BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION

PRE-BOARD EXAMINATION 2023 – 24

SUBJECT: MATHEMATICS

SET – 1

CLASS XII **TIME: 3 HOURS**

DATE: **MAX. MARKS: 80**

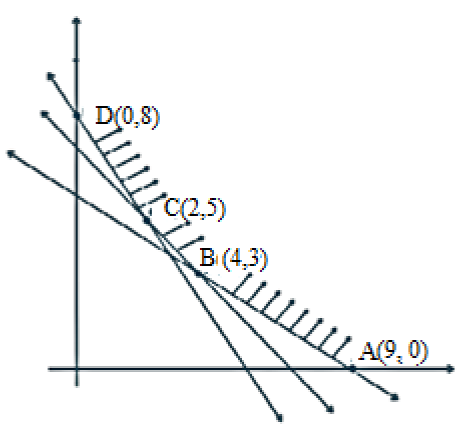
GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is Compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts

	SECTION A	
1	Total number of possible matrices of order 3×2 with each entry 2 or 0 or -1 is a) 27 b) 81 c) 729 d) 512	**[1]**
2	If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then x + y equals a) 3 b) 0 c) -1 d) 2	**[1]**
3	If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj(adj A)) is a) 14^3 b) 14	**[1]**

	c) 14^4 d) 14^2	
4	If $y = x\sqrt{1-x^2} + \sin^{-1}x$, then $\frac{dy}{dx}$ is equal to a) x b) $\frac{1}{\sqrt{1-x^2}}$ c) $2\sqrt{1-x^2}$ d) $\sqrt{1-x^2}$	**[1]**
5	$\int_0^2 [x^2]\, dx$ equals, where [ ] denotes Greatest Integer Function a) $5-\sqrt{3}-\sqrt{2}$ b) 3 c) $\sqrt{5}-4$ d) $\sqrt{5}+4$	**[1]**
6	When x is positive, the minimum value of x^x is a) $e^{\frac{-1}{e}}$ b) $e^{\frac{1}{e}}$ c) $\frac{1}{e}$ d) e^e	**[1]**
7	If the function $f(x) = x^3 - 9kx^2 + 27x + 30$ is increasing on R, then a) $0 < k < 1$ b) $-1 < k < 1$ c) $k < -1$ or $k > 1$ d) $-1 < k < 0$	**[1]**
8	If $y = \log_2 x$ then $\frac{dy}{dx} =$ a) $\frac{1}{x}\log 2$ b) $\frac{1}{x\log 2}$ c) $\frac{1}{x}$ d) $\frac{1}{x\log e}$	**[1]**

9	Integrating factor of the differential equation $x\frac{dy}{dx} - 2y = \ sinx$ is a) x^2 b) $\frac{1}{x^2}$ c) e^{x^2} d) $\frac{2}{x^2}$	**[1]**
10	If $\left\|\vec{a} \times \vec{b}\right\| = 4, \left\|\vec{a} \cdot \vec{b}\right\| = 2$, then $\|\vec{a}\|^2\left\|\vec{b}\right\|^2 =$ a) 2 b) 20 c) 8 d) 6	**[1]**
11	Consider the following statements in respect of the differential equation: $\frac{d^2y}{dx^2}$ + cos $\left(\frac{dy}{dx}\right)$ = 0 1. The degree of the differential equation is not defined. 2. The order of the differential equation is 2. Which of the above statement(s) is/are correct? a) Both (i) and (ii) b) Only (ii) c) Only (i) d) Neither (i) nor (ii)	**[1]**
12	If $\vec{a} + \vec{b} + \vec{c} = \vec{0},\ \|\vec{a}\| = 3,\ \ \left\|\vec{b}\right\| = 5,\ \|\vec{c}\| = 7$, then the angle between $\vec{a}\ and\ \vec{b}$ is a) $\frac{\pi}{6}$ b) $\frac{5\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{3}$	**[1]**
13	If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}\ with\ \hat{\imath},\ \ \frac{\pi}{4}\ with\ \hat{\jmath}\ and\ an\ acute\ angle\ \theta\ with\ \hat{k}$, then the components of $\vec{a}$ are a) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{3}$ b) $\frac{1}{3}, \frac{1}{\sqrt{2}}, \frac{1}{2}$	**[1]**

	c) $\frac{1}{3}, \frac{1}{\sqrt{3}}, \frac{1}{2}$ d) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$	
14	The Cartesian equations of a line are $\frac{3x-6}{12} = \frac{2-y}{3} = \frac{z-5}{-1}$. Its vector equation is a) $\vec{r} = (6\hat{\imath} - \hat{\jmath} + 5\hat{k}) + \lambda(12\hat{\imath} - 3\hat{\jmath} - \hat{k})$ b) $\vec{r} = (6\hat{\imath} + 3\hat{\jmath} - \hat{k}) + \lambda(12\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$ c) $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$ d) $\vec{r} = (2\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(4\hat{\imath} - 3\hat{\jmath} - \hat{k})$	**[1]**
15	The direction ratios of the line perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to a) 4, 5, 7 b) − 4, 5, 7 c) 4, − 5, − 7 d) 4, − 5, 7	**[1]**
16	The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q > 0. Condition on ‘p’ and ‘q’ so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is a) p = q b) p = 3q c) 2p = q d) 3p = q	**[1]**
17	Feasible region (shaded) for a LPP is shown in the Figure. Minimum of Z = 4x + 3y occurs at the point. 	**[1]**

