



**BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION  
(BSSCA)  
Class XII**

**MARKING SCHEME MATHEMATICS (CODE-041)**

**SET B**

**SECTION:A (MULTIPLE CHOICE QUESTIONS- 1 MARK EACH)**

Question No	Hints/Solution	Answer
1.	$A^2 = \lambda A$ $\begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \Rightarrow \lambda = 6$	a
2.	$A = A'$ $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix}$ On comparing $2b = 3$ , $3a = -2$ $b = \frac{3}{2}$ , $a = \frac{-2}{3}$ $\therefore a + b = b + a = \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$	b
3.	$\det(A^{-1}) = (\det A)^k$ $\frac{1}{ A } =  A ^k \Rightarrow 1 =  A ^{k+1}$ , $\Rightarrow  A ^0 =  A ^{k+1}$ , $\Rightarrow k+1=0$ , $\Rightarrow k = -1$	d
4.	$A(\text{adj}A) =  A I$ , But A is a singular matrix, $\therefore  A =0$ , $\therefore A(\text{adj}A)=0$	a
5.	$ A  = \lambda^2 - 4$ , and $ A^3  = 125$ , $\therefore  A ^3 = 125$ , $\therefore  A  = 5$ $\therefore 5 = \lambda^2 - 4$ , $\Rightarrow \lambda = \pm 3$ .	b
6.	$(\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$ , $\therefore (2 - \lambda)3 + (2 + 2\lambda) = 0$ , $\therefore \lambda = 8$	b
7.	Median is $2i + 2k$ . And its length is $ 2i + 2k  = 2\sqrt{2}$ .	c
8.	Area = $\frac{1}{2}  \vec{d}_1 \times \vec{d}_2  = \frac{1}{2}  -2i - 14j - 10k  = \frac{\sqrt{300}}{2} = 5\sqrt{3}$	a
9.	Line is $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$ . The direction ratios are a , 1 , c.	b
10.	The probability that both the tickets will show even numbers is $\frac{12}{25} \times \frac{11}{24} = \frac{11}{50}$	a
11.	Order=2 and degree=2. The sum of order and degree=4	c
12.	Integrating factor is $e^{\int (\frac{1}{x} + \cot x) dx} = e^{\log x \sin x} = x \sin x$	b
13.	$I = \int_0^{\frac{\pi}{2}} \log(\tan x) dx$ . Also $I = \int_0^{\frac{\pi}{2}} \log(\tan[\frac{\pi}{2} - x]) dx$ $\therefore 2I = \int_0^{\frac{\pi}{2}} \log 1 = 0$	b
14.	On integrating $f(x) = \frac{x^2}{2} + \frac{1}{x} + C$ , and $f(1) = \frac{1}{2} + 1 + C$ , $C = -1$ . $\therefore f(x) = \frac{x^2}{2} + \frac{1}{x} - 1$	a
15.	$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (-1 + a) = -1 + a$ $\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} (1 + b) = 1 + b$	b

	$f(x) = a + b$ $\therefore a + b = a - 1$ and $a + b = 1 + b, \Rightarrow a = 1, b = -1$	
16.	$y' = \frac{\cos(\log x)}{x}, y'' = \frac{-\sin(\log x) - \cos(\log x)}{x^2},$ $\therefore x^2 y'' + xy' = -y$	b
17.	Minimum value is at (4,3)=-24	d
18.	Minimum value is at (0,8)=-24	c
19.	Both A and R are true and R is the correct explanation of A. i.e $-1 \leq 2x - 1 \leq 1$ $0 \leq 2x \leq 2$ $0 \leq x \leq 1 \Rightarrow x \in [0,1]$	a
20.	Both A and R are true and R is the correct explanation of A. i.e $\cos \theta = \left  \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{1+1+4} \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16}} \right  = \frac{1}{2}$ $\theta = \frac{\pi}{3}$	a

### SECTION B

This section comprises of VSA of 2 marks each

21.	$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$ $-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}(\sqrt{3}) + \tan^{-1}(-1)$ $\frac{-\pi}{6} - \frac{\pi}{3} - \frac{\pi}{4} = \frac{-3\pi}{4}$	1 1
21.(or)	For one-one: Let $x_1, x_2 \in A,$ $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$ $x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 3x_2 - 2x_1 + 6$ $x_1 = x_2.$ Hence, function is one-one. For onto: Let for $y \in B,$ there exist $x \in A$ such that $y = f(x) \Rightarrow y = \frac{x - 2}{x - 3} \Rightarrow xy - 3y = x - 2$ $x(y - 1) = 3y - 2$ $x = \frac{3y - 2}{y - 1} \in A$ Hence, function is onto. Therefore, f is 1-1 and onto.	1 1

