Solution
PRE-BORAD-1 (2023-24)
Class 12 - Mathematics
Section A

1.
(c) nk
Explanation: $\because$ A = $[a_{ij}]_{n\times n}$
Trace of A, i.e., tr (A) = $\sum a_{ij}^{n} i$ = 1 = a_{11} + a_{22} + ... + a_{nm}
= k + k + k + k + k + ...(n times)
= k(n)
= nk

2.
(b) 80
Explanation: f(z) = $\begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix}$ = 5(z^2 - 2) - 2(3z - 16) +1(3 - 8z)
= $5z^2$ - 10 - 6z + 32 + 3 - 8z = $5z^2$ - 14z + 25
f(5) = 5 $\times$ 5^2 - 14 $\times$ 5 + 25 = 125 - 70 + 25
= 150 - 70 = 80

3.
(b) adj A
Explanation: $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
|A| = $\cos^2\theta$ - ($-\sin^2\theta$)
= $\cos^2\theta$ + ($\sin^2\theta$)
= 1 ...(i)
We know that $A^{-1} = \frac{1}{|A|}$adj A
= adj A [From I]

4.
(b) 7
Explanation: $\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0
$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x+4\tan x}{x}$
$\Rightarrow f(x) = \lim\limits_{x\to 0} \frac{3x}{x} + \frac{4\tan x}{x}$
$\Rightarrow f(x) = 3 + 4\lim\limits_{x\to 0} \frac{\tan x}{x}$
$\Rightarrow$ f(x) = 3 + 4
$\therefore$ k = 7

5.
(c) $\frac{8}{\sqrt{29}}$
Explanation: The given equations can be reduced as: $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t\left(-\hat{i} + \hat{j} - 2\hat{k}.\right)$and
$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s\left(\hat{i} + 2\hat{j} - 2\hat{k}.\right)$
On comparing them with:
$\overrightarrow{r} = \overrightarrow{a_1} + t\overrightarrow{b_1}$, and $\overrightarrow{r} = \overrightarrow{a_2} + t\overrightarrow{b_2}$,
We get:
$\overrightarrow{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$
and $\overrightarrow{a_2} = \hat{i} - \hat{j} - \hat{k}, \overrightarrow{b_2} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore S.D = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_2})}{\left|\vec{b_1} \times \vec{b_2}\right|}\right|$

$= \left|\frac{(2\hat{i} - 4\hat{j} - 3\hat{k}).(\hat{j} - 4\hat{k})}{\sqrt{29}}\right|$

$= \left|\frac{-4+12}{\sqrt{29}}\right| = \left|\frac{8}{\sqrt{29}}\right|$

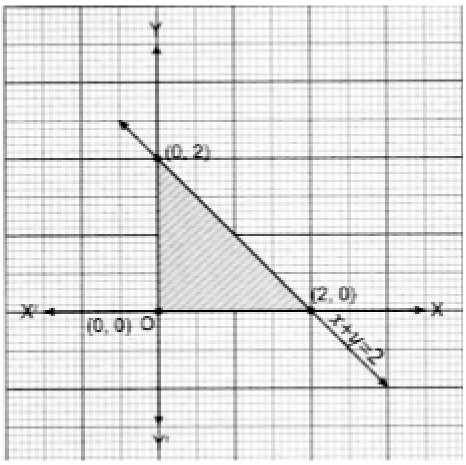
6.
(b) parabolas
Explanation: Given equation can be written as
$\frac{2dy}{y+3} = \frac{dx}{x}$
$\Rightarrow$ 2log (y + 3) = log x + log c
$\Rightarrow$ $(y + 3)^2$ = cx which represents the family of parabolas

7. **(a)** convex polygon
Explanation: Feasible region for an LPP is always a convex polygon.

