SAPPHIRE INTERNATIONAL SCHOOL, NOIDA

PRE-BOARD 1 (2023-2024)

SUBJECT- MATHEMATICS (041)

Name: ________________ **Roll No.:** ___________ **Date: 14 Dec 2023**

Duration: 3Hrs **Class XII Set: B** **Max. Marks: 80**

- This question paper has 06 printed sides.
- There are 38 questions in this question paper.

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If A and B are 2-rowed square matrices such that (A + B)= $\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ and (A - B)= $\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$ then AB = ? **[1]**

   a) $\begin{bmatrix} 7 & -5 \\ 1 & 5 \end{bmatrix}$ b) $\begin{bmatrix} -7 & 5 \\ 1 & -5 \end{bmatrix}$

   c) $\begin{bmatrix} 7 & -1 \\ 5 & -5 \end{bmatrix}$ d) $\begin{bmatrix} 7 & -1 \\ -5 & 5 \end{bmatrix}$

2. The value of the determinant $\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$ is **[1]**

   a) -5 b) 4

   c) 5 d) 0

3. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to **[1]**

   a) -6 b) 0

   c) 6 d) ± 6

4. $\frac{d^4}{dx^4}(\sin^3 x)$ is equal to **[1]**

   a) $\frac{3}{4}\cos x - \frac{3^4 \cos 3x}{4}$ b) None of these

   c) $\frac{3\sin x - 3^4 \sin 3x}{4}$ d) $\frac{3}{4}\sin x - \frac{3^4 \cos 3x}{4}$

5. The Cartesian equations of a line are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation? **[1]**

   a) none of these b) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

d) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k})$

6. The solution of $x^2 + y^2 \frac{dy}{dx} = 4$, is **[1]**

a) $x^3 + y^3 = 12x + C$

b) $x^3 + y^3 = 3x + C$

c) $x^2 + y^2 = 12x + C$

d) $x^2 + y^2 = 3x + C$

7. In a LPP, the linear inequalities or restrictions on the variables are called **[1]**

a) Limits

b) Inequalities

c) Linear constraints

d) Constraints

8. If the position vectors of P and Q are $\hat{i} + 3\hat{j} - 7\hat{k}$ and $5\hat{i} - 2\hat{j} + 4\hat{k}$ respectively , then the cosine of the angle between $\overrightarrow{PQ}$ and y-axis is **[1]**

a) $\frac{4}{\sqrt{162}}$

b) $\frac{11}{\sqrt{162}}$

c) $\frac{5}{\sqrt{162}}$

d) $-\frac{5}{\sqrt{162}}$

9. $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx = ?$ **[1]**

a) $2 \sin\left(e^{\sqrt{x}}\right) + C$

b) None of these

c) $\sin\left(e^{\sqrt{x}}\right) + C$

d) $\frac{1}{2}\sin\left(e^{\sqrt{x}}\right) + C$

10. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then A^n = **[1]**

a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, if n $\in$ N

b) none of these

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if n is an even natural number

d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if n is an odd natural number

11. Solution of LPP maximize Z = 2x - y subject to x + y $\leq$ 2, x, y $\geq$ 0 **[1]**

a) 0

b) None of these

c) 4

d) 2

12. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ **[1]**

a) three

b) infinite

c) two

d) one

13. If A is a 3-rowed square matrix and I3AI = k|A| then k = ? **[1]**

a) 9

b) 1

c) 3

d) 27

14. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the the probability that exactly two of the three balls were red, the first ball being red, is **[1]**

a) $\frac{4}{7}$

b) $\frac{15}{28}$

c) $\frac{5}{28}$

d) $\frac{1}{3}$

15. The general solution of the DE $x\frac{dy}{dx} = y + x \tan \frac{y}{x}$ is **[1]**

a) $\sin\left(\frac{y}{x}\right) = C$

b) $\sin\left(\frac{y}{x}\right) = Cy$

c) none of these

d) $\sin\left(\frac{y}{x}\right) = Cx$

16. Let $\vec{a}, \bar{b}, \bar{c}$ be three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$ and $\vec{a}$ is perpendicular to $\vec{b}$. If $\vec{c}$ makes angle α and β with $\vec{a}$ and $\vec{b}$ respectively, then $\cos \alpha + \cos \beta =$ **[1]**