22.	<p>Given: <math>\frac{dA}{dt} = k</math></p> $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow \frac{k}{2\pi r} = \frac{dr}{dt} \dots\dots\dots(1)$ <p>Now,</p> $P = 2\pi r \Rightarrow \frac{dP}{dt} = 2\pi \frac{dr}{dt} \Rightarrow \frac{dP}{dt} = 2\pi \times \frac{k}{2\pi r} = \frac{dP}{dt} = \frac{k}{r}$ $\frac{dP}{dt} \propto \frac{1}{r}$	1  1
23.	<p><math>\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda</math>, is the equation of the line</p> <p>General point, <math>(x = 3\lambda - 2; y = 2\lambda - 1; z = 2\lambda + 3)</math></p> <p>The distance between the point on the line and given point P(1,3,3) is 5 units, then</p> $\sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2} = 5$ <p>On solving, <math>\lambda = 0, 2</math></p> <p>When <math>\lambda = 0</math>, the point is (-2,-1,3); <math>\lambda = 2</math>, the point is (4,3,7).</p>	1  1
23. (or)	<p>Let <math>\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}</math>,</p> <p>Given, <math>\vec{r} \cdot \vec{a} = 0 \Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + \hat{k}) = 0 \Rightarrow 2x + z = 0 \dots (1)</math></p> <p>Given, <math>\vec{r} \times \vec{b} = \vec{c} \times \vec{b} \Rightarrow \begin{vmatrix} i &amp; j &amp; k \\ x &amp; y &amp; z \\ 1 &amp; 1 &amp; 1 \end{vmatrix} = \begin{vmatrix} i &amp; j &amp; k \\ 4 &amp; -3 &amp; 7 \\ 1 &amp; 1 &amp; 1 \end{vmatrix}</math></p> $(y - z)i + (z - x)j + (x - y)k = -10i + 3j + 7k$ <p><math>y - z = -10; z - x = 3; x - y = 7 \dots\dots(2)</math></p> <p>On solving (1) and (2)</p> <p><math>x = -1, y = -8, z = 2</math>, therefore; <math>\vec{r} = -\hat{i} - 8\hat{j} + 2\hat{k}</math></p>	$\frac{1}{2}$  1  $\frac{1}{2}$
24.	<p><math>\log y = a \tan^{-1} x</math></p> $\frac{1}{y} \frac{dy}{dx} = \frac{a}{1+x^2} \Rightarrow (1+x^2) \frac{dy}{dx} = ay$ <p>Again differentiate,</p> $(1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = a \frac{dy}{dx} \Rightarrow (1+x^2) \frac{d^2y}{dx^2} + (2x-a) \frac{dy}{dx} = 0$	1  1
25.	<p><math>\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j}) = -\hat{i} + \hat{j} + \hat{k}</math></p> <p><math>\vec{c} - \vec{b} = (3\hat{i} - 4\hat{j} - 5\hat{k}) - (2\hat{i} + \hat{j}) = \hat{i} - 5\hat{j} - 5\hat{k}</math></p> <p>Unit vector perpendicular to both <math>\vec{a} - \vec{b}</math> and <math>\vec{c} - \vec{b}</math></p> $\Rightarrow \frac{(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})}{ (\vec{a} - \vec{b}) \times (\vec{c} - \vec{b}) }$ $(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b}) = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix}$ $= ((-5+5)\hat{i} - (5-1)\hat{j} + (5-1)\hat{k})$ <p><math>= -4\hat{j} + 4\hat{k}</math></p> $ (\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})  = \sqrt{16+16} = 4\sqrt{2}$	$\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$

