Solution
PRE-BORAD-2 (2023-24)
Class 12 - Mathematics

Section A

1. **(a)** $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$

Explanation: Given that,

$(A+B)=\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ ...(i)

$(A-B)=\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$ ...(ii)

$(i)+(ii) \Rightarrow 2A=\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}+\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow 2A=\begin{bmatrix} 2 & -4 \\ 6 & 8 \end{bmatrix}$

Dividing the matrix by 2

$\Rightarrow A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

$(i)-(ii) \Rightarrow 2B=\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}-\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow 2\mathrm{B}=\begin{bmatrix} 6 & -2 \\ -4 & 4 \end{bmatrix}$

Dividing the matrix by 2

$\Rightarrow \mathrm{B}=\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$A \times B=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$=\begin{bmatrix} 1 \times 3+(-2) \times(-2) & (1) \times(-1)+(-2) \times(2) \\ 3 \times 3+4 \times(-2) & 3 \times(-1)+4 \times 2 \end{bmatrix}$

$=\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$

2.

(d) 0

Explanation: $\Delta=\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Taking 3x common from R_3, we get

$=(3x)\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}=3x \times 0\ [\because R_1=R_3]$

= 0

3.

(d) ± 6

Explanation: We have $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

We know that determinant of A is calculated as $|A|=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad - bc

$\Rightarrow$ x(x) – 2(18) = 6(6) – 2(18)

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 = 36 - 36 + 36$

$\Rightarrow x^2 = 36$
$\Rightarrow x = \pm 6.$

4.

(c) $\frac{3 \sin x - 3^4 \sin 3x}{4}$

Explanation: $\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cos x$

$\frac{d^2}{dx^2}(\sin^3 x) = \frac{d}{dx}(3\sin^2 x \cos x) = 6 \sin x \cos^2 x - 3\sin^3 x$

$\frac{d^3}{dx^3}(\sin^3 x) = \frac{d}{dx}(6\sin^2 x \cos^2 x - 3\sin^3 x)$

$= 6\cos^3 x - 12\sin^2 x \cos x - 9\sin^2 x \cos x = 6\cos^3 x - 21\sin^2 x \cos x$

$\frac{d^4}{dx^4}(\sin^3 x) = \frac{d}{dx}(6\cos^3 x - 21\sin^2 x \cos x) = -18\cos^2 x \sin x - 42 \sin x \cos^2 x + 21\sin^3 x$

$= 60 \sin x \cos^2 x + 21\sin^3 x = -60 \sin x(1 - \sin^2 x) + 21\sin^3 x$

$= -60 \sin x + 60\sin^3 x + 21\sin^3 x = -60 \sin x + 81\sin^3 x$

$= -60 \sin x + 81\left[\frac{3 \sin x - \sin 3x}{4}\right] = \frac{3 \sin z - 3^4 \sin 3z}{4}$

5.

(c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

Explanation: Fixed point is $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and the vector is $2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Equation $(2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

6. **(a)** $x^3 + y^3 = 12x + C$

Explanation: We have ,

$x^2 + y^2 \frac{dy}{dx} = 4$

$y^2 \frac{dy}{dx} = 4 - x^2$

$y^2 dy = (4 - x^2)\, dx$

$\Rightarrow \int y^2 dy = \int (4 - x^2)\, dx$

$\Rightarrow \frac{y^3}{3} = 4x - \frac{x^3}{3} + C$

$\Rightarrow y^3 + 12x - x^3 + C$

$\Rightarrow x^3 + y^3 = 12x + C$

7.

(c) Linear constraints

Explanation: In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

8.

(d) $-\frac{5}{\sqrt{162}}$

Explanation: Given position vectors $\overrightarrow{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$ and $\overrightarrow{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$\implies drs$ of $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = 4\hat{i} - 5\hat{j} + 11\hat{k}$ and drs along with y-axis are (0,1,0) 0r $\hat{j}$

direction cosines between $\overrightarrow{PQ}$ and y-axis is $\frac{(4\hat{i} - 5\hat{j} + 11k).\hat{j}}{\sqrt{16+25+121}} = \frac{-5}{\sqrt{162}}$

9. **(a)** $2 \sin\left(e^{\sqrt{x}}\right) + C$

Explanation: Formula :- $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c; \int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t$

$\Rightarrow \left(\cos e^{\sqrt{x}}\right) \times \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2\sqrt{x}}\right) dx = dt$

$= \int 2dt$

$= 2t + c$

$= 2 \sin e^{\sqrt{x}} + c$

10.

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if n is an even natural number

Explanation: $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

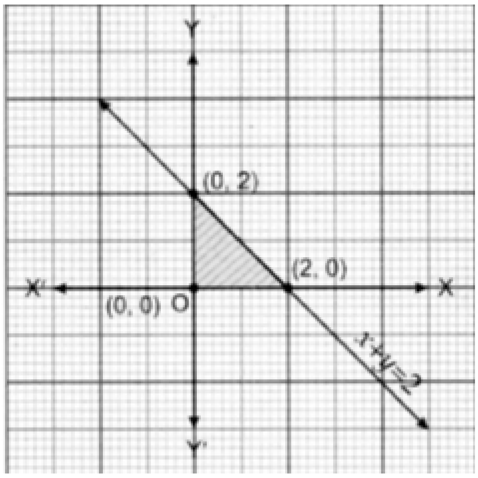
$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

If n is an even natural number.