8.
(b) 20
Explanation: We know that
$(\vec{a} \cdot \vec{b})^2 + (\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$
$|\vec{a}|^2 \cdot |\vec{b}|^2 = 2^2 + 4^4$
$|\vec{a}|^2 \cdot |\vec{b}|^2 = 20$

9. **(a)** 0
Explanation: If f is an odd function,
$\int_{-a}^{a} f(x)dx = 0$
as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$
here f(x) = x^4 sin x
we will see f(-x) = $(-x)^4$ sin (-x)
= - x^4 sin x
Therefore, f(x) is a odd function,
$\int_{-\pi}^{\pi} x^4 \sin x dx = 0$

10.
(d) skew-symmetric matrix
Explanation: We have matrices A and B of same order.
Let P = (AB' − BA')
Then, P′ = (AB′ − BA′)'
= (AB′)′ − (BA′)′
= (B′)′ (A)′ − (A′)′B′ = BA′ − AB′ = −(AB′ − BA′) = −P
Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

11. **(b)** two points
Explanation: 

Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)
Z(0, 0) = 0

Z(2, 0) = 2⟵ maximise
Z(0, 2) = 2 ⟵ maximise
Z_{max} = 2 obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

12.
(c) $-10\hat{i} - 3\hat{j} + 4\hat{k}$
Explanation: The vector perpendicular to both the vectors $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 4 & -4 & 7 \end{vmatrix}$
$= i(-14+4) - \hat{j}(7-4) + \hat{k}(-4+8)$
$= -10\hat{i} - 3\hat{j} + 4\hat{k}$

13.
(d) 25
Explanation: The property states that |adj A| = $|A|^{n-1}$
Here n = 3 and |A| = 5
|adj A| = $|5|^{3-1}$
= $|5|^2$
= 25.

14.
(b) A and B are mutually exclusive
Explanation: If A and B are independent events, i.e., the occurrence of one does not affect the occurrence of the other. Hence A and B are not Mutually Exclusive.

15.
(b) x (y + cos x) = sin x + c
Explanation: We have, $\frac{dy}{dx} + \frac{1}{x}y$ = sin x
Which is linear differential equation.
Here, P = $\frac{1}{x}$ and Q = sin x
$\therefore$ I.F. = $e^{\int \frac{1}{x}dx} = e^{\log x} = x$
$\therefore$ The general solution is
$y \cdot x = \int x \cdot \sin x dx + C$
= - x cos x - $\int -\cos x dx$
= -x cos x + sin x
⇒ x(y + cos x) = sin x + C

16. **(a)** 3 sq units
Explanation: Given, in $\triangle ABC$, $\overrightarrow{AB} = 2\hat{i} + 3\hat{j} + 2\hat{k}$
and $\overrightarrow{AC} = -4\hat{i} + 5\hat{j} + 2\hat{k}$
To find Area of △ABC.
Clearly, area of
$\triangle ABC = 1/2|\overrightarrow{AB} \times \overrightarrow{AC}|$ ...(i)
Let us first find $\overrightarrow{AB} \times \overrightarrow{AC}$, which is given by,
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 2 \\ -4 & 5 & 2 \end{vmatrix}$
= $\hat{i}$(6 - 10) - $\hat{j}$(-4 + 8) + $\hat{k}$(-10 + 12)
= $-4\hat{i} - 4\hat{j} + 2\hat{k}$
⇒ $|AB \times AC| = \sqrt{(-4)^2 + (-4)^2 + (2)^2}$
$= \sqrt{32+4} = \sqrt{36}$ = 6

$\therefore$ From Eq. (i),
Area of $\triangle$ABC = $\frac{1}{2}\times 6 = 3$ sq units

17.

(c) $\frac{y}{x} \cdot \left(\frac{x \log y - y}{y \log x - x}\right)$

Explanation: $x^y = y^x \Rightarrow y \log x = x \log y$

$\frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$

$\Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \left(\log y - \frac{y}{x}\right) = \frac{x \log y - y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{x \log y - y}{y \log x - x} \times \frac{y}{x} = \frac{y}{x} \times \frac{x \log y - y}{y \log x - x}$

18.