**SECTION C (Short Answer Questions of 3 Marks each)**

26.	$\int \frac{\cos(x+a)}{\sin(x+b)} dx = \int \frac{\cos\{(x+b) + (a-b)\}}{\sin(x+b)} dx$ $= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)} dx$ $= \cos(a-b) \int \cot(x+b) dx - \sin(a-b) \int 1 dx$ $= \cos(a-b) \cdot \log \sin(x+b)  - \sin(a-b)x + c$	1 1 1								
27.	<p>Group of 50 people , 20 believe in non-violence  X; Number of Persons who are non-violent  X: 0,1,2</p> <table border="1" data-bbox="309 566 1099 1151"> <thead> <tr> <th>X</th> <th>P(X)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td> <math display="block">P(0) = \frac{{}^{30}C_2 \times {}^{20}C_0}{{}^{50}C_2} = \frac{30 \times 29}{50 \times 49}</math> <math display="block">P(0) = \frac{87}{245}</math> </td> </tr> <tr> <td>1</td> <td> <math display="block">P(1) = \frac{{}^{30}C_1 \times {}^{20}C_1}{{}^{50}C_2}</math> <math display="block">= \frac{30 \times 20 \times 2}{50 \times 49}</math> <math display="block">P(1) = \frac{120}{245}</math> </td> </tr> <tr> <td>2</td> <td> <math display="block">P(2) = \frac{{}^{30}C_0 \times {}^{20}C_2}{{}^{50}C_2} = \frac{20 \times 19}{50 \times 49}</math> <math display="block">P(2) = \frac{38}{245}</math> </td> </tr> </tbody> </table> <p>Mean = <math>\sum X.P(X) = 0 \times P(0) + 1 \times P(1) + 2P(2) = \frac{196}{245}</math></p>	X	P(X)	0	$P(0) = \frac{{}^{30}C_2 \times {}^{20}C_0}{{}^{50}C_2} = \frac{30 \times 29}{50 \times 49}$ $P(0) = \frac{87}{245}$	1	$P(1) = \frac{{}^{30}C_1 \times {}^{20}C_1}{{}^{50}C_2}$ $= \frac{30 \times 20 \times 2}{50 \times 49}$ $P(1) = \frac{120}{245}$	2	$P(2) = \frac{{}^{30}C_0 \times {}^{20}C_2}{{}^{50}C_2} = \frac{20 \times 19}{50 \times 49}$ $P(2) = \frac{38}{245}$	½  1 ½  1
X	P(X)									
0	$P(0) = \frac{{}^{30}C_2 \times {}^{20}C_0}{{}^{50}C_2} = \frac{30 \times 29}{50 \times 49}$ $P(0) = \frac{87}{245}$									
1	$P(1) = \frac{{}^{30}C_1 \times {}^{20}C_1}{{}^{50}C_2}$ $= \frac{30 \times 20 \times 2}{50 \times 49}$ $P(1) = \frac{120}{245}$									
2	$P(2) = \frac{{}^{30}C_0 \times {}^{20}C_2}{{}^{50}C_2} = \frac{20 \times 19}{50 \times 49}$ $P(2) = \frac{38}{245}$									
27.OR	<p>X: Bolts produced by Machine X ;P(X)=1/6  Y: Bolts produced by Machine Y; P(Y)=2/6=1/3  Z: Bolts produced by Machine Z; P(X)=3/6=1/2  E: defective bolts produced</p> <p>i. <math>P(E/Y) = \frac{1.5}{100} = \frac{3}{200} = 0.015</math></p> <p>ii. <math>P(\text{defective by Z}) = P(Z) \times P(E/Z) = \frac{3000}{6000} \times \frac{2}{100} = 0.01</math></p> <p>iii. <math>P(E) = \frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{1.5}{100} + \frac{1}{2} \times \frac{2}{100} = \frac{1}{60}</math></p>	1 1 1								
28.	$\int_1^4 ( x-1  +  x-2  +  x-4 ) dx =$ $\int_1^4 (x-1) dx + \int_1^2 (2-x) dx + \int_2^4 (x-2) dx + \int_1^4 (4-x) dx$ $= \left[ \frac{x^2}{2} - x \right]_1^4 + \left[ 2x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} - 2x \right]_2^4 + \left[ 4x - \frac{x^2}{2} \right]_1^4 = \frac{23}{2}$	1 ½ 1 ½								

