SAPPHIRE INTERNATIONAL SCHOOL, NOIDA

PRE-BOARD 1 (2023-2024)

SUBJECT- MATHEMATICS (041)

Name: ________________ **Roll No.:** __________ **Date: 14 Dec 2023**

Duration: 3Hrs **Class XII Set: A** **Max. Marks: 80**

- This question paper has 06 printed sides.
- There are 38 questions in this question paper.

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A = [a_{ij}] is a scalar matrix of order n × n such that a_{ij} = k for all i, then trace of A is equal to **[1]**

   a) $\frac{n}{k}$ b) none of these

   c) nk d) n + k

2. Let f(z) = $\begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix}$ then f(5) is equal to **[1]**

   a) 10 b) 80

   c) -20 d) None of these

3. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then A^{-1} = ? **[1]**

   a) -adj A b) adj A

   c) -A d) A

4. The value of k for which f(x) = $\begin{cases} \dfrac{3x + 4\tan x}{x}, \text{when } x \neq 0 \\ k, \text{when } x = 0 \end{cases}$ is continuous at x = 0, is **[1]**

   a) 3 b) 7

   c) None of these d) 4

5. Find the shortest distance between the lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ **[1]**

   a) $\frac{8}{\sqrt{31}}$ b) $\frac{8}{\sqrt{35}}$

   c) $\frac{8}{\sqrt{29}}$ d) $\frac{8}{\sqrt{33}}$

6. The solution of the differential equation $2x \cdot \frac{dy}{dx} - y = 3$ represents a family of **[1]**

a) circles

b) parabolas

c) straight lines

d) ellipses

7. The feasible region for an LPP is always a **[1]**

a) convex polygon

b) none of these

c) concave polygon

d) type of polygon

8. If $|\vec{a} \times \vec{b}| = 4, |\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2 |\vec{b}|^2 =$ **[1]**

a) 2

b) 20

c) 8

d) 6

9. $\int_{-\pi}^{\pi} x^4 \sin x dx = ?$ **[1]**

a) 0

b) None of these

c) π

d) 2π

10. If A and B are matrices of same order, then (AB′ – BA′) is a **[1]**

a) null matrix

b) unit matrix

c) symmetric matrix

d) skew-symmetric matrix

11. By graphical method solution of LLP maximize Z = x + y subject to x + y $\leq$ 2x; y $\geq$ 0 obtained at **[1]**

a) at infinite number of points

b) only two points

c) only one point

d) none of these

12. Consider the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and b = $4\hat{i} - 4\hat{j} + 7\hat{k}$. **[1]**
What is the vector perpendicular to both the vectors?

a) $-10\hat{i} + 3\hat{j} + 4\hat{k}$

b) $10\hat{i} - 3\hat{j} + 4\hat{k}$

c) $-10\hat{i} - 3\hat{j} + 4\hat{k}$

d) None of these

13. If A is a 3-rowed square matrix and IAI = 5 then Iadj AI = ? **[1]**

a) None of these

b) 5

c) 125

d) 25

14. If A and B are independent events such that 0 < P (A) < 1 and 0 < P (B) < 1, then which of the following is not correct? **[1]**

a) A′ and B′ are independent

b) A and B are mutually exclusive

c) A and B′ are independent

d) A′ and B are independent

15. Solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}$ = sin x is: **[1]**

a) x (y + cos x) = cos x + c

b) x (y + cos x) = sin x + c

c) x (y - cos x) = sin x + c

d) xy cos x = sin x + c

16. The adjacent sides AB and AC of a $\triangle$ABC are represented by the vectors $-2\hat{i}+3\hat{j}+2\hat{k}$ and $-4\hat{i}+5\hat{j}+2\hat{k}$, respectively. The area of the $\triangle$ABC is **[1]**

a) 3 sq units

b) 4 sq units

c) 5 sq units

d) 6 sq units

17. If $x^y = y^x$, find $\frac{dy}{dx}$ **[1]**

a) x log x

b) 0

c) $\frac{y}{x}\cdot\left(\frac{x\log y-y}{y\log x-x}\right)$

d) None of these

18. The perpendicular distance of the point P (1, 2, 3) from the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ is **[1]**

a) none of these

b) 5

c) 7

d) 0

19. **Assertion (A):** $y=\frac{e^x+e^{-x}}{2}$ is an increasing function on [0, ∞). **[1]**
Reason (R): $y=\frac{e^x-e^{-x}}{2}$ is an increasing function on (- ∞, ∞).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Every function is invertible. **[1]**
Reason (R): Only bijective functions are invertible.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. **[2]**