(c) 7

Explanation: We have,

$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Let point (1, 2, 3) be P and the point through which the line passes be Q(6, 7, 7). Also, the line is parallel to the vector

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Now,

$\overrightarrow{PQ} = 5\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 5 & 5 & 4 \end{vmatrix}$

$= 18\hat{i} - 22\hat{j} + 5\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{18^2 + (-22)^2 + 5^2}$

$= \sqrt{324 + 484 + 25}$

$= \sqrt{833}$

$\therefore d = \frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \frac{\sqrt{833}}{\sqrt{17}}$

$= \sqrt{49}$

$= 7$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion Let $f(x) = \frac{e^x + e^{-x}}{2}$

$\Rightarrow f'(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

$= \frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right)$ ...(i)

Now, for $x \geq 0$, we have

$2x \geq 0 \Rightarrow e^{2x} \geq e^0$ [$\because e^x$ is an increasing function]

$\Rightarrow e^{2x} \geq 1$

Also, for $x \geq 0$

$\Rightarrow e^x \geq 1$

$\therefore$ From Eq. (i), we have

$f'(x) = \frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right) \geq 0$

So, f(x) is an increasing function on $[0, \infty)$.

Reason: Let $g(x) = \frac{e^x - e^{-x}}{2}$

$\Rightarrow g'(x) = \frac{e^x + e^{-x}}{2} > 0$ [$\because e^x$ and e^{-x} both are greater than zero in $(-\infty, \infty)$]

So, g(x) is an increasing function on $(-\infty, \infty)$.

Hence, both Assertion and Reason are true.

20.

(d) A is false but R is true.

Explanation: Assertion is false because every function is not invertible. The function which is one-one and onto i.e. bijective functions are invertible so reason is true.

Section B

21. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$=\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}(-1)$.

$=\tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right]+tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$

$=\tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\begin{bmatrix}\because \tan^{-1}(\tan x)=x, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\\ \cot^{-1}(\cot x)=x, x\in(0,\ \pi)\\ and\ \tan^{-1}(-x)=-\tan^{-1}x\end{bmatrix}$

$=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}=\frac{-2\pi+4\pi-3\pi}{12}$

$=\frac{-5\pi+4\pi}{12}=\frac{-\pi}{12}$

OR

The domain of $\sin^{-1} x$ is [-1,1]. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$\Rightarrow\quad 1\geq x^2\geq -1$

$\Rightarrow 0\leq x^2\leq 1$

$\Rightarrow x^2\leq 1$

$\Rightarrow x^2-1\leq 0$

$\Rightarrow (x - 1)(x + 1) \leq 0$

$\Rightarrow -1\leq x\leq 1$

Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is [-1, 1].

22. Given:- $f(x) = x^9 + 4x^7 + 11$

$f'(x)=\frac{d}{dx}\left(x^9+4x^7+11\right)$

$f'(x) = 9x^8 + 28x^6$

$f'(x) = x^6(9x^2 + 28)$

as given in question $x \in R$,

$\Rightarrow x^6 > 0$ and $9x^2 + 28 > 0$

$\Rightarrow x^6(9x^2 + 28) > 0$

$\Rightarrow f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

23. At any instant t, let r be the radius, V the volume and S the surface area of the balloon. Then,

$\frac{dV}{dt} = 20\text{cm}^3$ /sec ... (given) ... (i)

Now, $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=\frac{dV}{dr}.\frac{dr}{dt}$

$\Rightarrow 20=\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\cdot\frac{dr}{dt}$

$\Rightarrow 20=\frac{4}{3}\pi\times 3r^2\times\frac{dr}{dt}=4\pi r^2\cdot\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt}=\frac{5}{\pi r^2}$ .... (ii)

$\therefore S = 4\pi r^2 \Rightarrow \frac{dS}{dt}=\frac{dS}{dr}.\frac{dr}{dt}$

$=\frac{d}{dr}\left(4\pi r^2\right)\cdot\frac{5}{\pi r^2}$

$=\left(8\pi r\times\frac{5}{\pi r^2}\right)=\frac{40}{r}$

$\Rightarrow \left[\frac{dS}{dt}\right]_{r=8\text{cm}}=\left(\frac{40}{8}\right)\text{cm}^2$ /sec = 5 cm^2 /sec

Hence, the rate of change of surface area at the instant when r = 8 cm is 5 cm^2/sec.