11.

(c) 4

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2).

Z(0, 0) = 0

Z (2, 0) = 4 $\longleftarrow$ maximum

Z(0, 2) = - 2

Z_{max} = 4 and obtained at (2, 0)

12.

(c) two

Explanation: Given that, $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}, \vec{\mathrm{b}} = \hat{\mathrm{j}} + \hat{\mathrm{k}}$

Now, a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ is given by

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \vec{i}(1 - 2) - \vec{j}(2 - 0) + \vec{j} = k(2 - 0) = -\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

Now, $\vec{a} \times \vec{b} = \sqrt{(-1)^2 + (-2)^2 + (2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore$ the required unit vector

$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-i - 2\hat{j} + 2\hat{k}}{3} = \frac{-1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}k$

There are two perpendicular directions to any plane. Thus, another unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is given by $-\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{\vec{b} \times \vec{a}}{|\vec{b} \times \vec{a}|}$

$\Rightarrow \frac{\vec{b} \times \vec{a}}{|\vec{b} \times \vec{a}|} = \frac{1}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} - \frac{2}{3}k$

Hence, there are two unit length vectors perpendicular to the $\vec{a}$ and $\vec{b}$.

13.

(d) 27

Explanation: Since the matrix is of order 3 so 3 will be taken common from each row or column.

So, k = 27

14. **(a)** $\frac{4}{7}$

Explanation: Let E_1 = Event that first ball is red = (RRR, RRB, RBR, RBB)

And E_2 = Event that exactly two of three balls being red = (RRR, RRB)

$\therefore P(E_1) = P_R \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_B + P_R \cdot P_B \cdot P_R + P_R \cdot P_B \cdot P_B$

$= \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6}$

$= \frac{60+60+60+30}{336} = \frac{210}{336}$

$P(E_1 \cap E_2) = P_B \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_R$

$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{120}{336}$

$\therefore P\left(E_2/E_1\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{120/336}{210/336} = \frac{4}{7}$

15.

(d) $\sin\left(\frac{y}{x}\right) = Cx$

Explanation: Given DE: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

Now,Dividing both sides by x, we obtain $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

Let y = v x Differentiating both sides,we get

$\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$

Now, our differential equation becomes,

$v + x\frac{dv}{dx} = v + \tan v$

On separating the variables, we obtain $\frac{dv}{\tan v} = \frac{dx}{x}$

Integrating both sides, we get, sin v = Cx

Substituting the value of v we get, $\sin\left(\frac{y}{x}\right) = Cx$

16. **(a)** -1

Explanation: We know that,

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\overrightarrow{a}^2| + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$ ... (i)

Since,

$\vec{a}$ is perpendicular to $\vec{b}$

$\Rightarrow\vec{a}\cdot\vec{b}$ =0

And according to question

$|\vec{a}| = |\vec{b}| = |\vec{c}|$= 1

We can rewrite equation (i) as

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 2\cos\beta + 2\cos\alpha$

1 = 1 + 1 + 1 + 0 + 2 (cos α + cos β)

1=3+2($cos\alpha + cos\beta$)

-2=2($cos\alpha + cos\beta$)

$\Rightarrow$cos α + cosβ = -1

17.

(d) continuous at x = 0

Explanation: Given f(x) = sin^{-1}(cos x),

Checking differentiability and continuity,

LHL at x = 0,

$\lim\limits_{\mathbf{x}\to 0^-} \mathbf{f}(\mathbf{x}) = \lim\limits_{\mathbf{h}\to 0} \mathbf{f}(0 - \mathbf{h}) = \lim\limits_{\mathbf{h}\to 0} \sin^{-1}(\cos(0 - \mathbf{h})) = \lim\limits_{\mathbf{h}\to 0} \sin^{-1}(\cos(-\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$

RHL at x =0,

$\lim\limits_{\mathbf{x}\to 0^+} \mathbf{f}(\mathbf{x}) = \lim\limits_{\mathbf{h}\to 0} \mathbf{f}(0 + \mathbf{h}) = \lim\limits_{\mathbf{h}\to 0} \sin^{-1}(\cos(0 + \mathbf{h})) = \lim\limits_{\mathbf{h}\to 0} \sin^{-1}(\cos(\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$

And $f(0) = \frac{\pi}{2}$

Hence, f(x) is continuous at x =0.

LHD at x = 0,

$\lim\limits_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim\limits_{h\to 0} \frac{\sin^{-1}(\cos(0-h))-\left(\frac{\pi}{2}\right)}{-h} = 1$

RHD at x =0,

$\lim\limits_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$

$= \lim\limits_{h\to 0} \frac{\sin^{-1}(\cos(0+h))-\left(\frac{\pi}{2}\right)}{h} = -1$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

18.