a) -1

b) $\frac{3}{2}$

c) $-\frac{3}{2}$

d) 1

17. The function $f(x) = \sin^{-1}(\cos x)$ is **[1]**

a) None of these

b) differentiable at x = 0

c) discontinuous at x = 0

d) continuous at x = 0

18. The line $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{4}$ meets the plane 2x + 3y - z = 14 in the point. **[1]**

a) (6, 5, 3)

b) (5, 7, 3)

c) (2, 5, 7)

d) (3, 5, 7)

19. **Assertion (A):** The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval $(2, \infty)$. **[1]**
Reason (R): The function $f(x) = x^2 - 4x + 6$ is strictly decreasing in the interval $(-\infty, 2)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If A = {0, 1} and N be the set of natural numbers. Then, the mapping $f : N \to A$ defined by f (2n - 1) = 0, f (2n) = 1, $\forall$ n$\in$ N, is onto. **[1]**
Reason (R): Range = Codomain

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ **[2]**

OR

Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

22. Sand is pouring from a pipe at the rate of 18 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is one-sixth of the radius of the base. How fast is the height of the sand cone increasing when its height is 3 cm? **[2]**

23. Prove that the function f(x) = e^{2x} is strictly increasing on R. **[2]**

OR

Find the intervals in which the function f given by f(x) = $2x^2 - 3x$ is increasing.

24. Evaluate: $\int_0^1 xe^{x^2}$ dx **[2]**

25. Water is dripping out from a conical funnel at a uniform rate of 4 cm/sec through a tiny hole at the vertex at the bottom. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of the funnel is 120°. **[2]**

Section C

26. Evaluate: $\int e^{ax} \sin(bx + c)\, dx$ **[3]**

27. Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective. **[3]**

28. Evaluate: $\int_0^1 \frac{1-x^2}{x^4+x^2+1}$dx **[3]**

OR

Evaluate: $\int_0^{\pi/2} \frac{dx}{(5+4\sin x)}$

29. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1 \ (x \neq 0)$ **[3]**

OR

Solve $(1 + y^2)dx + (x - e^{-\tan^{-1} y})dy = 0$, given that when y = 0, then x = 0.

30. Minimize Z = 3x + 2y subject to the constraints: **[3]**
$x + y \geq 8$
$3x + 5y \leq 15$
$x \geq 0,\ y \geq 0$

OR

Solve the Linear Programming Problem graphically:
Minimize Z = 2x + 4y Subject to
$x + y \geq 8$
$x + 4y \geq 12$
$x > 3, y \geq 2$

31. Show that the function $f(x) = |x - 3|$, $x \in R$, is continuous but not differentiable at x = 3. **[3]**

Section D

32. Using integration, find the area of the region bounded by the line x - y + 2 = 0, the curve x = $\sqrt{y}$ and Y-axis. **[5]**

33. Show that the function f: $R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0? **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b ∈ A and $|a^2 - b^2| \leq 5$. Write R as set of ordered pairs. Mention whether R is

i. reflexive
ii. symmetric
iii. transitive

Give reason in each case.

34. An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method. **[5]**

35. Find the shortest distance between the following lines: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$. **[5]**

OR

Find the shortest distance between the pairs of lines whose Cartesian equations are:
$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

Section E

36. **Read the text carefully and answer the questions:** **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.
The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(ii) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.

(iii) If P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6, then P(A ∪ B) is equal to

OR

If A and B are two independent events with
P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then find P(A' ∩ B').

37. **Read the text carefully and answer the questions:** **[4]**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

(i) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then find $\vec{a} + \vec{b} + \vec{c}$.

(ii) What is the Area of $\triangle$ABC.

(iii) Suppose, if the given slogans are to be placed on a straight line, then find the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then find the unit vector in the direction of vector $\vec{a}$.

38. **Read the text carefully and answer the questions:** **[4]**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?