I (c) 4I (d) $-4I$

5. The vectors 2j+k and 3i-j+4k represent the two series AB and AC respectively of ΔABC the lenght of the median through A is

(a) $\frac{\sqrt{35}}{2}$ (b) $\sqrt{48}$ (c) $\sqrt{18}$ (d) $\sqrt{\frac{35}{2}}$

6. if p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$ then the value of 2p-3q is

(a) 7 (b) -7 (c) 3 (d) -3

7. The corner points of the feasible region determined by the system of liner constants are (0,10) (5. 15 (15,15) (0,20).Let $z = px + qy$, where p, q>0 , conditions on p and q so that the maximum of z occurs at be the points (15, 15) or (0, 20) is.

(a) p=q (b) p=2q (c) q=2q (d) q=3p

8. For which value of p, is $(\hat{i}+\hat{j}+\hat{k})p$ a unit vector?

(a) $\pm\frac{1}{\sqrt{3}}$ (b) $\pm\sqrt{3}$ (c) ± 1 (d) $\pm \frac{1}{3}$

9. $\int_0^{\frac{\pi}{2}} \log(\tan x)dx$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2}\log 2$ (c) 0 (d) $\frac{\pi}{8}\log 2$

10. value of determinant $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$ is

(a) $\frac{15}{2}$ (b) 9 (c) $\frac{3}{2}$ (d) $\frac{13}{2}$

11. The objective funtion of an LPP is

(a) Constant (c) a linear function to be optimized

(b) An inequality (d) a quadratic expression

12. Consider points A, B, C and D with position vectors $7\hat{i} - 4j + 7k, \hat{i} - 6\hat{j} + 10\hat{k}, -\hat{i} - 3j + 4k$ and 5i-j+k respecting. Then ABCD is a

(a) Square (b) Rhombus

(b) Rectangle (d) Parallelogram but not a rhombus

13. The matrix A = $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

(a) Diagonal Matrix (b) Symmetric Matrix

(b) Skew Symmetric (d) Scalar Matrix

14. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and spade card is.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{3}{4}$

15. Which of the following differential equations has $y = c_1 e^{x} + c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

16. The value of P for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9k$ and $\hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors is-

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{43}$

17. The function $f(x) = \tan x - x, x \in R$ is

(a) Always increasing (c) always decreasing

(b) Neither increasing nor decreasing (d) none of these

18. A line makes angle α, β, γ with x axis , y axis z axis respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to .

(a) 2 (b) 1 (c) -2 (d) -1

ASSERTION REASON BASED QUESTION-

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

a) Both (A) and (R) are true and (R) is the correct explanation of (A).

b) Both (A) and (R) are true and (R) is not the correct explanation of (A).

c) (A) is true but (R) is false.

d) A) is false but (R) is true.

19. Let f: $[-1,3] \rightarrow R$ be defined as f(x) $4x^3$-12x

Assertion (A)- The image of [-1,3] under f is not the interval [f(-1), f(3)].

Reason (R) - f is not an injective function.

20. Assertion- The function $\begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x=0

Reason- The function $\begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ is differentiable at x=0.

Section-B

(This section comprises of very short answer type-question (VSA) of 2 marks each)

21. Find the value $\tan^{-1}$ [2 sin ($2\cos^{-1}\frac{\sqrt{3}}{2}$)]

OR

Find the domain of $\cos^{-1}$ (2x-3) ?

22. Find the interval on which x^4-4x ;x∈R is strictly increasing.

23. The total revenue in rupees received from the sale of x national flags on the occasion of REPUBLIC DAY' is given by R(x) = $13x^2$ +26x+15.Find the marginal revenue when x=7.

OR

At what point the slope the curve $y = -x^3 + 3x^2 + 9x - 27$ is maximum? Also find maximum slope.

24. Evaluate $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$.

25. Find the least value of the function $f(x) = x^3 + 18x^2 + 96x$ in the interval [0,9] is.

Section-C

(This section comprises of short answer type-question (SA) of 3 marks each)

26. Evaluate $\int \frac{x}{x^4 - x^2 + 1} dx$.

27. Let X denote the number of colleges where you will apply after your results and P(X=x) denote your probability of getting admission in x number of colleges. It is given that

$$P(X=x) = \begin{cases} kx & \text{if } x = 0 \text{ or } 1 \\ 2kx & \text{if } x = 2 \\ k(5-x) & \text{if } x = 3 \text{ or } 4 \\ 0 & \text{if } x > 4 \end{cases}$$

Where k is a positive constant. Find the value of k. Also find the probability that you will after admission in :

(i) exactly one college ,

(ii) at most 2 college,

(iii) at least 2 colleges.

28. $\int \frac{1}{x(x^n + 1)} dx$.

