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BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION 2023 – 24
SUBJECT: MATHEMATICS
SET – 1 - MARKING SCHEME

CLASS XII
DATE:

TIME: 3 HOURS
MAX. MARKS: 80

Section – A

1) (c) $3^6 = 729$

2) (b) $AB = I$

$$\begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x+y=0$

3) (c) $|A| = 14$

$|adj\,A| = 14^2$

$|adj(adj\,A)| = (14^2)^2 = 14^4$

4) (c) $y = x\sqrt{1-x^2} + \sin^{-1}x$

$$\frac{dy}{dx} = \frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

$$\frac{1-x^2+1-x^2}{\sqrt{1-x^2}} = 2\sqrt{1-x^2}$$

5) (a) $\int_0^2 [x^2]\,dx$

$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx$

$= 0 + \sqrt{2} - 1 + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3})$

$= 5 - \sqrt{3} - \sqrt{2}$

⑥ a) $f(x) = x^x$

$f'(x) = x^x(1+\log x)$

$x = \frac{1}{e}$

$f(e) = e^{\frac{-1}{e}}$.

⑦ b) $f(x) = x^3 - 9kx^2 + 27x + 30$

$f'(x) = 3x^2 - 18kx + 27$

$= 3(x^2 - 6kx + 9)$

$36k^2 - 4(9) < 0$

$36(k^2 - 1) < 0$

$-1 < k < 1$

⑧ $y = \log_2 x$

$\frac{dy}{dx} = \frac{1}{x\log 2}$

⑨ $x\frac{dy}{dx} - 2y = \sin x$

$\frac{dy}{dx} - \frac{2}{x} = \frac{\sin x}{x}$

$I.F = e^{\int -\frac{2}{x}dx}$

$= \frac{1}{x^2}$

10 b) $(\vec{a}.\vec{b})^2 + (\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

$2^2 + 4^2 = |\vec{a}|^2|\vec{b}|^2$

$|\vec{a}|^2 * |\vec{b}|^2 = 20$

11) a

12 d) $\vec{a}+\vec{b}+\vec{c} = 0$

$\vec{a}+\vec{b} = -\vec{c}$

$|\vec{a}|^2+|\vec{b}|^2+2\,\vec{a}.\vec{b} = |\vec{c}|^2$

$2\,\vec{a}.\vec{b} = 49-34 = 15$

$\cos\theta = \frac{15}{30} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

13 d)

14 d) $\frac{3(x-2)}{12} = -\frac{(y-2)}{3} = \frac{z-5}{-1}$

$\frac{x-2}{4} = \frac{y-2}{-3} = \frac{z-5}{-1}$

15 a) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

$= 4\hat{i} + 5\hat{j} + 7\hat{k}$

(16) d) $3p = q$

(17) d)

(18) c probability of getting atleast one head $= 1 - \frac{1}{8}$

$= \frac{7}{8}$

(19) c

(20) a

Section - B

(21) $\cos(\tan^{-1}x) = \sin(\cot^{-1}\frac{3}{4})$

$\frac{1}{\sec(\tan^{-1}x)} = \frac{1}{\operatorname{cosec}(\cot^{-1}\frac{3}{4})}$

$\sec(\tan^{-1}x) = \operatorname{cosec}(\cot^{-1}\frac{3}{4})$ ——— $\frac{1}{2}$

$\sqrt{1+\tan^2(\tan^{-1}x)} = \sqrt{1+\cot^2(\cot^{-1}\frac{3}{4})}$ ——— $\frac{1}{2}$

$1+x^2 = 1+\frac{9}{16}$

$x^2 = \frac{9}{16}$ ——— $\frac{1}{2}$

$x = \pm\frac{3}{4}$ ——— $\frac{1}{2}$

(OR)

③

LHS

$= \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right)$ ———— $\frac{1}{2}$

$= \tan^{-1} \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$ ———— $\frac{1}{2}$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)$ ———— $\frac{1}{2}$