	a) (4, 3) b) (9, 0) c) (0, 8) d) (2, 5)	
18	A die tossed thrice. Then the probability of getting atleast one head is a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{7}{8}$ d) $\frac{1}{8}$	**[1]**
19	**Assertion (A):** A function f: Z → Z defined as $f(x) = x^3$ is injective. **Reason (R):** A function f: A → B is said to be injective if every element of B has a pre - Image in A. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**
20	**Assertion (A):** If $f(x) = \frac{\sin(ax+b)}{\cos(cx+d)}$, then $f'(x) = a\cos(ax+b)\sec(cx+d) + c\sin(ax+b)\tan(cx+d)\sec(cx+d)$ **Reason (R):** If f(x) = $\frac{u}{v}$, then f ′ (x) = $\frac{vu' - uv'}{v^2}$. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**
	SECTION B	
21	Solve the following equation $\cos(\tan^{-1}x) = \sin(\cot^{-1}\frac{3}{4})$. **OR** Prove that, $\tan^{-1}\left(\frac{cosx}{1+sinx}\right) = \frac{\pi}{4} - \frac{x}{2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.	**[2]**
22	If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = O$. hence find A^{-1}.	**[2]**

23	Find the intervals in which $f(x)$ is increasing: $f(x) = \sin x(1 + \cos x), \quad 0 < x < \frac{\pi}{2}$ **OR** The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface, when the radius is 2cm.	**[2]**
24	Prove that: $\int_0^{\pi/2} \frac{dx}{(1+\sqrt{\tan x})} = \frac{\pi}{4}$	**[2]**
25	Solve the differential equation : $e^x \tan y\, dx + (1 - e^x) \sec^2 y dy = 0$	**[2]**
	SECTION C	
26	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$. **OR** If $x = \sin\left(\frac{1}{a} log y\right)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a^2y$.	**[3]**
27	Evaluate $\int_e^2 \left[\log(\log x) + \frac{1}{\log x}\right] dx$. **OR** Evaluate $\int_0^2 \lvert x^3 - x \rvert\, dx$.	**[3]**
28	Evaluate $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\, dx$.	
29	Solve the differential equation : $(x \cos \frac{y}{x})\frac{dy}{dx} = (y \cos \frac{y}{x}) + x$ **OR** Solve the differential equation $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$	**[3]**
30	Solve the following LPP graphically: Maximize $Z = 5x + 10y$ Subject to $x + 2y \leq 120, \quad x + y \geq 60, \quad x - 2y \geq 0, \; x, y \geq 0$ **OR** Determine graphically the minimum value of the objective function $Z = -50x + 20y$ Subject to constraints $2x - y \geq -5, \; 3x + y \geq 3, \; -2x + 3y \geq -12, \; x, y \geq 0$	**[3]**
31	A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.	**[3]**

	SECTION- D	
32	Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)R(c, d) \iff ad(b + c) = bc(a + d)$. Prove that R is an equivalence relation. **OR** Let A = {1, 2, 3} and R = {(a, b): a, b∈ A and $\lvert a^2 - b^2 \rvert \leq 5$. Write R as set of ordered pairs. Mention whether R is 1. reflexive 2. symmetric 3. transitive Give reason in each case.	[5]
33	If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ find A^{-1} , using A^{-1} solve the system of equations $2x - 3y + 5z = 11;\ 3x + 2y - 4z = -5;\ x + y - 2z = -3.$	[5]
34	Using method of integration find the area of the region: $\{(x, y) : 9x^2 + y^2 \leq 36,\ 3x + y \geq 6\}$.	[5]
35	Show that the straight lines whose direction cosines are given by the equations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular, if $a^2(v + w) + b^2(u + w) + c^2(u + v) = 0$ and, parallel, if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$ **OR** Find the length and equation of shortest distance between the following lines: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.	[5]
	SECTION – E	
	This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)	
36	Family photography is all about capturing group of people who have family ties. These range from the small group, such as parents and their children. New-born photography	

also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

I) Find the probability that daughter is at one end, given that father and mother are in the middle — 1

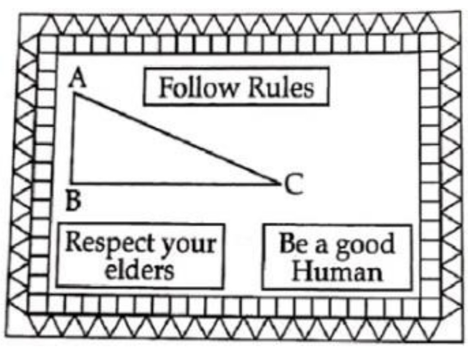
II) Find the probability that mother is at right end, given that son and daughter are together. — 1

III) Find the probability that father and mother are in the middle, given that son is at right end.

OR

Find the probability that father and son are standing together, given that mother and daughter are standing together — 2

37 The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, $-$ 3, $-$ 2) and (-2, 2, 6), respectively.



i) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find the unit vector in the direction of $\vec{a} + \vec{b}$. — 1

ii) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find the projection of $\vec{c}$ on $\vec{b}$ — 1

iii) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively Find area of $\triangle$ ABC.

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively Then find length of the median through A. — 2

38	In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.  I) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x. II) find the length and width of the soccer field (in terms of a and b) that maximize its area.	 2 2

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