(d) (3, 5, 7)

Explanation: Let $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda(\text{ say })$

A general point on this line is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

For some value of λ, let the given line meet the plane 2x + 3y - z = 14 at a point $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

Then, $2(2\lambda+1)+3(3\lambda+2)-(4\lambda+3)=14$

$\Rightarrow 9\lambda=9 \Rightarrow \lambda=1$

So, the required point is P(2 + 1, 3 + 2, 4 + 3),

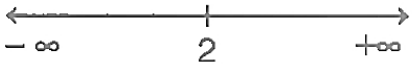
i.e., P(3, 5, 7).

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We have, f(x) = x^2 - 4x + 6

or f'(x) = 2x - 4 = 2 (x - 2)



Therefore, f'(x) = 0 gives x = 2.

Now, the point x = 2 divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, f'(x) = 2x - 4 < 0.

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, f'(x) > 0 and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Given A = {0, 1}

f(2n - 1) = 0, f(2n) = 1 $\forall$ n $\in$ N

$\Rightarrow$ every element in A has its preimage in N.

so A is true.

and we know range is subset or equal to codomain

so R is true.

and for onto function, Range = Codomain

so R is correct explanation of A.

Section B

21. We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right)+\frac{\pi}{6}\right]$

$=\cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right)+\frac{\pi}{6}\right]$

$=\cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)\{\because \cos^{-1}\cos x=x,\ x\in[0,\pi]\}$

$=\cos\left(\frac{6\pi}{6}\right)$

$=\cos(\pi)=-1$

OR

Given $\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta)=\pi-\cos^{-1}\theta$

$=\sin^{-1}\left(\frac{1}{3}\right)-\left[\pi-\cos^{-1}\left(\frac{1}{3}\right)\right]$

$=\sin^{-1}\left(\frac{1}{3}\right)-\pi+\cos^{-1}\left(\frac{1}{3}\right)$

$=\sin^{-1}\left(\frac{1}{3}\right)+\cos^{-1}\left(\frac{1}{3}\right)-\pi$

$=\frac{\pi}{2}-\pi$

$=-\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)=-\frac{\pi}{2}$

22. Given: $h=\frac{r}{6} \Rightarrow r=6h$

So, V = $\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi(6h)^2 h=12\pi h^3$

$\therefore \frac{dV}{dt} = 36\pi h^2 \cdot \frac{dh}{dt} \Rightarrow 18 = 36\pi h^2 \cdot \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{18}{36\pi h^2} - \frac{1}{2\pi h^2}$

$\Rightarrow \left[\frac{dh}{dt}\right]_{h=3} = \frac{1}{(2\pi\times 9)} cm/s$

$\frac{dh}{dt} = \frac{1}{18\pi} cm/s$

23. Here, Domain of the function is R

finding derivative i.e f'(x) = $2e^X$

As we know e^X is strictly increasing its domain

f'(x) > 0

Therefore, f(x) is strictly increasing in its domain

OR

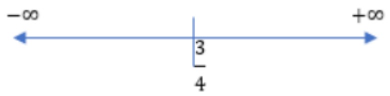
It is given that function, f(x) = $2x^2$ - 3x

⇒ f'(x) = 4x - 3

If f'(x) = 0, then we get,

$x = \frac{3}{4}$

So, the point x = $\frac{3}{4}$, divides the real line into two disjoint intervals, $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$



So, in interval $\left(\frac{3}{4}, \infty\right)$, f'(x) = 4x -3 > 0

Therefore, the given function (f) is strictly increasing in interval $\left(\frac{3}{4}, \infty\right)$

24. Let I = $\int_0^1 xe^{x^2}$ dx

Also let x^2 = t ⇒ 2x dx = dt

Also, when x = 0, t = 0

and

When x = 1, t = 1

Hence,

I = $\frac{1}{2}\int_0^1 e^t$ dt

$= \frac{1}{2}|e^t|_0^1$

$= \frac{1}{2}$ (e - 1)

25. Let at any time t, V be the volume of the water in the cone i.e., the volume of the water-cone VA' B', and let l be the slant height. Then,

$AO' = l\sin 60^\circ = \frac{\sqrt{3}l}{2}$ and $VO' = l\cos 60^\circ = \frac{l}{2}$

$\therefore V = \frac{1}{3}\pi\left(\frac{\sqrt{3}l}{2}\right)^2\left(\frac{l}{2}\right) = \frac{\pi l^3}{8}$

$\Rightarrow \frac{dV}{dt} = \frac{3\pi l^2}{8}\frac{dl}{dt}$

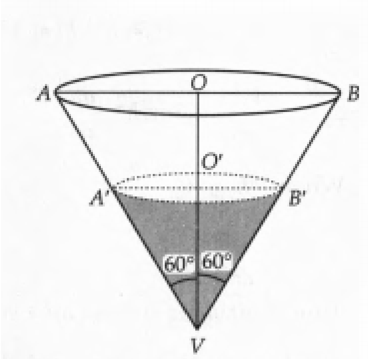
Given that $\frac{dV}{dt}$ = - 4 cm^3 sec (negative sign due to decrease)