OR

We have Local max. value is 251 at x = 8 and local min. value is -5 at x = 0

Also $F'(x) = -3x^2 + 24x = 0$

$\Rightarrow$ -3x(x - 8) = 0
$\Rightarrow$ x = 0, 8
$F''(x) = -6x + 24$
$F''(0) > 0$, 0 is the point of local min.
$F''(8) < 0$, 8 is the point of local max.
F(8) = 251 and f(0) = -5

24. $I = \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$
$= \int \frac{e^{\log x^5} - e^{\log x^4}}{e^{\log x^3} - e^{\log x^2}} dx$
$= \int \left(\frac{x^5 - x^4}{x^3 - x^2}\right) dx \ \left[\because e^{\log \theta} = \theta\right]$
$= \int \frac{x^4(x-1)}{x^2(x-1)} dx$
$= \int x^2 dx$
$= \frac{x^3}{3} + c$

25. It is given that f(x) = |x + 2| - 1
Now, we can see that |x + 2| $\geq$ 0 for every x $\in$ R
$\Rightarrow f(x) = |x+2| - 1 \geq -1$ for every x $\in$ R
Clearly, the minimum value of f is attained when |x + 2| = 0
i.e, |x + 2| = 0
$\Rightarrow$ x = -2
Then, Minimum value of f = f(-2) = |-2 + 2| - 1 = -1
Therefore, function f does not have a maximum value.

Section C

26. let the given integral be,
$l = \int \frac{1}{\cos(x-a)\cdot\cos(x-b)} dx$
$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\cos(x-a)\cdot\cos(x-b)} dx$
$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cdot\cos(x-b)} dx$
$= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cdot\cos(x-a) - \cos(x-b)\cdot\sin(x-a)}{\cos(x-a)\cdot\cos(x-b)}$
$= \frac{1}{\sin(a-b)} \int \left[\frac{\sin(x-b)\cdot\cos(x-a)}{\cos(x-a)\cdot\cos(x-b)} - \frac{\cos(x-b)\cdot\sin(x-a)}{\cos(x-a)\cdot\cos(x-b)}\right] dx$
$= \frac{1}{\sin(a-b)} \int [\tan(x-b) - \tan(x-a)] dx$
$= \frac{1}{\sin(a-b)} \int \tan(x-b) dx - \int \tan(x-a) dx$
$= \frac{1}{\sin(a-b)}$ [ln |sec(x - b)| - ln|sec(x - a)|] + C
$= \frac{1}{\sin(a-b)}$[ln |cos(x - a) - ln|cos(x - b|] + C
$= \frac{1}{\sin(a-b)} \left[\ln\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right] + C$

27. Given, bag A = 4 black and 6 red balls
bag B = 7 black and 3 red balls.
Let E_1 = The event that die show 1 or 2
E_2 = The event that die show 3 or 4 or 5 or 6
E = The event that among two drawn balls, one of them is red and other is black
$P(E_1) = \frac{2}{6}$, $P(E_2) = \frac{4}{6}$ [$\because$ total number in a die is six]
$\therefore P\left(\frac{E}{E_1}\right)$ = P (getting one red and one black from)
bag A = $\frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \frac{4\times6\times2}{10\times9}$
$\Rightarrow P\left(\frac{E}{E_2}\right)$ = P(getting one red and one black from bag B)
$= \frac{7C_1 \times {}^3C_1}{{}^{10}C_2}$

$= \frac{7\times3\times2}{10\times9}$

Now, by theorem of total probability

P(E) = $P\left(E_1\right)\cdot P\left(\frac{E}{E_1}\right)+P\left(E_2\right)\cdot P\left(\frac{E}{E_2}\right)$

$=\frac{2}{6}\cdot\left(\frac{4\times6\times2}{10\times9}\right)+\frac{4}{6}\cdot\left(\frac{7\times3\times2}{10\times9}\right)$

$=\frac{4\times6}{6\times10\times9}(4+7)=\frac{4\times6\times11}{6\times10\times9}=\frac{22}{45}$

28. Let the given integral be I = $\int\frac{x^3}{x^4+3x^2+2}dx$

Take, x = $\lambda\frac{d}{dx}$(x^2 + 3x + 2) + μ

= λ(2x + 3) + μ

$\Rightarrow$ x = (2λ) x + (3λ + μ)