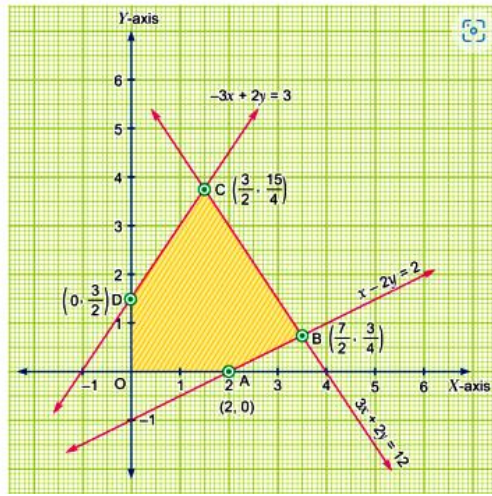
28.(or)	<p>Let <math>I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x \sin x}{e^x + 1} dx \dots \dots \dots (i)</math></p> <p>Also, <math>I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\left(\frac{-\pi}{2} + \frac{\pi}{2} - x\right) \sin\left(\frac{-\pi}{2} + \frac{\pi}{2} - x\right)}{e^{\left(\frac{-\pi}{2} + \frac{\pi}{2} - x\right)} + 1} dx</math></p> <p><math>I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{-x \sin(-x)}{e^{-x} + 1} dx, I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x \sin x e^x}{e^x + 1} dx \dots \dots \dots (ii)</math></p> <p>Adding (i) and (ii), we obtain <math>2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x \sin x}{e^x + 1} + \frac{x \sin x e^x}{e^x + 1}\right) dx</math></p> <p><math>2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin x dx, 2I = [-x \cos x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx</math></p> <p><math>2I = [-x \cos x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}</math></p> <p><math>2I = (0 - 0) + 1 - (-1),</math></p> <p><math>\therefore 2I = 2, I = 1</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>1 \frac{1}{2}</math></p>
Q29.	<p><math>(1 + y^2)dx = (\tan^{-1}y - x)dy, y(0) = 0</math></p> <p><math>\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1 + y^2}, \Rightarrow \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{\tan^{-1}y}{1 + y^2} \dots \dots (i)</math>, This is a linear differential equation with <math>P = \frac{1}{1 + y^2}</math> and <math>Q = \frac{\tan^{-1}y}{1 + y^2}</math>,</p> <p><math>\therefore I.F = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1}y}</math></p> <p>G.S</p> <p><math>x e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1 + y^2} e^{\tan^{-1}y} dy + C</math></p> <p><math>\Rightarrow x e^{\tan^{-1}y} = \int t e^t dt + C, \text{ where } t = \tan^{-1}y,</math></p> <p><math>\Rightarrow x e^{\tan^{-1}y} = e^{\tan^{-1}y} (\tan^{-1}y - 1) + C \dots \dots \dots (ii)</math></p> <p>It is given that <math>y = 0</math> when <math>x = 0</math>. From (ii), we get <math>0 = e^0(0 - 1) + C</math></p> <p><math>\Rightarrow C = 1</math></p> <p>From (ii) we get <math>x e^{\tan^{-1}y} = e^{\tan^{-1}y} (\tan^{-1}y - 1) + 1</math></p> <p><math>\Rightarrow (x - \tan^{-1}y + 1) e^{\tan^{-1}y} = 1</math>, which is the required solution.</p>	<p>1</p> <p>1</p> <p>1</p>
29.(or)	<p>The given differential equation is <math>(x^2 + xy)dy = (x^2 + y^2)dx</math></p> <p><math>\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy} \dots (i)</math></p> <p>Putting <math>y = xv</math> and <math>\frac{dy}{dx} = v + x \frac{dv}{dx}</math> in (i),</p> <p>we get <math>v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{x^2 + vx^2}, \Rightarrow v + x \frac{dv}{dx} = \frac{1 + v^2}{1 + v}</math></p> <p><math>\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2 - v - v^2}{1 + v}</math></p> <p><math>\Rightarrow x \frac{dv}{dx} = \frac{1 - v}{1 + v} \Rightarrow x(1 + v)dv = (1 - v)dx</math></p> <p><math>\Rightarrow \frac{1 + v}{1 - v} dv = \frac{dx}{x} \Rightarrow \frac{2 - (1 - v)}{1 - v} dv = \frac{dx}{x}</math>. Integrating both sides we get</p> <p><math>\int \left(\frac{2}{1 - v} - 1\right) dv = \int \frac{dx}{x} \Rightarrow \int \frac{2}{1 - v} dv - \int 1 \cdot dv = \int \frac{dx}{x}</math></p> <p><math>\Rightarrow -2 \log 1 - v  - v = \log x  + \log C</math></p>	<p>1</p> <p>1</p>

$\Rightarrow \log|x| + \log C + 2\log|1 - v| = -v$   
 $\Rightarrow \log\{C|x|(1 - v)^2\} = -v \Rightarrow C|x|(1 - v)^2 = e^{-v}$   
 $\Rightarrow C|x|\left(1 - \frac{y}{x}\right)^2 = e^{-\frac{y}{x}}$   
 $\Rightarrow C(x - y)^2 = |x|e^{-\frac{y}{x}}$  .....(ii) ,It is given that when  $y=0, x=1, \therefore$   
 from (ii) we get  
 $C(1 - 0)^2 = e^0 \Rightarrow C = 1,$   
 $\therefore$  The particular solution is  $(x - y)^2 = |x|e^{-\frac{y}{x}}$ .