$= \frac{\pi}{2} - \frac{x}{2}$ ———— $\frac{1}{2}$

22

$A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$A^2 - 4A + 7I$

$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$ ———— 1

Finding A^{-1} ———— 1m

23

(23) $f(x) = \sin x(1+\cos x)$ $\quad 0 < x < \frac{\pi}{2}$

$f'(x) = 2\cos^2 x + \cos x - 1$ ——— $\frac{1}{2}$

$f'(x)$ $(2\cos x - 1)(\cos x + 1) > 0$ ——— $\frac{1}{2}$

$2\cos x - 1 > 0$ and $\cos x + 1 > 0$

$x \in (0, \frac{\pi}{3})$ and $x \in (0, \frac{\pi}{2})$ ——— 1

$x \in (0, \frac{\pi}{3})$

$\therefore$ $f(x)$ is increasing on $(0, \frac{\pi}{3})$

OR

$\frac{dV}{dt} = 3 \text{ cm}^3/\text{s}$

$\frac{4}{3}\pi r^3 = V$

$4\pi r^2 \cdot \frac{dr}{dt} = 3$

$\frac{dr}{dt} = \frac{3}{4\pi r^2}$ ——— 1

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$= 8\pi r \cdot \frac{3}{4\pi r^2}$ ——— 1

$\frac{dS}{dt} = \frac{6}{r}$

$\left.\frac{dS}{dt}\right|_{r=2} = \frac{6}{2} = 3 \text{ cm}^2/\text{sec}$

24

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad -(1)$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad -(2)$$

— 1

(1) + (2)

$$2I = \int_0^{\pi/2} 1\,dx$$

$$I = \frac{\pi}{4}$$

— 1

25

$$e^x \tan y\,dx = (e^x-1)\sec^2 y\,dy$$

$$\int \frac{e^x}{e^x-1}\,dx = \int \frac{\sec^2 y}{\tan y}\,dy$$

— 1

$$\log|e^x-1| = \log|\tan y| + \log c$$

$$\frac{e^x-1}{\tan y} = c$$

— 1

Section-C

(26) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$x^2(1+y) = y^2(1+x)$

$x^2 + x^2y = y^2 + xy^2$

$x^2 - y^2 + x^2y - xy^2 = 0$

$(x+y)(x-y) + xy(x-y) = 0$

$(x-y)(x+y+xy) = 0$ $\quad$ (as $x \neq y$)

$x + y + xy = 0$ $\quad$ 2m

$y = \frac{-x}{x+1}$

$\frac{dy}{dx} = -\frac{1}{(x+1)^2}$ $\quad$ 1m

OR

$x = \sin\left(\frac{1}{a}\log y\right)$

$\sin^{-1}x = \frac{1}{a}\log y$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{ay}\frac{dy}{dx}$ $\quad$ — 1

$\sqrt{1-x^2}\frac{dy}{dx} = ay$

$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{2x}{2\sqrt{1-x^2}}\frac{dy}{dx} = a\frac{dy}{dx}$ $\quad$ — 1

$$\left(\sqrt{1-x^2}\right)^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} = a\sqrt{1-x^2}\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a^2 y$$

-1

(27) $\int_e^2 \left[\log(\log x) + \frac{1}{\log x}\right] dx$

let $\log x = t$ -1

$$\frac{1}{x} dx = dt$$

$$dx = x\, dt$$

$$dx = e^t\, dt$$

$x = e \Rightarrow t = 1$ -1

$x = 2 \Rightarrow t = \log 2$

$$\int_1^{\log 2} e^t \left(\log t + \frac{1}{t}\right) dt$$

-1

$$= \left[e^t \log t\right]_1^{\log 2}$$

$$= e^{\log 2}\log(\log 2) - e^1 \log(1)$$

$$= 2\log(\log 2)$$

(28)

OR

$$I = \int_0^2 |x^3 - x|\, dx$$

$x^3 - x$

$x(x^2 - 1)$

$= x(x-1)(x+1)$

$= \; - \quad + \quad - \quad +$ at $-1 \quad 0 \quad 1$

$$= \int_0^1 |x^3 - x|\, dx + \int_1^2 |x^3 - x|\, dx$$

$$= \int_0^1 (x - x^3)\, dx + \int_1^2 (x^3 - x)\, dx$$

$$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2} - \frac{1}{4} + 4 - 2 - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= -\frac{3}{4} + 2 = \frac{5}{4}$$

