BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION 2023 – 24
SUBJECT: MATHEMATICS
SET – 2 MARKING SCHEME

CLASS XII | TIME: 3 HOURS
DATE: | MAX. MARKS: 80

1

(1) b

$(A^T A^{-1})^T = A^T A^{-1}$

$(A^{-1})^T A = A^T A^{-1}$

$A^T (A^{-1})^T A = (A^T)^2 A^{-1}$

$A A = (A^T)^2$

(2) d

$A^2 - A + I = 0$

$A - I + A^{-1} = 0$

$A^{-1} = I - A$

(3) d

$|A| = 8$

$|adj A| = 8^2 = 64$

(4) b

$m\frac{\pi}{2} + 1 = 1 + n$

$n = \frac{m\pi}{2}$

(5) a

$x = \tan\theta$

$y = \cot^{-1}(\cot^{-1}(\frac{\pi}{4}+\theta)$

$y = \frac{\pi}{4} + \theta$

$y = \tan^{-1} x$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

6 c) $\frac{da}{dt} = 2$ cm/sec

$A = \frac{\sqrt{3}}{4} a^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2} a \cdot \frac{da}{dt}$

$= \frac{\sqrt{3}}{2} \times 10 \times 2$

$= 10\sqrt{3}$ cm²/se

7 b $\int \frac{1}{x\sqrt{x^6-1}} dx$

$= \int \frac{x^2}{x^3\sqrt{x^6-1}} dx$

$= \frac{1}{3} \sec^{-1} x^3 + c$

8 b 3

9 b $e^{\int \frac{-x}{1-x^2} dx}$

$= e^{+\frac{1}{2}\log(1-x^2)}$

$= \sqrt{1-x^2}$

10 c $x = |\vec{a}+\vec{b}+\vec{c}|$

$x^2 = a^2+b^2+c^2+2(0)$

$x^2 = 3a^2$

$x = \sqrt{3}a$

11 d

$$\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2=1$$

$$\frac{3}{c^2}=1$$

$$c^2=3$$

$$c=\pm\sqrt{3}$$

12 a)

$$\alpha=1+\frac{r}{4} \quad \Big| \quad -1+\frac{r}{3}=\beta \quad \Big| \quad \gamma=2+0\cdot r$$

$$r=4\alpha-4 \quad \Big| \quad r=3\beta+3$$

$4\alpha-4=3\beta+3$ and $\gamma=2$

$$\frac{\alpha-1}{4}=\frac{\beta+1}{3}=\frac{\gamma-2}{0}$$

13 d)

$$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$(\vec{a}\times\hat{i})^2=a_2^2+a_3^2$$

$$(\vec{a}\times\hat{j})^2=a_1^2+a_3^2$$

$$(\vec{a}\times\hat{k})^2=a_2^2+a_3^2$$

$$2(a_1^2+a_2^2+a_3^2)$$

$$=2|\vec{a}|^2$$

14) b)

15) b) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2 - 3$

$= -1$

16) c) $z = ax + by$

$2a + 4b = 4a + 0$

$2a = 4b$

$a = 2b$

17) b)

18) c) $2a^2 + 4 + 2(4a) + 5(8) = 86$

$2a^2 + 8a - 42 = 0$

$a^2 + 4a - 21 = 0$

$(a+7)(a-3) = 0$

$a = -7$ & $a = 3$

Sum $= -7 + 3 = -4$

19) d

20) d

## Section-B

(21) $\sin^{-1}\left(\cos \frac{33\pi}{5}\right) + \cot^{-1}\left(\cot \frac{34\pi}{9}\right)$

$= \sin^{-1}\left(\cos\left(6\pi + \frac{3\pi}{5}\right)\right) + \cot^{-1}\left(\cot 4\pi - \frac{2\pi}{9}\right)$ —— $\frac{1}{2}$

$= \sin^{-1}\left(\cos \frac{3\pi}{5}\right) + \cot^{-1}\left(\cot\left(-\frac{2\pi}{9}\right)\right)$ —— $\frac{1}{2}$

$= \frac{\pi}{2} - \frac{3\pi}{5} + \pi - \frac{2\pi}{9}$ —— $\frac{1}{2}$

$= \frac{5\pi - 6\pi}{10} + \frac{7\pi}{9}$

$= -\frac{\pi}{10} + \frac{7\pi}{9}$ —— $\frac{1}{2}$

$= \frac{61\pi}{90}$

or.