1

30. Maximize and Minimize  $Z=5x+2y$   
 Subject to constraints;  $x - 2y \leq 2; 3x + 2y \leq 12; -3x + 2y \leq 3; x \geq 0; y \geq 0$

2



Corner Point	$Z = 5x + 2y$
$O(0, 0)$	0
$A(2, 0)$	10
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	19
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	15
$D\left(0, \frac{3}{2}\right)$	3

$\rightarrow$  Minimum  
 $\rightarrow$  Maximum

**Hence,**  $Z$  is minimum at  $x = 0, y = 0$  and minimum value = 0  
 $Z$  is maximum at  $x = \frac{7}{2}, y = \frac{3}{4}$  and maximum value = 19

2

1

31. Let  $I = \int \frac{\sqrt{x}}{\sqrt{x-3}\sqrt{x}} dx, \therefore I = \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}} - x^{\frac{3}{2}}} dx$ , put  $x = t^6$  and  $dx = 6t^5 dt$ , we get

$I = \int \frac{t^3}{t^3 - t^2} \cdot 6t^5 dt = 6 \int \frac{t^6}{t-1} dt = 6 \int \frac{t^6 - 1 + 1}{t-1} dt = 6 \int \frac{t^6 - 1}{t-1} + \frac{1}{t-1} dt$

$I = 6 \int t^5 + t^4 + t^3 + t^2 + t + 1 + \frac{1}{t-1} dt$

$I = 6 \left\{ \frac{t^6}{6} + \frac{t^5}{5} + \frac{t^4}{4} + \frac{t^3}{3} + \frac{t^2}{2} + t + \log(t-1) \right\} + C$

$I = 6 \left\{ \frac{x}{6} + \frac{x^{\frac{5}{6}}}{5} + \frac{x^{\frac{2}{3}}}{3} + \frac{x^{\frac{1}{2}}}{3} + \frac{x^{\frac{1}{3}}}{2} + x^{\frac{1}{6}} + \log\left(x^{\frac{1}{6}} - 1\right) \right\} + C$

$\frac{1}{2}$

$\frac{1}{2}$

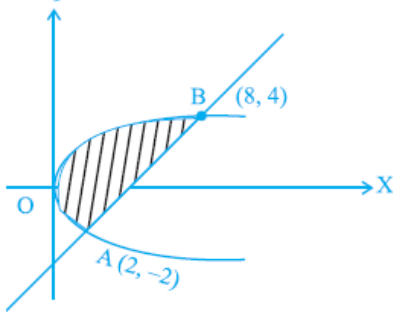
$\frac{1}{2}$

$\frac{1}{2}$

1

**SECTION D**

(This section comprises of long answer-type questions (LA) of 5 marks each)

32.	<div style="text-align: center;">  </div> <p>The intersecting points of the given curves are obtained by solving the equations <math>x - y = 4</math> and <math>y^2 = 2x</math> for <math>x</math> and <math>y</math>.</p> <p>We have <math>y^2 = 8 + 2y</math> i.e., <math>(y - 4)(y + 2) = 0</math> which gives <math>y = 4, -2</math> and <math>x = 8, 2</math>.</p> <p>Thus, the points of intersection are <math>(8, 4), (2, -2)</math>. Hence</p> $\text{Area} = \int_{-2}^4 \left( 4 + y - \frac{1}{2}y^2 \right) dy$ $= \left[ 4y + \frac{y^2}{2} - \frac{1}{6}y^3 \right]_{-2}^4 = 18 \text{ sq units.}$	1
33.	<p><b>Reflexive:</b>            Let <math>(a, b) \in N \times N</math>, then  <math>a + b = b + a</math>  <math>\Rightarrow (a, b)R(b, a)</math>  <math>\Rightarrow R</math> is reflexive.....(1)</p> <p><b>Symmetric:</b>            Let <math>(a, b), (c, d) \in N \times N</math> and <math>(a, b)R(c, d)</math>  <math>a + d = b + c</math>  <math>b + c = a + d</math>  <math>c + b = d + a</math>  <math>(c, d)R(a, b)</math>  <math>\Rightarrow R</math> is symmetric.....(2)</p> <p><b>Transitive:</b>            Let <math>(a, b), (c, d), (e, f) \in N \times N</math> and Let <math>(a, b)R(c, d), (c, d)R(e, f)</math>  <math>\Rightarrow a + d = b + c; c + f = d + e</math>  <math>a + d + c + f = b + c + d + e</math>  <math>a + f = b + e</math>  <math>(a, b)R(e, f)</math>  <math>\Rightarrow R</math> is transitive.....(3)</p> <p>From (1),(2) and (3)</p>	1  1 ½  1 ½  ½