Comparing the coefficients of like powers of x,

2λ = 1 $\lambda=\frac{1}{2}$

3λ + μ = 0 $\Rightarrow 3\left(\frac{1}{2}\right)+\mu=0$

$\mu=-\frac{3}{2}$

So, I = $\int\frac{\frac{1}{2}(2x+3)-\frac{3}{2}}{x^2+3x+2}dx$

I = $\frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx-\frac{3}{2}\int\frac{1}{x^2+3x+2}dx$

$=\frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx-\frac{3}{2}\int\frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2}dx$

$\Rightarrow I=\frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx-\frac{3}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$

$=\frac{1}{2}\log\left|x^2+3x+2\right|-\frac{3}{2}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right|+c$ ...[Since, $\int\frac{1}{a^2-x^2}dx=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|$ + c]

$I=\frac{1}{2}\log\left|x^2+3x+2\right|-\frac{3}{2}\log\left|\frac{x+1}{x+2}\right|+c$

OR

We have from LHS,

$I=\int_0^{2a}f(x)dx=\int_0^a f(x)dx+\int_a^{2a}f(x)dx$ ..(i)

Let x = 2a - t, then dx = -dt

$x=a\Rightarrow t=a,$ and $x=2a\Rightarrow t=0$

$\therefore\int_0^{2a}f(x)dx=-\int_a^0 f(2a-t)dt$

$\Rightarrow\int_0^{2a}f(x)dx=\int_0^a f(2a-t)dt$

$\Rightarrow\int_0^{2a}f(x)dx=\int_0^a f(x)dx$

Substituting $\int_0^{2a}f(x)dx=\int_0^a f(x)dx$ in (i)

We get,

$\int_0^{2a}f(x)dx=\int_0^a f(x)dx+\int_0^a f(x)dx$

$\Rightarrow\int_0^{2a}f(x)dx=2\int_0^a f(x)dx$

Hence proved

29. According to the question,

Given differential equation is

$\frac{dy}{dx}=1+x+y+xy$

$\Rightarrow\frac{dy}{dx}=1(1+x)+y(1+x)$

$\Rightarrow\quad\frac{dy}{dx}=(1+x)(1+y)$

$\frac{1}{(1+y)}dy=(1+x)dx$

On integrating both sides, we get

$\int\frac{1}{1+y}dy=\int(1+x)dx$

$\Rightarrow\log|1+y|=x+\frac{x^2}{2}+C$ ...(i)

Also, given that y = 0, when x = 1.

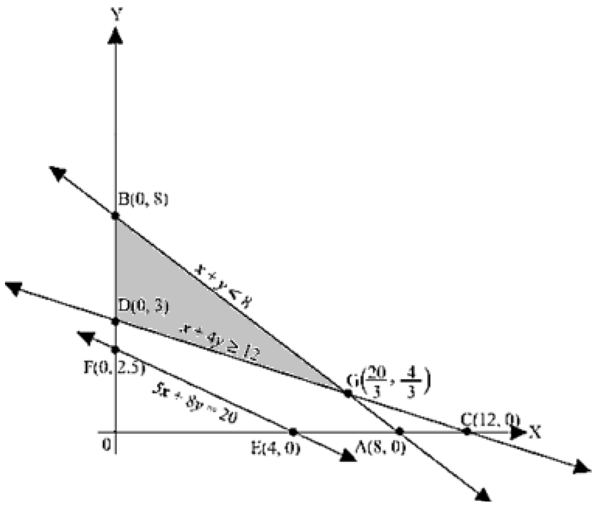
On substituting x = 1, y = 0 in Eq. (i), we get

$\log |1+0| = 1 + \frac{1}{2} + C \Rightarrow C = -\frac{3}{2} [\because \log 1 = 0]$
Now, on substituting the value of C in Eq. (i), we get
$\log |1+y| = x + \frac{x^2}{2} - \frac{3}{2}$
which is the required particular solution of given differential equation.