$\therefore -4 = \frac{3\pi}{8}l^2\frac{dl}{dt}$

$\Rightarrow \frac{dl}{dt} = -\frac{32}{3\pi l^2}$

$\frac{dl}{dt} = -\frac{32}{3\pi(3)^2} = -\frac{32}{27\pi}\text{cm/sec}$

Thus, the slant height of the water - cone is decreasing at the rate of $\frac{32}{27\pi} cm/sec$



Section C

26. Let I = ∫ e^{ax} sin (bx + C) dx
solving this by the method of integration as parts.
Considering sin (bx + C) as first function and e^{ax} as second function
I = sin (bx + C) $\frac{e^{ax}}{a}$ $-$ ∫ cos (bx + C) $b\frac{e^{ax}}{a}$ dx
$\Rightarrow I = \frac{e^{ax}\sin(bx+C)}{a} - \frac{b}{a}$ ∫ e^{ax} cos (bx + C) dx
$\Rightarrow I = \frac{e^{ax}\sin(bx+C)}{a} - \frac{b}{a}I_1$ ...(i)
where I_1 = ∫ e^{ax} cos (bx + C) dx
Now, I_1 = ∫ e^{ax} cos (bx + C) dx
Considering cos (bx + C) as first function e^{ax} as second function
I_1 = cos (bx + C) $\frac{e^{ax}}{a}$ $-$ ∫ - sin (bx + C) $b\frac{e^{ax}}{a}$ dx
$\Rightarrow I_1 = \frac{e^{ax}\cos(bx+C)}{a} + \frac{b}{a}$ ∫ e^{ax} sin (bx + C) dx
$\Rightarrow I_1 = \frac{e^{ax}\cos(bx+C)}{a} + \frac{b}{a}I$ ...(ii)
from (i) and (ii)
$I = \frac{e^{ax}\sin(bx+C)}{a} - \frac{b}{a}\left[\frac{e^{ax}\cos(bx+C)}{a} + \frac{b}{a}I\right]$
$\Rightarrow I = \frac{e^{ax}\sin(bx+C)}{a} - \frac{b}{a^2}$ e^{ax} cos (bx + C) $-\frac{b^2}{a^2}$ I
$\Rightarrow I\left(1 + \frac{b^2}{a^2}\right) = \frac{e^{ax}a\sin(bx+C) - be^{ax}\cos(bx+C)}{a^2} + C_1$
$\Rightarrow I = e^{ax}\frac{[a\sin(bx+C) - b\cos(bx+C)]}{a^2+b^2} + C_1$
Where C_1 is the integration constant

27. Let D be the event that the picked up tube is defective.
Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3, respectively.
P(D) = $P(A_1)P(D|A_1) + P(A_2)P(D|A_2) + P(A_3)P(D|A_3)$
$P(A_1)$, $\frac{50}{100} = \frac{1}{2}$, $P(A_2) = \frac{1}{4}$, $P(A_3)\frac{1}{4}$
Also $P(D|A_1) = P(D|A_2) = \frac{4}{100} = \frac{1}{25}$
$P(D|A_3) = \frac{5}{100} = \frac{1}{20}$
Putting these values in (1), we get
P(D) = $\frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$
$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{17}{400}$ = 0.0425

28. The given integral can be written as:
I = ∫ $\frac{1-x^2}{x^4+x^2+1}$dx = $-$ ∫ $\frac{x^2-1}{x^4+x^2+1}$dx
I = $-$ ∫ $\frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}$dx [Dividing the numerator and denominator by X^2]
$\Rightarrow$ I = $-$ ∫ $\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1^2}$dx

Let, $x + \frac{1}{x} = u$

Then, $d(x + \frac{1}{x}) = du \Rightarrow (1 - \frac{1}{x^2})dx = du$

$I = -\int \frac{du}{u^2 - 1^2}$

$\Rightarrow l = -\frac{1}{2(1)} \log \left| \frac{u-1}{u+1} \right| + C$

$\Rightarrow I = -\frac{1}{2} \log \left| \frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1} \right| + C$

$= -\frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| + C$

Therefore, $\int\limits_0^1 \frac{1-x^2}{x^4+x^2+1} dx = \left[-\frac{1}{2}\log \left| \frac{x^2-x+1}{x^2+x+1}\right]\right|_0^1$

$= (-\frac{1}{2} \log\left|\frac{1}{3}\right|) - (-\frac{1}{2} \log |1|) = \log \sqrt{3}$

$= \log 3^{\frac{1}{2}}$

$= \frac{1}{2} \log 3$

OR

Let the given integral be ,$l = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x} dx$

Using $\sin x = \frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$, we get

$I = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\left(\frac{x}{2}\right)}{5+5\tan^2\left(\frac{x}{2}\right)+8\tan\left(\frac{x}{2}\right)} dx$

Let $\tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx = dt,$

when $x = 0$, $t = 0$ and when $x = \frac{\pi}{2'}$ $t = 1$

Hence, $I = \int_0^1 \frac{2}{5+5t^2+8t} dt$

$= \frac{2}{5} \int_0^1 \frac{1}{t^2+\frac{8}{5}t+\frac{16}{25}+\frac{9}{25}} dt$

