

Pre-Board Examination

Class – XII

Session: 2023-24

Applied Mathematics (Code – 241)

Time Allowed: 3 hrs.

M.M – 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) questions of 2 marks each.
4. Section C has 6 Short Answer (SA) questions of 3 marks each.
5. Section D has 4 Long Answer (LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternative in all such questions.
8. If you wish to re-answer any question then, cancel the previous attempt first.
9. Attempt the questions in the order of best known to least known.

SECTION – A

(All questions are compulsory. No internal choice is provided in this section)

1. If $a > b$ and $c < 0$, then
 - (a) $ac > bc$
 - (b) $\frac{a}{c} < \frac{b}{c}$
 - (c) $ac \geq bc$
 - (d) $\frac{a}{c} \geq \frac{b}{c}$
2. $-41 \pmod{7}$ is equal to
 - (a) -6
 - (b) 5
 - (c) 1
 - (d) -1
3. If A is a 3×4 matrix and B is a matrix such that $A'B$ and $B'A$ are both defined, then the order of matrix B is
 - (a) 3×4
 - (b) 4×3
 - (c) 4×4
 - (d) 3×3
4. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|\text{adj } A|$ is
 - (a) a^2
 - (b) a^6
 - (c) a^9
 - (d) a^{24}
5. The intervals in which the function $f(x) = x^2 - 4x + 6$ is increasing is
 - (a) $(2, \infty)$
 - (b) $(-\infty, 2) \cup (2, \infty)$
 - (c) $(-\infty, 2)$
 - (d) $(-\infty, 2] \cup [2, \infty)$
6. The value of $\int \frac{dx}{x+\sqrt{x}}$ is
 - (a) $\log|\sqrt{x}| + c$
 - (b) $\log|1 + \sqrt{x}| + c$
 - (c) $2 \log|1 + \sqrt{x}| + c$
 - (d) $4 \log|1 + \sqrt{x}| + c$
7. The order of the differential equation corresponding to the family of curves $y = Ae^{3x} + Be^{-3x}$ is
 - (a) 4
 - (b) 3
 - (c) 2
 - (d) 1

8. The side of an equilateral triangle is increasing at the rate of 2 cm/sec. The rate at which the area is increasing when the side is 10 cm is
- (a) $5\sqrt{3} \text{ cm}^2/\text{sec}$ (b) $5\sqrt{3} \text{ cm}^3/\text{sec}$
(c) $20\sqrt{3} \text{ cm}^3/\text{sec}$ (d) $10\sqrt{3} \text{ cm}^2/\text{sec}$
9. If X is a Poisson variate such that $3P(X=2) = 2P(X=1)$ then the mean of the distribution is equal to
- (a) $\frac{4}{3}$ (b) $\frac{3}{4}$
(c) $-\frac{4}{3}$ (d) $-\frac{3}{4}$
10. For predicting the straight-line trend in the sales of scooters (in thousands) on the basis of 6 consecutive years data, the company makes use of 4-year moving averages method. If the sales of scooters for respective years are a, b, c, d, e and f respectively, then which of the following average will not be computed?
- (a) $\frac{a+b+c+d}{4}$ (b) $\frac{b+c+d+e}{4}$
(c) $\frac{a+c+d+e}{4}$ (d) $\frac{c+d+e+f}{4}$
11. t-test for two independent groups (when variances are equal) is
- (a) $t = \frac{\bar{x}_1 + \bar{x}_2}{S_p \sqrt{\frac{1}{n_1} - \frac{1}{n_2}}}$ (b) $t = \frac{\bar{x}_1 + \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} - \frac{s_2^2}{n_2}}}$
(c) $t = \frac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ (d) $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
12. Standard deviation of a sample from a population is called a
- (a) standard error (b) parameter
(c) statistic (d) central limit
13. If we reject the null hypothesis, we might be making
- (a) Type-I error (b) Type-II error
(c) A correct decision (d) A wrong decision
14. Time series analysis helps to
- (a) understand the behaviour of variables in the past
(b) predict the future behaviour of a variable
(c) plan future operations
(d) all of the above
15. At what rate converted semi-annually will be the present value of a perpetuity of ₹ 450 payable at the end of each 6 months be ₹ 20,000?
- (a) 4 % (b) 4.5 %
(c) 5 % (d) 5.5 %
16. Rohan Invested ₹ 300000 in a fund for two years. At the end of two years, the investment was worth ₹ 327000. Rohan's rate of return is
- (a) 6% (b) 7%
(c) 8% (d) 9%
17. A machine costing ₹ 50,000 has a useful life of 4 years. The estimated scrap value is ₹ 10,000, then the annual depreciation (in ₹) is
- (a) 20,000 (b) 10,000
(c) 5,000 (d) 2,500
18. The maximum value of the function $z = 7x + 5y$, subject to the constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0$ is
- (a) 21 (b) 10
(c) 31 (d) 37

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

- (a) Both A and R are true and R is the correct explanation of the assertion
- (b) Both A and R are true but R is not the correct explanation of the assertion
- (c) A is true, but R is false
- (d) A is false, but R is true

19. **Assertion(A):** The standard normal variable is given by $Z = \frac{x-\mu}{\sigma}$

Reasoning (R): The mean and standard deviation of a standard normal variate are 1 and 0 respectively.

20. **Assertion(A):** For the curve $x^3 + y^3 = 6xy$, the slope of the tangent at (3,3) is 2

Reasoning (R): $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$ gives the slope of the tangent to the curve $y = f(x)$ at (x_1, y_1)

SECTION – B

(All questions are compulsory. In case of internal choice, attempt any one question only)

21. (a) In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.

