BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION

PRE-BOARD EXAMINATION 2023 – 24

SUBJECT: MATHEMATICS

SET – 2

CLASS XII **TIME: 3 HOURS**

DATE: **MAX. MARKS: 80**

GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is Compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts

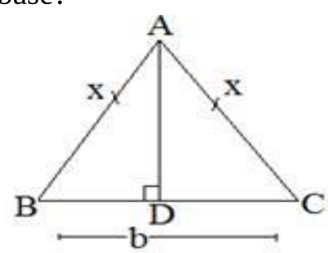
	SECTION A	
1	If $A^T A^{-1}$ is symmetric, then a) $A = (A^T)^2$ b) $A^2 = (A^T)^2$ c) $A^2 = A^T$ d) $A^2 \neq (A^T)^2$	**[1]**
2	If $A^2 - A + I = O$, then the inverse of A is a) $A + I$ b) A^{-2} c) $A - I$ d) $I - A$	**[1]**

3	If $A.(adj\ A) = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ then det(adjA) = a) 8 b) 4 c) 0 d) 64	**[1]**
4	If $f(x) = \begin{cases} mx+1, & if x \leq \frac{\pi}{2} \\ \sin x + n, & if x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then a) $m = n = \frac{\pi}{2}$ b) $n = \frac{m\pi}{2}$ c) m = 1, n = 0 d) $m = \frac{n\pi}{2} + 1$	**[1]**
5	If y = $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx} = ?$ a) $\frac{1}{(1+x^2)}$ b) $\frac{-1}{(1+x^2)}$ c) $\frac{-x}{(1+x^2)}$ d) $\frac{1}{(1+x^2)^{3/2}}$	**[1]**
6	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm is a) 10 cm^2 /s b) $\sqrt{3}$ cm^2 /s c) $10\sqrt{3}$ cm^2 /s d) $\frac{10}{3}$ cm^2 /s	**[1]**

7	$\int \frac{dx}{x\sqrt{x^6-1}}\ dx = ?$ a) $\frac{1}{3} cosec^{-1}x^3 + C$ b) $\frac{1}{3}\sec^{-1}x^3 + C$ c) $\frac{1}{3}\cos^{-1}x^3 + C$ d) $\frac{1}{3}\cot^{-1}x^3 + C$	**[1]**
8	The sum of degree and order of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$ is: a) 1 b) 3 c) 2 d) 5	**[1]**
9	Integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is a) $-$ x b) $\sqrt{1-x^2}$ c) $\frac{x}{1+x^2}$ d) $\frac{1}{2}\log(1-x^2)$	**[1]**
10	If $\vec{a}, \vec{b}, \vec{c}$ are any three mutually perpendicular vectors of equal magnitude ‘a’, then $\left\|\vec{a}+\vec{b}+\vec{c}\right\|$ is equal to a) 2a b) $\sqrt{2}a$ c) $\sqrt{3}a$ d) a	**[1]**
11	If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$,then a) $0 < c < 1$ b) $c > 2$ c) $c = \pm\sqrt{2}$	**[1]**

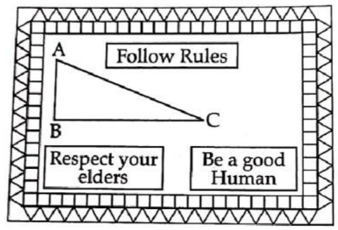
	d) $c = \pm\sqrt{3}$	
12	The equations of the locus of the point $\left(1+\frac{r}{4}, -1+\frac{r}{3}, 2\right)$ where $r \in R$ given by a) $\frac{x-1}{4} = \frac{y+1}{3} = \frac{z-2}{0}$ b) $\frac{x-1}{3} = \frac{y+1}{4} = \frac{z-2}{0}$ c) $\frac{x+1}{4} = \frac{y-1}{3} = \frac{z-2}{0}$ d) z = 2	**[1]**
13	For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{\imath})^2 + (\vec{a} \times \hat{\jmath})^2 + (\vec{a} \times \hat{k})^2$ is equal to a) $3(\vec{a})^2$ b) $4(\vec{a})^2$ c) $(\vec{a})^2$ d) $2(\vec{a})^2$	**[1]**
14	If $\lvert\vec{a}\rvert = 4$ and $-3 \leq \lambda \leq 2$, then the range of $\lvert\lambda\vec{a}\rvert$ is a) [0, 12] b) [0, 8] c) [8, 12] d) [− 12, 8]	**[1]**
15	If a line makes angles $\alpha, \beta\ and\ \gamma$ with the axes respectively, then $\cos 2\alpha + cos2\beta + cos2\gamma =$ a) -2 b) -1 c) 1 d) 2	**[1]**

16	The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of Z = $ax + by$, where a, b > 0 occurs at both (2, 4) and (4, 0), then: a) 3a = b b) 2a = b c) a = 2b d) a = b	**[1]**
17	A die is thrown twice and the sum of the numbers appearing is observed to be 7. The conditional probability that the number 2 has appeared at least once is a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{2}{7}$ d) $\frac{3}{5}$	**[1]**
18	There are two values of a which makes determinant, Δ = $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then sum of these numbers is a) 4 b) 5 c) -4 d) 9	**[1]**
19	**Assertion (A):** A function f: N → N be defined by $f(n) = \begin{cases} \frac{n}{2} & if\ n\ is\ even \\ \frac{(n+1)}{2} & if\ n\ is\ odd \end{cases}$ for all n ∈ N; is one - one. **Reason (R):** A function f: A → B is said to be injective if a ≠ b then f(a) ≠ f(b). a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**