	R is an equivalence Relation.	
33. (or)	<p>Relation S is defined as <math>S = \{(a, b) \in R \times R : 1 + ab &gt; 0\}</math></p> <p><b>Reflexive:</b> Let <math>a \in R</math>  <math>1 + a \cdot a &gt; 0</math>  <math>\Rightarrow 1 + a^2 &gt; 0</math>, true for <math>a \in R</math>  <math>\Rightarrow (a, a)</math> is Reflexive.</p> <p><b>Symmetric:</b> Let <math>a, b \in R</math> and <math>(a, b) \in S</math>  <math>\Rightarrow 1 + ab &gt; 0</math>  <math>1 + ba &gt; 0</math>  <math>\Rightarrow (b, a) \in S</math>  <math>\Rightarrow R</math> is symmetric</p> <p><b>Transitive:</b> Let <math>a, b, c \in R</math> and <math>(a, b), (b, c) \in S</math>  <math>\Rightarrow 1 + ab &gt; 0</math> and <math>1 + bc &gt; 0</math> but this may not imply <math>1 + ac &gt; 0</math>.  i.e. <math>(a, c) \notin S</math>.</p> <p>For example,  Let <math>a = 1, b = \frac{1}{2}, c = -\frac{3}{2}</math> then <math>(a, b) \in S</math> i.e. <math>(1, \frac{1}{2}) \in S</math>  <math>\Rightarrow 1 + 1 \times \frac{1}{2} &gt; 0</math>, true</p> <p>Now, <math>(b, c) \in S</math> i.e. <math>(\frac{1}{2}, -\frac{3}{2}) \in S</math>  <math>\Rightarrow 1 + \frac{1}{2} \times -\frac{3}{2} &gt; 0 \Rightarrow 1 - \frac{3}{4} &gt; 0 \Rightarrow \frac{1}{4} &gt; 0</math>, true</p> <p>Now for <math>(a, c) \in S</math>  i.e. <math>(1, -\frac{3}{2}) \in S</math>  <math>\Rightarrow 1 + 1 \times -\frac{3}{2} &gt; 0 \Rightarrow -\frac{1}{2}</math>  <math>\Rightarrow -\frac{1}{2} \not&gt; 0</math>, hence false</p> <p><math>\therefore (a, b) \in S, (b, c) \in S</math> but <math>(a, c) \notin S</math>.  <math>\Rightarrow S</math> is not transitive</p> <p>Therefore, S is reflexive, symmetric and but not transitive.  <math>\Rightarrow</math> therefore, S is not an equivalence Relation.</p>	<p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
34.	<p><math>L_1: \vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})</math></p> <p><math>L_2: \vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})</math></p> <p>Any point on <math>L_1, P(3 + \lambda, 2 + 2\lambda, -4 + 2\lambda)</math></p> <p>Any point on <math>L_2, Q(5 + 3\mu, -2 + 2\mu, 6\mu)</math></p> <p>If the <math>L_1</math> and <math>L_2</math> intersect then, they must have a common point on them,  i.e. P and Q must coincide for some values of <math>\lambda</math> and <math>\mu</math></p> <p>now, <math>3 + \lambda = 5 + 3\mu</math>.....(1)</p> <p>similarly, <math>2 + 2\lambda = -2 + 2\mu</math>.....(2)</p> <p>Also, <math>-4 + 2\lambda = 6\mu</math></p> <p>solving 1 and 2,  we get <math>\lambda = -4, \mu = -2</math></p> <p>substitute the values in 3,  <math>-4 + 2(-4) = 6(-2)</math>  <math>-12 = -12</math></p> <p>LHS=RHS</p> <p>Therefore the point of intersection <math>(-1, -6, -12)</math></p>	<p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p><math>\frac{1}{2}</math></p>