$= \frac{2}{5} \int_0^1 \frac{1}{\left(t+\frac{4}{5}\right)^2+\frac{9}{25}} dt$

Let $t + \frac{4}{5} = u$

$\Rightarrow dt = du.$

when $t = 0$, $u \frac{4}{5}$ and when $t = 1$, $u = \frac{9}{5}$

$I = \frac{2}{5} \int_{\frac{4}{5}}^{\frac{9}{5}} \frac{1}{(u)^2+\frac{9}{25}} du$

$= \frac{2}{5} \times \frac{5}{3}\tan^{-1}\left(\frac{5x}{3}\right)\Big|_{\frac{4}{5}}^{\frac{9}{5}}$

$= \frac{2}{3}\left(\tan^{-1} 3 - \tan^{-1}\left(\frac{4}{3}\right)\right)$

$= \frac{2}{3} \times \tan^{-1}\left(\frac{3-\frac{4}{3}}{5}\right)$

$= \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$

(Using $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$)

29. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$

$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Given differential equation is of the form:
$\frac{dy}{dx}+Py=Q$
$I.F=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$
Solution is,
$y\times e^{2\sqrt{x}}=\int e^{2\sqrt{x}}\times\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx+c$
$ye^{2\sqrt{x}}=\int\frac{1}{\sqrt{x}}dx+C$
$ye^{2\sqrt{x}}=2\sqrt{x}+C$

OR

The given differential equation is,
$\left(1+y^2\right)dy+\left(x-e^{-\tan^{-1}y}\right)dy=0$
$\Rightarrow\left(1+y^2\right)\cdot\frac{dx}{dy}+x-e^{-\tan^{-1}y}=0$
$\Rightarrow\left(1+y^2\right)\cdot\frac{dx}{dy}+x=e^{-\tan^{-1}y}$
$\Rightarrow\frac{dx}{dy}+\frac{1}{1+y^2}\cdot x=\frac{e^{-tan^{-1}y}}{1+y^2}$
This is of the from $\frac{dx}{dy}+Px=Q$
Where $P=\frac{I}{1+y^2}$ and $Q=\frac{e^{-\tan^{-1}y}}{1+y^2}$
Now, $IF=e^{\int Pdx}=e^{\int\frac{1}{1+y^2}}=e^{\tan^{-1}y}$
Solution is $y\cdot(IF)=\int(IF)Q\cdot dy+C$
$\Rightarrow y\cdot e^{\tan^{-1}y}=\int e^{\tan^{-1}y}\cdot\frac{e^{-\tan^{-1}y}}{1+y^2}dy+C$
$\Rightarrow y\cdot e^{-tan^{-1}y}=\int\frac{dy}{1+y^2}+C$
$\Rightarrow y\cdot e^{\tan^{-1}y}=\tan^{-1}y+C$ ....(iii)
Putting x = 0 and y = 0 in equation (i) we have
$0\times e^{\tan^{-1}0}=\tan^{-1}0+C$
⇒ C = 0
Hence, the required solution is $y.\,e^{\tan^{-1}y}=\tan^{-1}y$

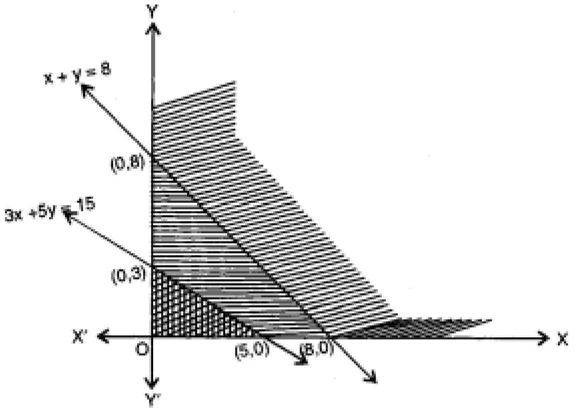
30. Linear constraints
$x+y\geq 8$
$3x+5y\leq 15$
$x\geq 0,y\geq 0$
Objective function is $\min(Z)=3x+2y$
Reducing the all inequations into equations and finding their point of intersections, i.e.,
$x+y=8$ ... (i)
$3x+5y=15$ ... (ii)
$x=0,y=0$ ... (iii)

Equations	Point of Intersection
(i) and (ii)	$x=\frac{25}{2}$ and $y=\frac{-9}{2}$
	$\Rightarrow\left(\frac{25}{2},\frac{-9}{2}\right)$
(i) and (iii)	when x = 0 ⇒ y = 8 ⇒ (0, 8)
	when y = 0 ⇒ x = 8 ⇒ (8, 0)
(ii) and (iii)	when x = 0 ⇒ y = 3 ⇒ (0, 3)
	when y = 0 ⇒ x = 5 ⇒ (5, 0)

For feasible region.
For $x+y\geq 8$ let $x=0,y=0$
$\Rightarrow 0\geq 8$ i.e., Not true
⇒ The shaded region will be away from origin
Again, for $3x+5y\leq 15$, let $x=0,y=0$