OR

Find the domain of f(x) = $\sin^{-1}(-x^2)$.

22. Show that f(x) = $x^9 + 4x^7 + 11$ is an increasing function for all $x \in R$ **[2]**

23. The volume of a spherical balloon is increasing at the rate of 20 cm^3/sec. Find the rate of change of its surface area at the instant when its radius is 8 cm. **[2]**

OR

Find the point of local maxima or local minima and the corresponding local maximum and minimum values of a function: $f(x) = -x^3 + 12x^2 - 5$.

24. Integrate the function $\int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx$ **[2]**

25. Find the maximum and minimum value, $f(x) = |x + 2| - 1$ **[2]**

Section C

26. Evaluate the integral: $\int \frac{1}{\cos(x-a) \cos(x-b)} dx$ **[3]**

27. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find, the probability of one of them being red and another black. **[3]**

28. Evaluate: $\int \frac{x}{x^4+3x+2} dx$ **[3]**

OR

If f is an integrable function such that f(2a - x) = f(x), then prove that $\int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx$

29. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y = 0 when x = 1. **[3]**

OR

Find the general solution for differential equation: $x \frac{dy}{dx} + y = y^2$

30. Solve the Linear Programming Problem graphically: **[3]**
Minimize Z = 30x + 20y Subject to
$x + y \leq 8$
$x + 4y \geq 12$
$5x + 8y = 20$
$x, y \geq 0$

OR

Exhibit graphically the solution set of the system of linear inequations x + y > 1, 7 x + 9y $\leq 63, y \leq 5, x \leq 6, x \geq 0$ and $y \geq 0$

31. Differentiating the function w.r.t. x: $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$. **[3]**

Section D

32. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. **[5]**

33. Let A = R – {3} and B = R – {1}. Consider the function f: A $\Rightarrow$B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A $\rightarrow$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A (adj A) = (adj A) A= $|A| I_3$. **[5]**

35. Find the shortest distance between the lines l_1 and l_2 whose vector equations are **[5]**

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ ...(1)

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ ...(2)

OR

Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) How is Bayes' theorem different from conditional probability?

(ii) Write the rule of Total Probability.

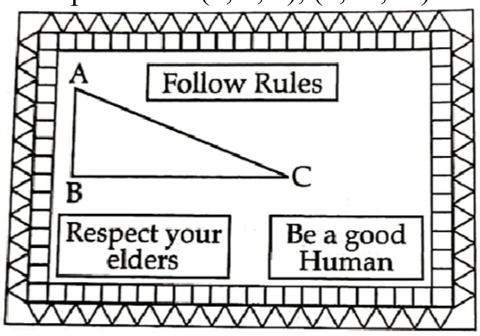
(iii) What is the probability that the shell fired from exactly one of them hit the plane?

OR

If it is know that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

37. **Read the text carefully and answer the questions:** **[4]**

The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6), respectively.



(i) If $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a} + \vec{b} + \vec{c}|$.

(ii) If $\vec{a} = 4\hat{i} + 6\hat{j} + 12\hat{k}$, then find the unit vector in direction of $\vec{a}$.

(iii) Find area of $\triangle$ABC.

OR

Write the triangle law of addition for $\triangle ABC$. Suppose, if the given slogans are to be placed on a straight line, then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. **Read the text carefully and answer the questions:** **[4]**

The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x < 12$, m being a constant, where f(x) is the temperature in ^{o}F at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant m.