34.(or)	<p>Shortest distance between lines with vector equations <math>\vec{r} = \vec{a}_1 + \lambda \vec{b}_1</math></p> <p>and <math>\vec{r} = \vec{a}_2 + \mu \vec{b}_2</math> is <math>\frac{ \vec{b}_1 \times \vec{b}_2 \cdot (\vec{a}_2 - \vec{a}_1) }{ \vec{b}_1 \times \vec{b}_2 }</math></p> $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ $= 1\hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k} \quad = s\hat{i} + 1\hat{i} + 2s\hat{j} - 1\hat{j} - 2s\hat{k} - 1\hat{k}$ $= (1\hat{i} - 2\hat{j} + 3\hat{k}) + t(-1\hat{i} + 1\hat{j} - 2\hat{k}) \quad = (1\hat{i} - 1\hat{j} - 1\hat{k}) + s(1\hat{i} + 2\hat{j} - 2\hat{k})$ <p>Comparing with <math>\vec{r} = \vec{a}_1 + t\vec{b}_1</math>, <span style="margin-left: 100px;">Comparing with <math>\vec{r} = \vec{a}_2 + s\vec{b}_2</math>,</span></p> $\vec{a}_1 = 1\hat{i} - 2\hat{j} + 3\hat{k} \quad \vec{a}_2 = 1\hat{i} - 1\hat{j} - 1\hat{k}$ $\& \vec{b}_1 = -1\hat{i} + 1\hat{j} - 2\hat{k} \quad \& \vec{b}_2 = 1\hat{i} + 2\hat{j} - 2\hat{k}$ <p>Now, <math>(\vec{a}_2 - \vec{a}_1) = (1\hat{i} - 1\hat{j} - 1\hat{k}) - (1\hat{i} - 2\hat{j} + 3\hat{k})</math></p> $= (1-1)\hat{i} + (-1+2)\hat{j} + (-1-3)\hat{k}$ $= 0\hat{i} + 1\hat{j} - 4\hat{k}$ $(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$ $= \hat{i}(1 \times -2 - (-2 \times -2)) - \hat{j}((-1 \times -2) - (1 \times -2)) + \hat{k}((-1 \times 2) - (1 \times 1))$ $= \hat{i}[-2 + 4] - \hat{j}[2 + 2] + \hat{k}[-2 - 1]$ $= 2\hat{i} - 4\hat{j} - 3\hat{k}$ <p>Magnitude of <math>(\vec{b}_1 \times \vec{b}_2) = \sqrt{2^2 + (-4)^2 + (-3)^2}</math></p> $ \vec{b}_1 \times \vec{b}_2  = \sqrt{4 + 16 + 9} = \sqrt{29}$ $d = \frac{8}{\sqrt{29}} \text{ units.}$	<p>1</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>
35.	$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ $= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$ $= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ <p>If <math>A = \begin{bmatrix} 1 &amp; -1 &amp; 1 \\ 1 &amp; -2 &amp; -2 \\ 2 &amp; 1 &amp; 3 \end{bmatrix}</math></p> $A^{-1} = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}, X = A^{-1}B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ $X = \frac{1}{8} \begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x = 3, y = -2, z = -1$	<p>1</p> <p>1/2</p> <p>1 1/2</p> <p>1</p> <p>1</p>