$\Rightarrow 0 \leq 15$ i.e. true

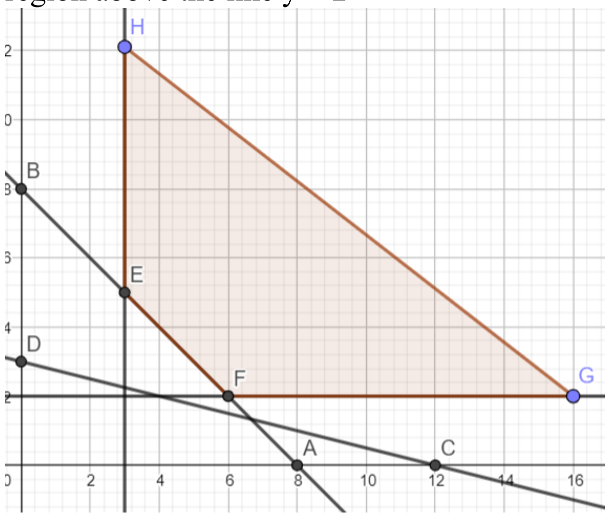
We have, no negative restriction, $x \geq 0, y \geq 0$ indicates that the shaded region will exist in first quadrant only.



The problem will not have any feasible region. Therefore there will be no feasible solution.

OR

First, we will convert the given inequations into equations, we obtain the following equations:
x + y = 8, x + 4 y = 12, x = 3, y = 2 and solving we get values are as follows:
The region represented by x + y $\geq$ 8: The line x + y = 8 meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line x + y = 8 Clearly (0,0) does not satisfies the inequation x + y $\geq$ 8 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + y $\geq$ 8
The region represented by x + 4 y $\geq$ 12 :
The line x + 4y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4y = 12.
Clearly (0,0) satisfies the inequation x + 4y $\geq$ 12 . So, the region in x y plane which contains the origin represents the solution set of the inequation x + 4y $\geq$ 12
The line x = 3 is the line that passes through the point (3,0) and is parallel to Y axis x $\geq$ 3 is the region to the right of the line x = 3
The line y = 2 is the line that passes through the point (0,12) and is parallel to X axis.y $\geq$ 2 is the region above the line y = 2



The corner points of the feasible region are E(3,5) and F(6,2)
The values of Z at these corner points are as follows.

Corner point	Z = 2x + 4y
E(3, 5)	$2 \times 3 + 4 \times 5 = 26$
F(6, 2)	$2 \times 6 + 4 \times 2 = 20$

Therefore, the minimum value of objective function Z is 20 at point F(6,2). Hence, x = 6 and y = 2 is the optimal solution of the given LPP. Thus, the optimal value of objective function z is 20.

31. According to the question, $f(x) = |x-3|$
To Check the continuity of f(x) at x = 3.
Here, LHL = $\lim_{x \to 3^-} |x-3| = \lim_{h \to 0} |3-h-3|$

$= \lim_{h\to 0}|-h| = 0$

RHL = $\lim_{x\to 3^+}|x-3| = \lim_{h\to 0}|3+h-3|$

$= \lim_{h\to 0}|h| = 0$

$and\ \ f(3) = |3-3| = 0$

$\therefore$, LHL = RHL = f(3)

Hence, f is continuous at x = 3.

To check the differentiability of f(x) at x = 3.

LHD = $f'\left(3^-\right) = \lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$

$= \lim_{h\to 0}\frac{|3-h-3|-|3-3|}{-h}$

$= \lim_{h\to 0}\frac{|-h|}{-h} = \lim_{h\to 0}\frac{h}{-h} = -1$

RHD = f'(3^+) = $\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$= \lim_{h\to 0}\frac{|3+h-3|-|3-3|}{h}$

$= \lim_{h\to 0}\frac{|h|}{h} = \lim_{h\to 0}\frac{h}{h} = 1$

Since, LHD $\neq$RHD at x = 3.

$\therefore$ f(x) is not differentiable at x=3

Hence proved.

Section D

32. According to the question ,

Given curves are

x - y + 2 = 0....(i)

x = $\sqrt{y}$...(ii)

Consider x = $\sqrt{y}$ $\Rightarrow$$x^2$ = y, which represents the parabola

vertex of parabola is (0, 0)

axis of parabola is Y-axis.

Now, the point of intersection of Eqs.(i) and (ii) is given by

x = $\sqrt{x+2}$

Squaring on both sides ,

$\Rightarrow$$x^2$ = x + 2

$\Rightarrow$$x^2$ - x - 2 = 0

$\Rightarrow$(x - 2)(x + 1) = 0

$\Rightarrow$x = -1, 2

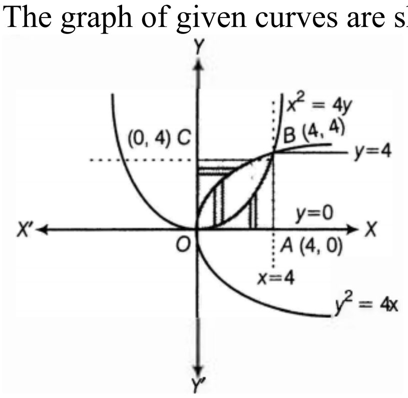
When x = -1 , does not satisfy the Eq. (ii).

When x = 2 , then 2 = $\sqrt{y}$ $\Rightarrow$ y = 4

Hence, the point of intersection is (2, 4).