OR

The given differential equation is,
$x \cdot \frac{dy}{dx} + y = y^2$
$x \cdot \frac{dy}{dx} = y^2 - y$
$\frac{1}{y^2-y} dy = \frac{1}{x} dx$
$\frac{1}{y(y-1)} dy = \frac{1}{x} dx$
Integrating on both the sides,
$\int \frac{1}{y(y-1)} dy = \int \frac{1}{x} dx$
LHS.
Let $\frac{1}{y(y-1)} dy = \frac{A}{y} + \frac{B}{(y-1)}$
$\frac{1}{y(y-1)} dy = A(y-1) + By$
1 = A(y - 1) + By
1 = Ay + By - A
Comparing coefficients in both the sides,we have,
A = - 1, B = 1
$\frac{1}{y(y-1)} dy = -\frac{1}{y} + \frac{1}{(y-1)}$
$\int \frac{1}{y(y-1)} dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right] dy$
= -y + log (y-1)
$= \log\left(\frac{y-1}{y}\right)$
RHS:
$\int \frac{1}{x} dx$
$\int \frac{1}{x} dx = \log x + \log C$
Therefore the solution of the given differential equation is
$\log\left(\frac{y-1}{y}\right) = \log x + \log C$
$\frac{y-1}{y} = x . c$
y - 1 = yxc
⇒y = 1 + xyc

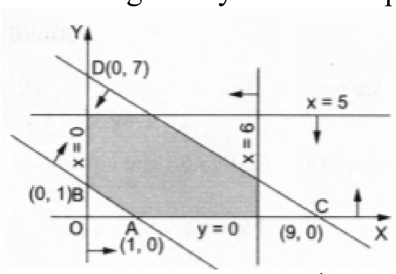
30. First, we will convert the given inequations into equations, we obtain the following equations:
x + y = 8, x + 4 y = 12, x = 0 and y = 0
5 x + 8 y = 20 is already an equation.
Region represented by x + y $\leq$ 8 The line x + y = 8 meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line x + y = 8 . Clearly (0,0) satisfies the inequation x + y $\leq$ 8.50, the region in x y plane which contain the origin represents the solution set of the inequation x + y $\leq$ 8.
Region represented by x + 4 y $\geq$ 12 :
The line x + 4 y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4 y = 12 . Clearly (0,0) satisfies the inequation x + 4 y $\geq$ 12 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + 4 y $\geq$ 12.
The line 5 x + 8 y = 20 is the line that passes through E(4,0) and $F\left(0, \frac{5}{2}\right)$ Region represented by x $\geq$ 0 and y $\geq$ 0 :
since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.
The feasible region determined by subject to the constraints arex + y $\leq$ 8, x + 4 y $\geq$ 12,5 x + 8 y = 20 and the non-negative restrictions , x $\geq$ 0 and y $\geq$ 0 are as follows.



The corner points of the feasible region are B(0,8), D(0,3), $G\left(\frac{20}{3}, \frac{4}{3}\right)$
The values of objective function at corner points are as follows:
Corner point: Z = 30x +20y
B(0,8): 160
D(0,3): 60
$G\left(\frac{20}{3}, \frac{4}{3}\right)$: 266.66
Therefore, the minimum value of objective function Z is 60 at the point D(0,3) . Hence, x = 0 and y = 3 is the optimal solution of the given LPP.
Thus, the optimal value of objective function Z is 60.

OR

Here, x + y = 1 meets the axes at A (1, 0) and B(0,1).
Join these points by a thick line. We note that the portion not containing O(0, 0) is the solution set of x + y > 1.So,
$7x + 9y = 63 \Rightarrow \frac{x}{9} + \frac{y}{7} = 1$
This line meets the axes at C (9,0) and D(0, 7). Join these points by a thick line. We note that the portion containing (0, 0) is the solution set of 7x + 9y < 63
y = 5 is a line parallel to the x-axis at a distance 5 from the x-axis and the portion containing O(0,0) is the solution set of the inequation y < 5. x = 6 is a line parallel to the y-axis at a distance 6 from the y-axis and the portion containing (0,0) is the solution set of x < 6
We note that, x > 0 has a solution represented by the y-axis and the portion on its right. Also, y > 0 has a solution represented by the x-axis and the portion above it. The shaded region represents the solution set of the given system of inequations



31. To find: Value of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

The formula used $\cos\theta = \sin\left(\frac{n}{2} - \theta\right)$

$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, tan-1 $\left(\frac{x}{\sqrt{1-x^2}}\right)$

Putting x = sin θ

$\theta = \sin^{-1} x$ .....(i)

Putting x = sin θ in the equation

$\tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-(\sin\theta)^2}}\right)$

$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta)$

$= \theta$

Now, we can see that $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \theta$

Now Differentiating

$= \frac{d\theta}{dx}$

$= \frac{d}{dx}\left(\sin^{-1} x\right)$

$= \frac{1}{\sqrt{1-x^2}}$

Section D

32. According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)

$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.