1

1

1

(5)

(28)

$$I = \int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$$

$$\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$$

$$A = \frac{3}{5} \quad B = \frac{2}{5} \quad C = \frac{1}{5}$$

— 1.5

$$I = \frac{3}{5}\int \frac{1}{x+2}\,dx + \frac{2}{5}\int \frac{x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}x + C$$

1.5

(29)

$$\frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}}$$

— 1

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\cos\frac{y}{x}}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

— 1

$$v + x\frac{dv}{dx} = v + \frac{1}{\cos v}$$

$$\int \cos v\,dv = \int \frac{1}{x}\,dx$$

$$\sin v = \log|x| + C$$

$\cancel{-\cos v}$

— 1

$$\sin\frac{y}{x} = \log|x| + C$$

(OR)

$$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$$

$$\frac{dy}{dx}+\frac{2x}{x^2+1}y=\frac{\sqrt{x^2+4}}{x^2+1}$$ — 1

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{2x}{x^2+1} \quad Q=\frac{\sqrt{x^2+4}}{x^2+1}$$

$$I.F=e^{\int P\,dx}$$

$$=x^2+1$$ — 1

G.S

$$y\,IF=\int Q\cdot IF\,dx+C$$

$$(x^2+1)y=\int\sqrt{x^2+4}\,dx$$

$$y(x^2+1)=\frac{x}{2}\sqrt{x^2+4}+2\log\left|x+\sqrt{x^2+4}\right|+C$$ — 1

(30)

$x+2y=120$

x	y	(x,y)
0	60	(0,60)
120	0	(120,0)

$x+y=60$

x	y	(x,y)
0	60	(0,60)
60	0	(60,0)

$x-2y=0$

x	y	(x,y)
0	0	(0,0)
60	30	(60,30)

put (0,0) in $x+2y\le 120$

$0\le 120$ (T)

∴ shade Towards (0,0)

put (0,0) in $x+y\ge 0$

$0\ge 60$ (F)