But actual equation of given parabola is x = $\sqrt{y}$, it means a semi-parabola which is on right side of Y - axis.

The graph of given curves are shown below:

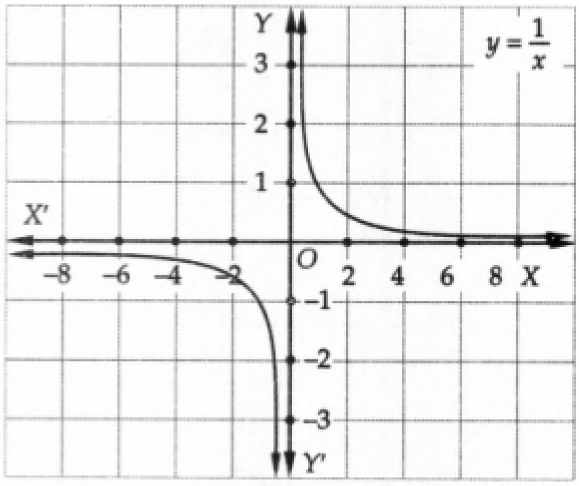


Clearly, area of bounded region = Area of region OABO

$= \int_0^2 \left[y_{(\text{ line })} - y_{(\text{ parabola })}\right] dx$

$= \int_0^2 (x+2)dx - \int_0^2 x^2 dx$
$= \left[\frac{x^2}{2} + 2x\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$
$= \left[\frac{4}{2} + 4 - 0\right] - \left[\frac{8}{3} - 0\right]$
$= 6 - \frac{8}{3}$
$= \frac{18-8}{3}$
$= \frac{10}{3}$sq.units.

33. We observe the following properties of f.
Injectivity: Let x, y ∈ R_0 such that f(x) = f(y). Then,
$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$
So, f : $R_0 \to R_0$ is one-one.
Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,
$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y ∈ R_0 (co-domain).
Thus, for each y ∈ R_0 (co-domain) there exits $x = \frac{1}{y} \in R_0$(domain) such that $f(x) = \frac{1}{x} = y$
So, f : $R_0 \to R_0$ is onto.
Hence, f: $R_0 \to R_0$ is one-one onto.
This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N → R_0 given by $f(x) = \frac{1}{x}$
For any x, y ∈ N, we find that
f(x) = f(y) ⇒ $\frac{1}{x} = \frac{1}{y} \Rightarrow x = y$
So, f: N → R_0 is one-one.
We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N →R_0 is not onto.
Thus, f: N →R_0 is one-one but not onto.

OR

Given that
Let A = {1, 2, 3} and R = {(a, b) : a, b ∈ A and $|a^2 - b^2| \leq 5$
Put a = 1, b = 1 $|1^2 - 1^2| \leq 5, (1,1)$ is an ordered pair.
Put a = 1, b = 2 $|1^2 - 2^2| \leq 5, (1,2)$ is an ordered pair.
Put a = 1, b = 3 $|1^2 - 3^2| > 5, (1,3)$ is not an ordered pair.
Put a = 2, b = 1 $|2^2 - 1^2| \leq 5, (2,1)$ is an ordered pair.
Put a = 2, b = 2 $|2^2 - 2^2| \leq 5, (2,2)$ is an ordered pair.
Put a = 2, b = 3 $|2^2 - 3^2| \leq 5, (2,3)$ is an ordered pair.
Put a = 3, b = 1 $|3^2 - 1^2| > 5, (3,1)$ is not an ordered pair.
Put a = 3, b = 2 $|3^2 - 2^2| \leq 5, (3,2)$ is an ordered pair.

Put a = 3, b = 3 $\left|3^2 - 3^2\right| \leq 5, (3, 3)$ is an ordered pair.
R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) ∈ R
$\left|a^2 - a^2\right| = 0 \leq 5$. Thus, it is reflexive.

ii. Let (a, b) ∈ R
$(a, b) \in R, \left|a^2 - b^2\right| \leq 5$
$\left|b^2 - a^2\right| \leq 5$
(b, a) ∈ R
Hence, it is symmetric

iii. Put a = 1, b = 2, c = 3
$\left|1^2 - 2^2\right| \leq 5$
$\left|2^2 - 3^2\right| \leq 5$
But $\left|1^2 - 3^2\right| > 5$
Thus, it is not transitive

34. Let x, y and z be the investments at the rates of interest of 6%, 7% and 8% per annum respectively.
Then, Total investment = Rs. 5000
⇒ x + y + z = 5000.
Now, Income from first investment of x = $\frac{6x}{100}$
Income from second investment of y = $\frac{7y}{100}$
Income from third investment of z = $\frac{8z}{100}$
∴ Total annual income =
$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$
⇒ 6x + 7y + 8z = 35800. [∵ Total annual income = Rs 358]
It is given that the combined income from the first two investments is Rs 70 more than the income from the third.
$\therefore \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100} \Rightarrow 6x + 7y - 8z = 7000$
Thus, we obtain the following system of simultaneous linear equations:
x + y + z = 5000
6x + 7y + 8z = 35800
6x + 7y - 8z = 7000
This system of equations can be written in matrix form as follows:
$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$
or, AX = B, where A = $\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$ = 1 (-56 - 56) - (-48 - 48) + (42 - 42) = - 1 6 $\neq$ 0.