To find the point of intersection of circle and line ,

substitute eq. (ii) in eq.(i) , we get

$x^2 + \frac{x^2}{3} = 16$

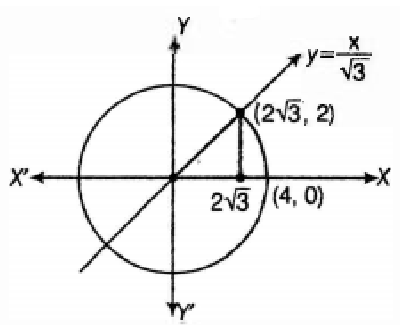
$\frac{3x^2+x^2}{3} = 16$

$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$

$\Rightarrow x = \pm 2\sqrt{3}$

When $x = 2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$



Required area (In first quadrant) = (Area under the line $y = \frac{1}{\sqrt{3}}x$ from x = 0 to $2\sqrt{3}$) + (Area under the circle from x = $2\sqrt{3}$ to x=4)

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}} x dx + \int_{2\sqrt{3}}^4 \sqrt{16 - x^2} dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^4$

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16-12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$

$= \frac{4\pi}{3}$ sq units.

33. A = R – {3} and B = R – {1} and $f(x)=\left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1)=\frac{x_1-2}{x_1-3}$ and $f(x_2)=\frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3)=(x_2-2)(x_1-3)$

$\Rightarrow x_1x_2-3x_1-2x_2+6=x_1x_2-2x_1-3x_2+6$

$\Rightarrow -3x_1-2x_2=-2x_1-3x_2$

$=x_1=x_2$

$\therefore f$ is one-one function.

Now $y=\frac{x-2}{x-3}$

$\Rightarrow y(x-3)=x-2$

$\Rightarrow xy-3y=x-2$

$\Rightarrow x(y-1)=3y-2$

$\Rightarrow x=\frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=\frac{3y-2-2y+2}{2y-2-3y+3}=y$

$\Rightarrow f(x)=y$

Therefore, f is an onto function.

OR

A = R – {3} and B = R – {1} and $f(x)=\frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1)=\frac{x_1-2}{x_1-3}$ and $f(x_2)=\frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3)=(x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one function.

Now $y=\frac{x-2}{x-3}$

$\Rightarrow y(x - 3) = x - 2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y - 1) = 3y - 2$

$\Rightarrow x=\frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right)=\frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}=\frac{3y-2-2y+2}{3y-2-3y+3}=y$

$\Rightarrow f(x)=y$

Therefore, f is an onto function.

34. Given, A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly, the co-factors of elements of |A| are given by,

$A_{11} = \cos\alpha$; $A_{12} = -\sin\alpha$; $A_{13} = 0$;

$A_{21} = \sin\alpha$; $A_{22} = \cos\alpha$; $A_{23} = 0$

$A_{31} = 0$; $A_{32} = 0$ and $A_{33} = 1$

$\therefore$ adj (A) = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$$\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}^T = \begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now, A (adj A)

$$= \begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\alpha + \sin^2\alpha & 0 & 0\\ 0 & \sin^2\alpha + \cos^2\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{...(i)}$$

$$(\text{ adj } A)\cdot(A) = \begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\alpha + \sin^2\alpha & 0 & 0\\ 0 & \sin^2\alpha + \cos^2\alpha & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{...(ii)}$$

$$\text{and } |A| = \begin{vmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{vmatrix}$$

$= 1\cdot(\cos^2\alpha + \sin^2\alpha) = 1$ [expanding along R_3]

$$\therefore \quad |A|I_3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

From qs. (i), (ii) and (iii), we get,

A (adj A) = (adj A).A = |A| I_3

35. $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$

Also, $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k}) = 3 + 7 + 0 = 10$

$$d = \left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right| = \frac{10}{\sqrt{59}}$$