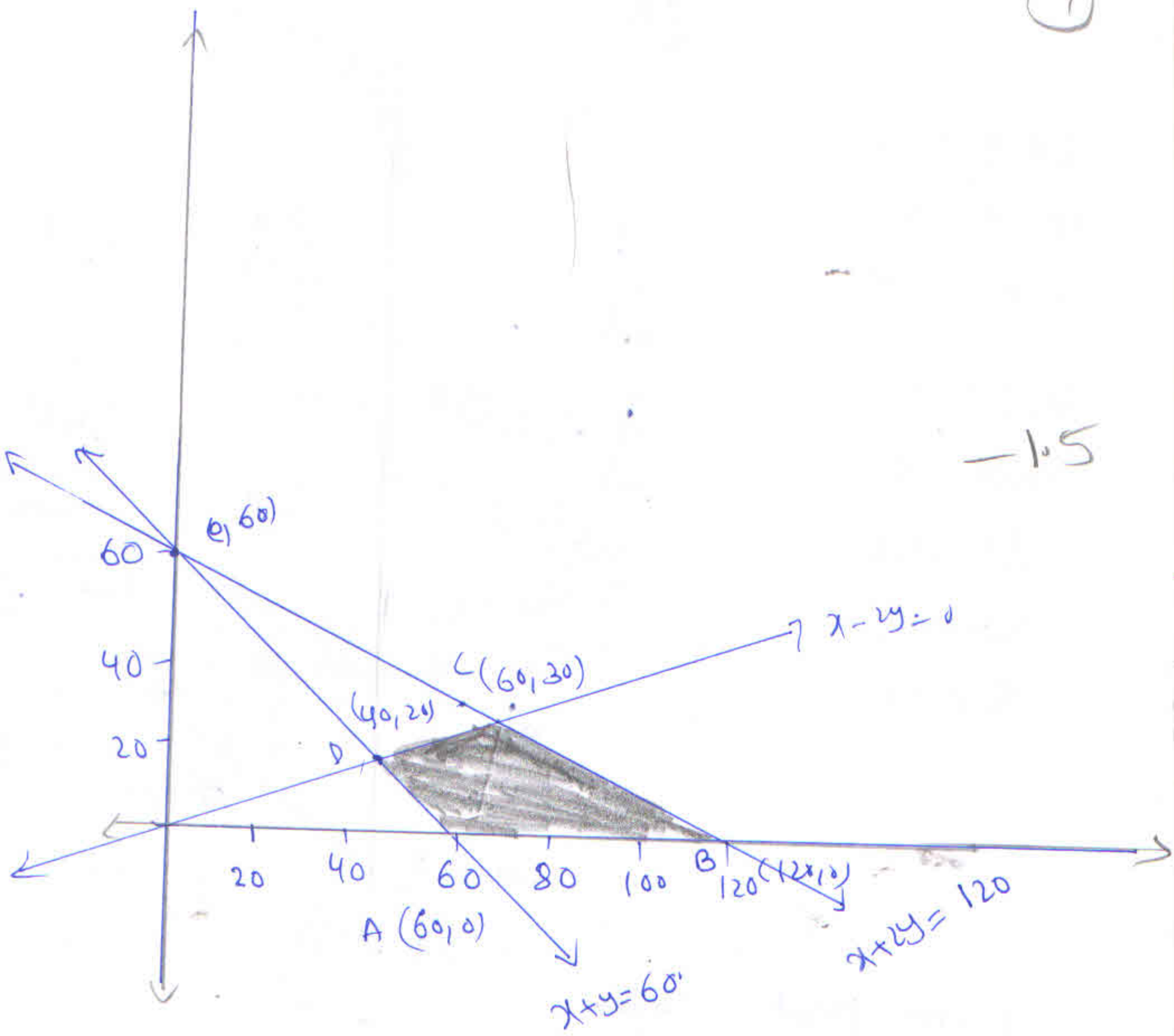
∴ shade away from origin — 1

put (0,30) in $x-2y\ge 0$

$-60\ge 0$ (false)

∴ shade away from (0,30)

(39) (7)



-1.5

vertex	value of $Z = 5x + 10y$
B (120, 0)	600
A (60, 0)	300
C (60, 30)	600
D (40, 20)	400