**Section E : (Case studies/Passage based questions of 4 marks each)**

36.	<p>1. <math>y = 4x - \frac{1}{2}x^2</math>  <math>\frac{dy}{dx} = 4 - x</math>            When <math>\frac{dy}{dx} = 0 \Rightarrow 4 - x = 0 \Rightarrow x = 4</math></p> <p>2. <math>x = 2 \text{ cm},</math>  <math>y = 4x - \frac{1}{2}x^2 \Rightarrow y = 4 \times 2 - \frac{1}{2}2^2 = 6 \text{ cm}</math>            Height of the plant will be 6 cm</p> <p>3. <math>f''(x) = -1 &lt; 0 \Rightarrow f(x)</math> attains maximum at <math>x = 4,</math>  <math>f(4) = 4 \times 4 - \frac{1}{2}4^2 = 8 \text{ cm}</math>            Local minimum value : not defined            Local maximum value <math>f(4) = 8 \text{ cm}</math>  <i>now, In <math>[0,5]</math></i>  <math>f(0) = 0 \text{ cm}</math>  <math>f(4) = 8 \text{ cm}</math>  <math>f(5) = 4 \times 5 - \frac{1}{2}5^2 = 20 - 12.5 = 7.5 \text{ cm}</math>            Therefore,            Absolute Maxima is <math>x=4</math> and Absolute Maximum value is 8cm            Absolute minima is <math>x=0</math> and Absolute Minimum value is 0cm.</p> <p>3. Or</p> <table border="1" data-bbox="300 1234 1262 1357"> <thead> <tr> <th>Interval</th> <th>Sign of <math>f'(x)</math></th> <th>Nature of the function</th> </tr> </thead> <tbody> <tr> <td><math>(-\infty, 4)</math></td> <td><math>&gt; 0</math></td> <td><math>f(x)</math> strictly increasing</td> </tr> <tr> <td><math>(4, \infty)</math></td> <td><math>&lt; 0</math></td> <td><math>f(x)</math> strictly decreasing</td> </tr> </tbody> </table>	Interval	Sign of $f'(x)$	Nature of the function	$(-\infty, 4)$	$> 0$	$f(x)$ strictly increasing	$(4, \infty)$	$< 0$	$f(x)$ strictly decreasing	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p>
Interval	Sign of $f'(x)$	Nature of the function									
$(-\infty, 4)$	$> 0$	$f(x)$ strictly increasing									
$(4, \infty)$	$< 0$	$f(x)$ strictly decreasing									
37.	<p>1. <math>P(x) = -5x^2 + 125x + 37500</math>  <math>P'(x) = -10x + 125, \text{ when } P'(x) = 0, x = 12.5</math>  <math>P''(x) = -10 &lt; 0</math>            Therefore, at <math>x = 12.5</math> the profit is maximum</p> <p>2. Maximum profit = <math>-5(12.5)^2 + 125 \times 12.5 + 37500 = ₹ 38281.25</math></p> <p>3. <math>P(x) = 38250; -5x^2 + 125x + 37500 = 38250</math>  <math>x^2 - 25x + 150 = 0</math>  <math>x = 10, x = 15</math></p> <p>3. or            When <math>x = 2, P(2) = -5 \times 2^2 + 125(2) + 37500 = ₹ 37750</math></p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>or</p> <p>2</p>									
38.	<p>Probability A will hit = <math>\frac{4}{5}</math>            Probability B will hit = <math>\frac{3}{4}</math>            Probability C will hit = <math>\frac{2}{3}</math>            i.. <math>P(\text{any two will hit}) = P(\text{A will not hit}) * P(\text{B will hit}) * P(\text{C will hit}) + P(\text{A will hit}) * P(\text{B will not hit}) * P(\text{C will hit}) + P(\text{A will hit}) * P(\text{B will hit}) * P(\text{C will not hit})</math></p>	1									

	<p>hit)*P(B will not hit)*P(C will hit)+ P(A will hit)*P(B will hit)*P(C will not hit)</p> $= \left(1 - \frac{4}{5}\right) \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \left(1 - \frac{3}{4}\right) \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \left(1 - \frac{2}{3}\right)$ $= \frac{1}{10} + \frac{2}{15} + \frac{1}{5} = \frac{13}{30}$ <p>ii.. P(at least one will hit)=P(exactly one will hit)+P(exactly two will hit)+P(all will hit)</p> <p>P(exactly one will hit) = P(A will hit)*P(B will not hit)*P(C will not hit)+ P(A will not hit)*P(B will hit)*P(C will not hit)+ P(A will not hit)*P(B will not hit)*P(C will hit)</p> $= \frac{4}{5} \times \left(1 - \frac{3}{4}\right) \times \left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{5}\right) \times \frac{3}{4} \times \left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{5}\right) \times \left(1 - \frac{3}{4}\right) \times \frac{2}{3}$ $= \frac{1}{15} + \frac{1}{20} + \frac{1}{30} = \frac{9}{60} \dots\dots\dots(1)$ <p>P(exactly two will hit)= <math>\frac{13}{30}</math>.....(2)</p> <p>P(all will hit)= <math>\frac{2}{5}</math>.....(3)</p> <p>Therefore , P(at least one will hit)= <math>\frac{9}{60} + \frac{13}{30} + \frac{2}{5} = \frac{59}{60}</math></p>	<p>1</p> <p>1</p> <p>1</p>
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