So, A^{-1} exists and the solution of the given system of equations is given by X = A^{-1} B.
here
$A11$ = -112, A_{12} = 96, A_{13} = 0, A_{21} = 15, A_{22} = -14, A_{23} = -1, A_{31} = 1, A_{32} = - 2 and A_{33} = 1

$\therefore$ adjA = $\begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

So, A-1 = = $\frac{1}{|A|}(\operatorname{adj} A) = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Hence, the solution is given by

$X = A^{-1}B = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -560000 & +537000 & +7000 \\ 480000 & -501200 & -14000 \\ 0 & -35800 & +7000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$

$\Rightarrow$ x = 1000, y = 2200 and 2 = 1800

Hence, three investments are of Rs1000, Rs 2200 and Rs1800 respectively.

35. $\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$

$\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{b}_1 = 7\hat{i} - 6\hat{j} + 1\hat{k}$

$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= -4\hat{i} - 6\hat{j} - 8\hat{k}$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (4\hat{i} + 6\hat{j} + 8\hat{k})(-4\hat{i} - 6\hat{j} - 8\hat{k}) = -16 - 36 - 64 = -116$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$

$= \sqrt{116}$

$= 2\sqrt{29}$

$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(b_1 \times b_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$

$= \left|\frac{-116}{2\sqrt{29}}\right|$

$= 2\sqrt{29}$

OR

$\overrightarrow{a}_1 = -\hat{i} - \hat{j} - \hat{k}$

$\overrightarrow{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\overrightarrow{b}_1 = 7\hat{i} - 6\hat{j} + 1\hat{k}$

$\overrightarrow{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$\overrightarrow{a}_2 - \overrightarrow{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$\overrightarrow{b}_1 \times \overrightarrow{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= -4\hat{i} - 6\hat{j} - 8\hat{k}$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (4\hat{i} + 6\hat{j} + 8\hat{k}).(-4\hat{i} - 6\hat{j} - 8\hat{k})$ = -16 - 36 - 64 = -116

$\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$

$= \sqrt{116}$

$= 2\sqrt{29}$

$d = \left|\frac{(\overrightarrow{a}_2 - \overrightarrow{a}_1).(b_1 \times b_2)}{\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right|}\right|$

$= \left|\frac{-116}{2\sqrt{29}}\right| = \frac{4 \times 29}{2\sqrt{29}}$

$= 2\sqrt{29}$

Section E

36. **Read the text carefully and answer the questions:**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.
The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) According to given information, we construct the following table.
Given, total applications = 2000

	Folk Genre	Classical Genre
	960 (given)	2000 - 960 = 1040
Below 18	192 (given)	1040 - 104 = 936
18 or Above 18	960 - 192 = 768	104 (given)

Let E_1 = Event that application for folk genre
E_2 = Event that application for classical genre
A = Event that application for below 18
B = Event that application for 18 or above 18
$\therefore P(E_2) = \frac{1040}{2000}$
and $P(B \cap E_2) = \frac{104}{2000}$
Required Probability $= \frac{P(B \cap E_2)}{P(E_2)}$
$= \frac{\frac{104}{2000}}{\frac{1040}{200}} = \frac{1}{10}$

(ii) Required probability $= P\left(\frac{\text{folk}}{\text{below 18}}\right)$
$= P\left(\frac{E_1}{A}\right)$
$= \frac{P(E_1 \cap A)}{P(A)}$
Now, $P(E_1 \cap A) = \frac{192}{2000}$
and $P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$
$\therefore$ Required probability $= \frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$

(iii)Here,
$P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$
$\because P(B|A) = \frac{P(B \cap A)}{P(A)}$
$\Rightarrow P(B \cap A) = P(B|A).P(A)$
$= 0.6 \times 0.4 = 0.24$
$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.4 + 0.8 - 0.24$
$= 1.2 - 0.24 = 0.96$

OR

Since, A and B are independent events, A' and B' are also independent. Therefore,
$P(A' \cap B') = P(A') \cdot P(B')$
$= (1 - P(A)(1 - P(B))$
$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right)$
$= \frac{2}{5} \cdot \frac{5}{9}$
$= \frac{2}{9}$

37. **Read the text carefully and answer the questions:**
Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes

first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

(i) $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$ and $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

(ii) We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$ and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

= $\hat{i}$(-28 - 8) - $\hat{j}$(8 - 12) + $\hat{k}$(-4 - 21) = -36$\hat{i}$ +4$\hat{j}$ - 25$\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

= $\sqrt{1296 + 16 + 625} = \sqrt{1937}$

$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{1937}$ sq. units

(iii)If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0

$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \bar{c} \times \vec{a}|$ = 0 [$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices A, B and C of $\triangle$ABC, then area of triangle = $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$]

OR

Here, $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\overrightarrow{a}| = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

Now unit vector $\widehat{a} = \frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$

$\widehat{a} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

38. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by V(x) = x(18 - x)^2

(ii) $V(x) = x(18 - 2x)^2$

$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$

For maxima or minima $= \frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is $x = 3$ cm