maximum of Z is attains at every point on the segment joint B and C

$\frac{1}{2}$

OR

$2x-y \geq -5$

$2x-y=-5$

$(0,5)\ (1,7)$

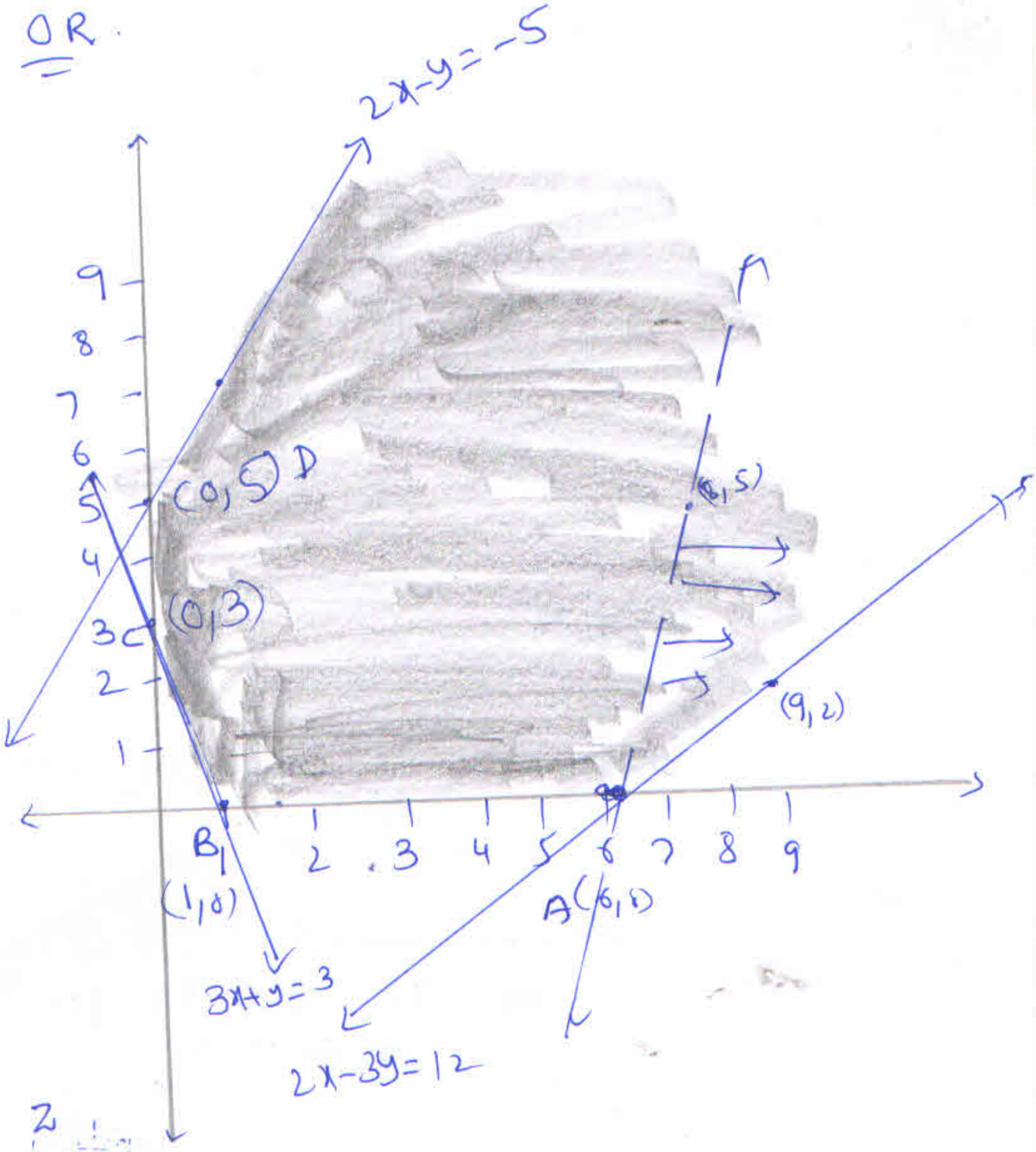
$3x+y \geq 3$

$3x+y=3$

$(1,0)\ (0,3)$

$2x-3y=12$

$(6,0)\ (9,2)$



Corner point	Z
(0,5)	100
(0,3)	60
(1,0)	-50
(6,0)	-300 → minimum

$-5x+2y<-30$

feasiable region and $-5x+2y<-30$ have common points

∴ Z has no minimum

(31) Let E_1 be the event that the lost card is diamond

E_2 be the even that the lost card is not diamond

$P(E_1) = \frac{1}{4}$ $P(E_2) = \frac{3}{4}$

Let 'A' be the event that the two cards drawn are both diamonds

$P(A|E_1) = \frac{^{12}C_2}{^{51}C_2}$ $P(A|E_2) = \frac{^{13}C_2}{^{51}C_2}$

1.5

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(A)}$$

$$= \frac{\frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2}}{\frac{1}{4}\frac{^{12}C_2}{^{51}C_2} + \frac{3}{4}\frac{^{13}C_2}{^{51}C_2}}$$

1.5

$$= \frac{66}{66 + 3 \times 78}$$

$$= \frac{11}{50}$$

(32)