OR

Here the equation of two planes are: $\vec{r}\cdot(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r}\cdot(3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

$\therefore$ Direction of line $\overrightarrow{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

$= -3\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore$ Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ .... (i)

Any point on line (i) is $(1-3\lambda, 2+5\lambda, 3+4\lambda)$

For this line to intersect the plane $\overrightarrow{r}\cdot(2\hat{i} + \hat{j} + \hat{k})$ we have

$(1-3\lambda)2 + (2+5\lambda)1 + (3+4\lambda)1 = 4$

$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection is (4, -3, -1)

Section E

36. **Read the text carefully and answer the questions:**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the condition probability, we find the reverse probabilities using Bayes' theorem.

(ii) Consider on event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

$P(E) = P(A \cap E) + P(B \cap E)$

$P(E) = P(A)\, P\left(\frac{E}{A}\right) + P(B)\left(\frac{E}{B}\right)$

(iii)Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06,$

$P(E_2) = 0.7 \times 0.8 = 0.56,$

$P(E_3) = 0.7 \times 0.2 = 0.14,$

$P(E_4) = 0.3 \times 0.8 = 0.24$

$P\left(\frac{E}{E_1}\right) = 0,\ P\left(\frac{E}{E_2}\right) + 0,\ P\left(\frac{E}{E_3}\right) = 1,\ P\left(\frac{E}{E_4}\right) = 1$

$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3).$

$P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$

$= 0.14 + 0.24 = 0.38$

OR

By Bayes' Theorem,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

Alternative way of writing the solution:

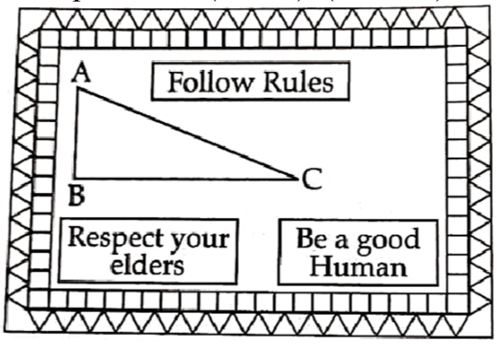
i. P(Shell fired from exactly one of them hits the plane)
= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]
$= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$

ii. $\frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$

$= \frac{P\text{ (Shell from only } B \text{ hit the plane)}}{P\text{ (Exactly one of them hit the plane)}}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

37. **Read the text carefully and answer the questions:**
The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6), respectively.



(i) Here,
Position vector of A is $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$
Position vector of B is $\vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$
Position vector of C is $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$
$\therefore \vec{a} + \vec{b} + \vec{c} = (1 + 3 - 2)\hat{i} + (4 - 3 + 2)\hat{j} + (2 - 2 + 6)\hat{k}$
$= 2\hat{i} + 3\hat{j} + 6\hat{k}$
Thus, $|\vec{a} + \vec{b} + \vec{c}| = \left|\sqrt{(2)^2 + (3)^2 + (6)^2}\right|$
$= |\sqrt{4 + 9 + 16}|$
$= \sqrt{29}$

(ii) Given, $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$,
$|\vec{a}| = \sqrt{4^2 + 6^2 + 12^2} = 14$
Therefore, the unit vector in direction of $\vec{a}$ is given by
$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{4\hat{i}+6\hat{j}+12\hat{k}}{14}$
$= \frac{4}{14}\hat{i} + \frac{6}{14}\hat{j} + \frac{12}{14}\hat{k}$
$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

(iii)We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)
Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$
and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$
$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$
$= \hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21)$
$= -36\hat{i} + 4\hat{j} - 25\hat{k}$
Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$
$= |\sqrt{1296 + 16 + 625}| = \sqrt{1937}$
$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
$= \frac{1}{2}\sqrt{1937}$ sq. units

OR

Triangle law of addition for $\triangle$ABC is given by
$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$
If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0
Then, $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$.

Also, if a, b, c are the position vector of the three vertices A, B and C of $\triangle$ABC, then area of triangle is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. **Read the text carefully and answer the questions:**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^oF at x days.

(i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

(ii) $f(x) = -0.2x + m$

At Critical point

$0 = -0.2 \times 6 + m$

$m = 1.2$