OR

(b) How much water must be added to 15 litres of milk, if the cost of $\frac{2}{3}$ litres of milk is ₹ 30 so as to have a mixture worth ₹ 25 per litre?

22. A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertising campaign, the mean sales per week per shop was 140 dozens. After a campaign a sample of 26 shops was taken and mean sales was found to be 147 dozens with standard deviation 16. Can you consider the advertisement effective? (Given $t_{25}(0.05) = 2.06$)

23. (a) Form the differential equation of the system of circles touching y-axis at the origin.

OR

(b) Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 5x + 6y$ given that $y = 0$, when $x = 0$

24. Suppose a person invested ₹ 15,000 in a mutual fund and the value of the investment at the time of redemption was ₹ 25,000. If CAGR for this investment is 8.88%. Calculate the number of years for which he invested the amount. (Given $\log(1.666) = 0.2216$ and $\log(1.0888) = 0.0369$)

25. Find the effective rate that is equivalent to a nominal rate of 8% compounded quarterly. (Given that $(1.02)^4 = 1.0824$)

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

26. Three pipes A, B and C are installed to fill a tank. Pipes A and B opened together can fill the tank in the same time in which C can alone fill the tank. If pipe B can fill the tank in 15 minutes faster than pipe A and 5 minutes slower than pipe C, then find the time required by pipe A to fill the tank alone.

27. (a) Express the matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

OR

(b) Using Cramer's rule, solve the following system of linear equations:

$$\frac{2}{x} + \frac{3}{y} = 2$$

$$\frac{5}{x} + \frac{8}{y} = \frac{31}{6}$$

28. (a) Find the consumer's surplus under pure competition for demand function $p_d = \frac{8}{x+1} - 2$ and the supply function $p_s = \frac{1}{2}(x + 3)$, where p is price and x is the quantity.

OR

(c) Evaluate: $\int \frac{x^3}{(x-1)(x-2)} dx$

29. A machine costs a company ₹ 40,000 and its effective life is estimated to be 10 years. A sinking fund is created for replacing the machine by a new model at the end of its life time when its scrap realises a sum of ₹ 5000 only. Calculate what amount should be retained out of profits at the end of each year to accumulate at 5% per annum with compound interest for 10 years to replace the machinery by a new one which is estimated to be 25% more than the present one. [Use $(1.05)^{10} = 1.62889$]
30. Solve the following linear programming problem graphically:
 Minimize $Z = 18x + 10y$
 Subject to $4x + 2y \geq 20$,
 $2x + 3y \geq 30$
 $x \geq 0, y \geq 0$
31. Assume that the probability that a bomb dropped from an aeroplane will strike a certain target is $\frac{1}{5}$. If 6 bombs are dropped, find the probability that
 (i) Exactly 2 will strike the target
 (ii) At least 2 will strike the target
 [Given $e^{-1.2} = 0.3012$]

SECTION – D

(All questions are compulsory. In case of internal choice, attempt any one question only)

32. A company produces three products every day. Their production on a certain day is 45 tons. It is found that the production of third product exceeds the production of first product by 8 tons while the total production of first and third product is twice the production of second product. Determine the production level of each product using matrix method.
33. (a) If X follows a binomial distribution with mean 4 and variance 2, find $P(|X - 4| \leq 2)$
- OR**
- (b) Two numbers are selected at random (without replacement) from first 6 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X . Also find the mean of the distribution
34. (a) A tour operator charges ₹ 136 per passenger with a discount of ₹ 4 for each 10 passengers in excess of 100. Find the number of passengers that will maximise the amount of money the tour operator receives. Also find the maximum profit.
- OR**
- (b) A company produces a certain commodity with 2400 fixed cost. The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of 8 per unit. Find the following
 (i) Cost Function. (ii) Revenue Function
 (iii) Breakeven Point (iv) Profit Function
35. Mahesh purchased a house from a company for ₹ 7,00,000 and made a down payment for ₹ 1,50,000. He repays the balance in 25 years by monthly instalments at 9% compounded monthly. Find
 (i) The monthly instalment
 (ii) The total interest paid
 [Given $(1.0075)^{-300} = 0.10629$]

SECTION – E
(Case-Based Questions)
(All questions are compulsory)

36. CASE STUDY – I

The following table gives the sales (in crore ₹) for years 2017 to 2021

Year	2017	2018	2019	2020	2021
Sales	5	8	12	20	25

Based on the above information, answer the following questions:

- (i) Fit a straight-line trend to the above data by the method of least squares. 3
(ii) (a) Find the difference in the trend values for the year 2024 and 2021 1

OR

- (ii) (b) Find the difference in the trend values for the year 2025 and 2020 1

37. CASE STUDY – II

A man can row against the current, three fourth of a kilometer in 15 minutes and returns the distance in 10 minutes



Based on the above information, answer the following questions. Show steps to support your answers.

- (i) Find the ratio of the speed in downstream to the speed in upstream. 1
(ii) Find the speed of the current and the speed of man in still water. 2
(iii) If a man can row a distance of 3 km against the stream, then find the time taken by the man to cover the distance. 1

38. CASE STUDY – III

A dietician has to develop a special diet using two foods P and Q. Each packet of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol.

Based on the above information, answer the following questions:

- (i) Formulate the problem as a linear programming problem 1
(ii) Represent the above LPP graphically and Shade the feasible region 2
(iii) (a) How many packets of each food should be used to minimise the amount of vitamin A in the diet? 1

OR

- (iii) (b) Find the minimum amount of Vitamin A 1