$(a,b) R (c,d) \iff ad(b+c) = bc(a+d)$

Reflexive:

$$ab(a+b) = ab(a+b)$$

$$\Rightarrow ab(b+a) = ba(a+b)$$

$$\Rightarrow (a,b) R (a,b)$$

$\therefore$ R is reflexive

— 1

Symmetric: $(a,b) R (c,d)$

$$\Rightarrow ad(b+c) = bc(a+d)$$

$$\Rightarrow bc(a+d) = ad(b+c)$$

$$\Rightarrow cb(d+a) = da(c+b)$$

$$\Rightarrow (c,d) R (a,b)$$

1.5

Transitive: $(a,b) R (c,d) \Rightarrow ad(b+c) = bc(a+d)$

(c,

$$\frac{b+c}{bc} = \frac{a+d}{ad}$$

$$\frac{1}{c} + \frac{1}{b} = \frac{1}{a} + \frac{1}{d} \quad \text{—(1)}$$

~~$((c,d) R (a,b) \Rightarrow bc(a+d) = ad(b+c)$~~

~~$\Rightarrow$~~

$(c,d) R (e,f)$

$\Rightarrow cf(d+e) = de(c+f)$

$\Rightarrow \frac{d+e}{de} = \frac{c+f}{fc}$

$\Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f}$ — ②

① − ②

$\frac{1}{a} + \cancel{\frac{1}{d}} - \cancel{\frac{1}{d}} - \frac{1}{e} = \cancel{\frac{1}{c}} + \frac{1}{b} - \cancel{\frac{1}{c}} - \frac{1}{f}$ —— 2.5

$\frac{1}{a} + \frac{1}{f} = \frac{1}{b} + \frac{1}{e}$

$\frac{a+f}{af} = \frac{b+e}{be}$

$\Rightarrow be(a+f) = af(b+e)$

$\Rightarrow af(b+e) = be(a+f)$

$(a,b) R (e,f)$

$\therefore$ R is transitive.

OR

$R = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ — 1.5

Reflexive: $(a,a) \in R \quad \forall\ a \in A$

$a - a = 0 \leq 5$ — 1

$\therefore (a,a) \in R$

$\therefore$ R is reflexive

Symmetric: $(a,b) \in R \Rightarrow |a^2 - b^2| \leq 5$

$\Rightarrow |b^2 - a^2| \leq 5$ — 1

$\therefore$ a $(b,a) \in R$

$\therefore$ R is symmetric

Transitive:

$(1,2) \in R$ as $|1^2 - 2^2| \leq 5$

$(2,3) \in R$ as $|2^2 - 3^2| \leq 5$ — 1.5

$(1,3) \notin R$ as $|1^2 - 3^2| > 5$

$\therefore$ R is not Transitive

(33) $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$

$|A| = 2(-4+4) - 3(6-5) + 1(12-10)$

$= -3+2$

$= -1$

$A_{11} = 0$ $\quad A_{21} = -(-6+4) = 2$ $\quad A_{31} = 3-2 = 1$

$A_{12} = -(6-5) = -1$ $\quad A_{22} = -4-5 = -9$ $\quad A_{32} = -(2+3) = -5$

$A_{13} = 12-10 = 2$ $\quad A_{23} = -(-8-15) = 23$ $\quad A_{33} = 4+9 = 13$

$\text{adj}A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{A} \text{adj}A$

$= \begin{bmatrix} 0 & -2 & -1 \\ +1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}$

By writing the given equations in matrix equation form

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$C \quad X = B$

$X = C^{-1} B$

$C^{-1} = (A^{-1})^T = \begin{bmatrix} 0 & -1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & +1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{0}1 \\ 2 \\ 3 \end{bmatrix}$$

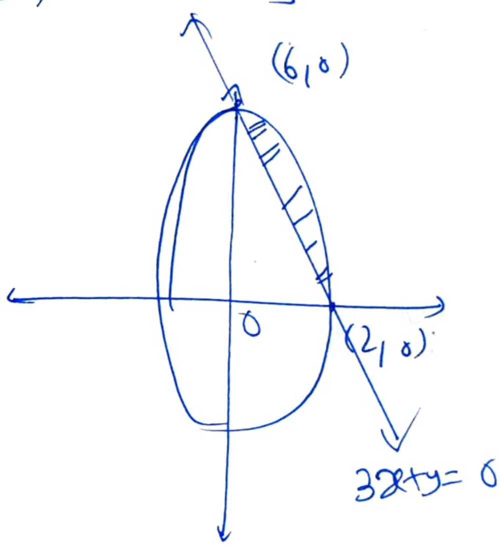
$\therefore\ x = \cancel{0}1, \quad y = 2, \quad z = 3$