20	**Assertion (A):** $f(x) = \frac{4-x^2}{4x-x^3}$ is not continuous at exactly two points. . **Reason (R):** $f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$ a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true.	**[1]**
	SECTION B	
21	Evaluate: - $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right) + \cot^{-1}\left(\cot\left(\frac{34\pi}{9}\right)\right)$ **OR** Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$	**[2]**
22	If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.	**[2]**
23	Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is increasing or decreasing. **OR** The two equal sides of an isosceles $\triangle$ ABC with fixed base 'b' are decreasing at the rate of 3cm/s. How fast is the area decreasing when the two equal sides are equal to the base?  	**[2]**
24	Evaluate: $\int \frac{8^{1+x} + 4^{1-x}}{2^x} dx$	**[2]**
25	Solve the following differential equation $\frac{dy}{dx} + y = cosx - sinx$.	**[2]**
	SECTION C	
26	If $y = \left(x + \sqrt{x^2+1}\right)^n$, then show that $(x^2+1)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = n^2y$. **OR**	**[3]**

	If $x = \sin\left(\frac{1}{a} log y\right)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx} =\ a^2y.$	
27	Evaluate $\int_0^{\frac{\pi}{2}} \frac{x}{sinx+\cos x}\ dx.$ **OR** Evaluate $\int_0^{\frac{3}{2}} \lvert xsin\pi x\rvert\ \ dx$	**[3]**
28	Evaluate $\int \frac{x^2}{x^4+x^2+1}\ dx.$	**[3]**
29	Solve the differential equation $(1+x^2)\left(\frac{dy}{dx}\right) + 2xy -\ 4x^2 = 0.$ **OR** Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0.$	**[3]**
30	Minimize Z = 400x + 200y subject to $5x + 2y\ \geq\ 30,\ \ 2x + y\ \leq\ 15,$ $x\ \leq\ y,\ \ x \geq 0,\ \ y \geq 0$ **OR** Determine graphically the minimum value of the objective function $Z = -50x + 20y$ Subject to constraints $2x\ - y\ \geq -5,\ \ 3x + y\ \geq 3,\ \ -2x + 3y\ \geq -12,\ x, y\ \geq\ 0$	**[3]**
31	Two numbers selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Write the probability distribution of X and also find E(X).	**[3]**
	SECTION- D	
32	If 'R' and 'S' are two equivalence relations on set 'A' then prove that R∩S is an equivalence relation on set 'A' **OR** Let A = R – {3} and B = R – {1}. Consider the function f : A→ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. Find the value(s) of 'a' which satisfies f(a) = 3	**[5]**
33	Given$A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations $y\ +\ 2z = 7,\ \ \ x - y\ =\ 3,\ \ 2x + 3y + 4z = 17.$	**[5]**
34	Make a sketch of the region $\{(x, y): 0\ \leq 2y \leq x^2\ , 0\ \leq x \leq 3\ \ and\ \ 0 \leq y\ \leq\ x\}$ and find its area using integration.	**[5]**

35	Show that the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r} = (4\hat{\imath} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$ intersect. Also, find their point intersection. **OR** A line with direction ratios (2, 2, 1) intersects the lines $\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1}$ *and* $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ.	**[5]**
	SECTION – E	
	This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)	
36	A card is lost from a pack of 52 cards. From the remaining cards, two cards are drawn at random.  i) Find the probability of drawing two diamonds, given that a card of diamond is missing. ii) Find the probability of drawing two diamonds, given that a card of the heart is missing. iii) Find the probability of drawing two diamond cards. **OR** The two cards drawn are found to be diamond then find the probability of the lost card being a diamond.	1 1 2
37	The slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, $-$ 3, $-$ 2) and ($-$ 2, 2, 6), respectively.	



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i) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find the unit vector in the direction of $\vec{a} + \vec{b}$. 1

ii) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find the projection of $\vec{c}$ on $\vec{b}$ 1

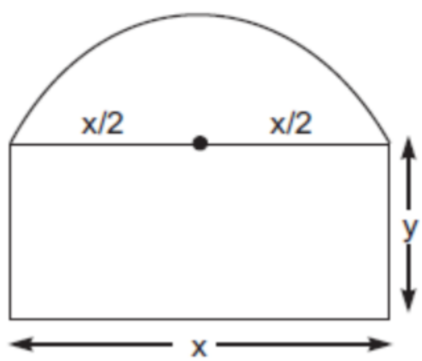
iii) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively Find area of $\triangle$ ABC.

OR

2

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively Then find length of the median through A.

38 Mr Prajwal, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below:



i) Express the combined area (A) of the rectangular region and semi-circular region of the window as a function in terms of 'x'. 2

ii) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, what is length of rectangular portion of the window. 2

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