OR

$$\int_o^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$$

29. Solve the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that y=1 when x=1 .

OR

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$$

30. Solve the Linear programming problem graphically-

Maximize z=3x+9y

Subject to the constrains

x+3y≤ 60, x+y≥10, x ≤ y, x,y≥0

OR

Solve the Linear programming problem graphically-

Manimize Z=3x+4y+370

Subject to the constrains

$x+y\leq 60, \quad x\leq 40, \quad y\leq 40, \quad x+y\geq 10, \quad y\geq 0$

31. If siny=x sin(a+y), then prove that

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

Section-D

(This section comprises of long answer type-question (LA) of 5 marks each)

32. Find the area enclosed by the parabola $4y=3x^2$ and the line 2y=3x+12.Using method of integration.

33. Show that the relation R defined by

(a,b) R (c,d) $\Rightarrow$ a+d=b+c

On A×A where A={1,2,3,……..10} is an equivalence relation.Hence write the equivalence class[(3,4)];a,b,c,dϵR.

OR

Let $f: R-\left\{\frac{3}{5}\right\}\to R-\left\{\frac{3}{5}\right\}$ be defined by $f(x)=\frac{3x+2}{5x-3}$ .So that function is bijective.

34. If A= $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ find A^{-1} and hence solve the system if equation

x-2y=10, 2x+y+3z=, -2y+z=7.

35. Find the length and the foot of the perpendicular drawn from the point (2,-1,5) on the line

$$\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}.$$

OR

Find the shortest between the lines

$$x+1=2y=-12z \quad \text{and} \quad x=y+2=6z-6.$$

Section-E

(This section comprises of 3 case-study/passage based questions of 4 marks each with sub parts.The first two case study questions have three sub parts (i) (ii) (iii) of marks 1,1,2 respectively.The third case study question has two sub parts of 2 marks each.)

36. In answering s multiple choice test for class XII,a student either knows or guesses or copies the answer to a multiple choice question with four choices.

The probability that he makes a guess is $1/3$ and the probability that he copies the answer is $1/6$. The probability that his answer is correct given that he copied is $1/8$.

Let E_1, E_2, E_3 be the events that the students guesses, copies or knows the answer respectively and a even that the students answer correctly.

i) What is the probability that students knows the answer?

ii) What is the probability that he answers correctly given that he knew the answer?

iii What is the probability that he knew the answer to the question, given that he answered it correctly?

OR

(iii) Find $\sum_{k=1}^{3} p(E_k / A)$

37. In solar panel have to be installed carefully so that the tilt of the roof and the direction to the sun, produce the largest possible electrical power in the solar panels.
A surveyor uese his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted.In the picture ,points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) P_2 (21,8,4) P_3 (21,16,10) and P_4 (6,16,10).

i. What are the components to the two edge vectors defined by $\vec{A} = P.V$ of $P_2 - P.V$ of P_1 and $\vec{B} = P.V$ of $P_4 - P.V$ of P_1 ?

ii. What are the components of vector $\vec{N}$ perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

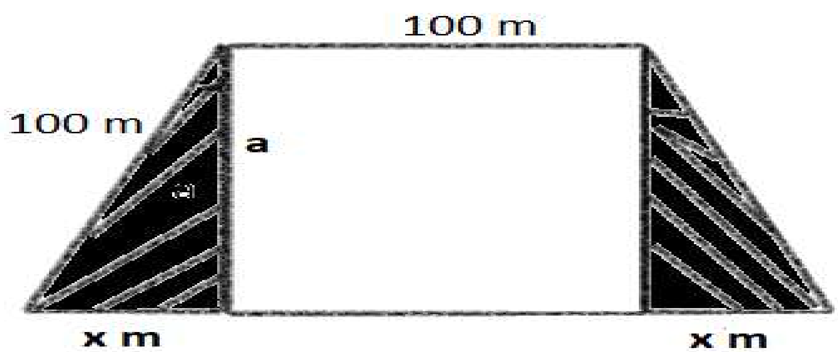
iii. What is the magnitude of $\vec{N}$ and its units?The sun is located along the unit vector

$\vec{S} = 2\frac{\sqrt{3}}{7}\hat{i} + \frac{6}{7}\hat{j} + \frac{1}{7}\hat{k}$. If the flow of solar energy is given by the vector $\vec{F} = 910, \vec{S}$ in units of watts/meter^2?, what is the dot product of $\vec{F}$ with $\vec{N}$, and the units for this quantity?

OR

(iii) What is the angle between vectors $\vec{N}$ and $\vec{S}$?

Q 38.Read the following passage and answer the question given below.



A resort at the top of a hill decided to, make a relaxation rectangular field with right triangular fields of equal shape and size , for planning flowers, attached to both sides as shown. They also are thinking of maximizing the total area.The length of rectangle and hypotenuse of right triangular fields are 100 m each.

(i) Find the perimeter of total enclosed area and total covered area of trapezium.

(ii) Find the value of x for which total area is maximum.