(34) $\{ (x, y) : 9x^2 + y^2 \le 36,\ 3x + y \ge 6 \}$

$9x^2 + y^2 = 36$

$\frac{x^2}{4} + \frac{y^2}{36} = 1$

$\frac{x^2}{2^2} + \frac{y^2}{6^2} = 1$



$3x + y = 6$

x,	y	(x, y)
0	6	(0, 6)
2	0	(2, 0)

(11)

Required Area

$$= 3\int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (6-3x)\,dx$$

$$= 3\left[\frac{x}{2}\sqrt{4-x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right] - \left[6x - \frac{3x^2}{2}\right]_0^2$$

$$= 3\left[2\frac{\pi}{2} - 2\right]$$

$$= 3(\pi - 2) \text{ Square units}$$

(35) $l = -\dfrac{(bm+cn)}{a}$

$$ul^2 + vm^2 + wn^2 = 0$$

$$\frac{u(bm+cn)^2}{a^2} + vm^2 + wn^2 = 0$$

$$u[b^2m^2 + c^2n^2 + 2bcmn] + vm^2a^2 + wa^2n^2 = 0$$

$$m^2[b^2u + va^2) + mn(2bcu) + n^2(c^2u + wa^2) = 0$$

$$(b^2u + a^2v)\left(\frac{m}{n}\right)^2 + 2bcu\left(\frac{m}{n}\right) + c^2u + a^2w = 0$$

which is a quadratic having roots $\frac{m_1}{n_1}$ and $\frac{m_2}{n_2}$

If lines are perpendicular

$$\frac{m_1}{n_1} \times \frac{m_2}{n_2} = \frac{c^2u + a^2w}{b^2u + a^2v}$$

$$\frac{m_1m_2}{c^2u + a^2w} = \frac{n_1n_2}{b^2u + a^2v} \quad \text{— (1)}$$

Similarly

$$\frac{l_1 l_2}{b^2 w + c^2 v} = \frac{m_1 m_2}{c^2 u + a^2 w} \quad - \text{(ii)}$$

from (1) & (2)

$$\frac{l_1 l_2}{b^2 w + c^2 v} = \frac{m_1 m_2}{c^2 u + a^2 w} = \frac{n_1 n_2}{b^2 u + a^2 v} = \lambda$$

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ [$\because$ (they are perpendicular)

$a^2 (v+w) + b^2 (w+u) + c^2 (u+v) = 0.$

~~(OR)~~

(ii) ||.

$\Delta = 0$

$(2bcu)^2 - 4(b^2 u + a^2 v)(c^2 u + a^2 w) = 0$

$$\Rightarrow \frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$$

(OR)

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = \lambda \quad - \text{(i)}$$

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \mu \quad - \text{(ii)}$$

Any point on (i) $(7\lambda - 1, -6\lambda - 1, \lambda - 1)$

Any point of (ii) $(\mu + 3, -2\mu + 5, \mu + 7)$

Direction ratios of PQ .

$(\mu - 7\lambda + 4, \ -2\mu + 6\lambda + 6, \ \mu - \lambda + 8)$

PQ is the shortest distance between (i) and (ii)

only when PQ is perpendicular to both (i) and (ii)

$\therefore \ 20\mu - 86\lambda = 0$ $\quad$ PQ $\perp$ line (i)

$3\mu - 10\lambda = 0$ $\quad$ PQ $\perp$ line (ii)

$\lambda = 0, \ \mu = 0$

$\therefore P(-1, -1, -1)$ $\quad$ $Q(3, 5, 7)$

$PQ = \sqrt{116}$

Equation $\quad \dfrac{x-3}{2} = \dfrac{y-5}{3} = \dfrac{z-7}{4}$

Section - E

38

(i) $n(S) = 24$

Daughter is at one end $\quad n(A) = 12$

Father and mother in the middle $\quad n(B) = 4$

$n(A \cap B) = 4$

$P(A|B) = \dfrac{4}{4} = 1$

(ii) mother at right end $n(A) = 6$

son and daughter together $n(B) = 12$

$n(A \cap B) = 4$

$n(A|B) = \frac{4}{12} = \frac{1}{3}$

(iii) Father and mother on the middle $= n(A) = 4$

Son on the right end $n(B) = 6$

$n(A \cap B) = 2$

$P(A|B) = \frac{2}{6} = \frac{1}{3}$

or

Father and son standing together $n(A) = 12$

mother and daughter standing together $n(B) = 12$

$n(A \cap B) = 8$

$P(A|B) = \frac{2}{3}$

(37) (i) $\vec{a} + \vec{b} = 4\hat{i} + \hat{j} + 0\hat{k}$

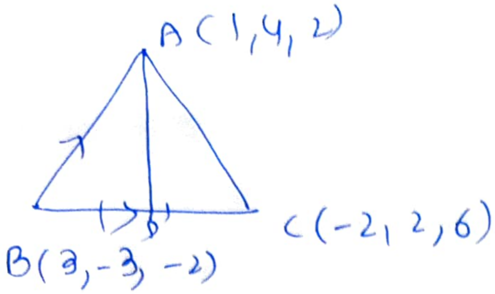
$\hat{c} = \frac{4}{\sqrt{17}}\hat{i} + \frac{1}{\sqrt{17}}\hat{j}$

(ii) projection of $\vec{c}$ on $\vec{b} = \frac{\vec{c} \cdot \vec{b}}{|\vec{b}|}$

$= \frac{-6 + (-6) + (-12)}{\sqrt{20}}$

$= \frac{-24}{2\sqrt{5}} = \frac{-12}{\sqrt{5}}$

13. (iii)



$\vec{a} = -2\hat{i} + 7\hat{j} + 4\hat{k}$

$\vec{b} = -5\hat{i} + 5\hat{j} + 8\hat{k}$

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 7 & 4 \\ -5 & 5 & 8 \end{vmatrix}$$

$= i(56 - 20) - \hat{j}(-16 + 20) + \hat{k}(-10 + 35)$

$= 36\hat{i} - 4\hat{j} + 25\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{1296 + 16 + 625}$

$= \sqrt{1937}$

Area $= \frac{1}{2}\sqrt{1937}$ Sq units

(or)

$$|\vec{AD}| = \left|\frac{\vec{AB} + \vec{AC}}{2}\right|$$

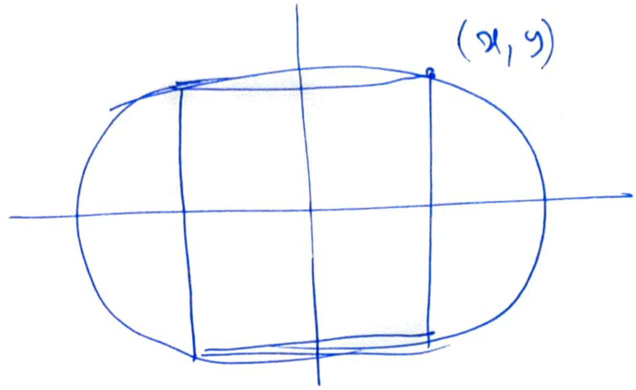
$$= \left|\frac{(2i - 7\hat{j} - 4\hat{k}) + -3\hat{i} + (-2\hat{j}) + 4\hat{k}}{2}\right|$$

$$= \frac{|\hat{i} - 9\hat{j} + 0\hat{k}|}{2}$$

$$= \frac{\sqrt{82}}{2}$$

(38)

(i)



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$A = 2x \cdot 2y$$

$$= 4\frac{b}{a}x\sqrt{a^2 - x^2}$$

(ii) $f(x) = 4\frac{b}{a}x\sqrt{a^2 - x^2}$

$$f'(x) = 4\frac{b}{a}\left(\frac{x(-2x)}{2\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}\right)$$

$$f'(x) = 0$$

$$x = \frac{a}{\sqrt{2}}$$

$$l = \sqrt{2}a$$

$$b = \sqrt{2}b$$