$RHS = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

let $x = \tan^2\theta$

$\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ —— $\frac{1}{2}$

$= \frac{1}{2}\cos^{-1}(\cos 2\theta)$ —— $\frac{1}{2}$

$= \frac{1}{2}(2\theta)$

$= \theta$

$= \tan^{-1}\sqrt{x}$

$\left[\because x = \tan^2\theta \Rightarrow \tan\theta = \sqrt{x},\ \theta = \tan^{-1}\sqrt{x}\right]$ —— 1

(22) $A^2 = kA - 2I$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$ — 1m

$$\begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$3k-2=1$ — 1m

$3k=3$

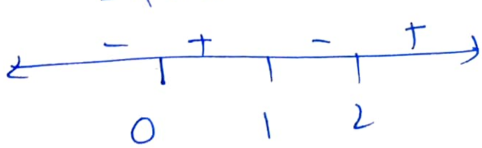
$k=1$

(23) $f(x) = x^4 - 4x^3 + 4x^2 + 15$

$f'(x) = 4x^3 - 12x^2 + 8x$ — $\frac{1}{2}$

$= 4x(x^2 - 3x + 2)$

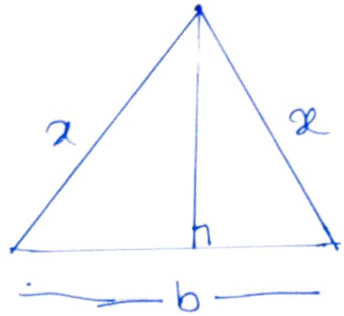
$= 4x(x-2)(x-1)$ — $\frac{1}{2}$

 — $\frac{1}{2}$

Increasing on $(0,1)$, $(2,\infty)$ — $\frac{1}{2}$

Decreasing on $(-\infty,0)$, $(1,2)$.

OR




$$\frac{dx}{dt} = 3\,cm/sec$$

$$h = \sqrt{x^2 - \frac{b^2}{4}} \quad \text{———} \quad \frac{1}{2}$$

$$A = \frac{1}{2}\, b \sqrt{x^2 - \frac{b^2}{4}} \quad \text{———} \quad \frac{1}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\, b \,\frac{1}{2\sqrt{x^2 - \frac{b^2}{4}}} \cdot 2x\, \frac{dx}{dt} \quad \text{———} \quad \frac{1}{2}$$

$$\left.\frac{dA}{dt}\right|_{x=b} = \frac{1}{2}\, b\, \frac{1}{\frac{\sqrt{3}\,b}{2}} \cdot 3b \quad \text{———} \quad \frac{1}{2}$$

$$= \sqrt{3}\, b \quad cm^2/sec.$$

(24)
$$\int \frac{8^{x+1} + 4^{1-x}}{2^x}\, dx$$

$$= \int \frac{8^{x+1}}{2^x} + \frac{4^{1-x}}{2^x}\, dx \quad \text{———} \quad \frac{1}{2}$$

$$= \int 2^{3x+3-x}\, dx + \int 2^{2-2x-x}\, dx \quad \text{———} \quad \frac{1}{2}$$

$$= \int 2^{2x+3}\, dx + \int 2^{2-3x}\, dx \quad \text{———} \quad \frac{1}{2}$$

$$= \frac{2^{2x+3}}{2\log 2} - \frac{2^{2-3x}}{3\log 2} + c \quad \text{———} \quad \frac{1}{2}$$

(25)

$$\frac{dy}{dx} + y = \cos x - \sin x$$

$$\frac{dy}{dx} + Py = Q$$

$$P = 1, \quad Q = \cos x - \sin x$$

$$IF = e^{\int P\,dx}$$

$$= e^{x}\ dx$$

$$I.F\ y = \int IF \times Q\,dx + C$$

$$y e^{x} = \int e^{x}(\cos x - \sin x)\,dx + C$$

$$y e^{x} = e^{x} \cos x + C$$

Secction–C

(26)

$$y = \left(x + \sqrt{x^2+1}\right)^n$$

$$\frac{dy}{dx} = n\left(x + \sqrt{x^2+1}\right)^{n-1}\left(1 + \frac{2x}{2\sqrt{x^2+1}}\right)$$

$$\frac{dy}{dx} = n\left(x + \sqrt{x^2+1}\right)^{n-1}\left(\frac{x + \sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$$

$$\sqrt{x^2+1}\,\frac{dy}{dx} = n\left(x + \sqrt{x^2+1}\right)^{n}$$

$$\sqrt{x^2+1}\,\frac{dy}{dx} = ny$$

$$\sqrt{x^2+1}\,\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{x^2+1}}\frac{dy}{dx} = n\frac{dy}{dx}$$

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{x^2+1}\,\frac{dy}{dx}$$

$$= n \cdot ny$$

$$= n^2 y$$

(OR)

$$x = \sin\left(\frac{1}{a}\log y\right)$$

$$\sin^{-1} x = \frac{1}{a}\log y$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{ay}\frac{dy}{dx}$$ — 1

$$\sqrt{1-x^2}\frac{dy}{dx} = ay$$ — 1

$$\frac{d^2y}{dx^2}\sqrt{1-x^2} + \frac{dy}{dx}\left(\frac{-2x}{2\sqrt{1-x^2}}\right) = a\frac{dy}{dx}$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = a\frac{dy}{dx}$$ — 1

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = a\sqrt{1-x^2}\frac{dy}{dx}$$

$$= a^2y$$

(27) $$I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\,dx \quad \text{—(1)}$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$ — 1

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x+\sin x}\,dx \quad \text{—(2)}$$

(1) + (2)

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x+\sin x}\,dx$$ — 1

$$2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}\,dx$$

$$2I = \frac{\pi}{2\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4\sqrt{2}}\left[\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\right] \qquad - 1$$

$$I = \frac{\pi}{4\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$$

(27) $I = \int_0^{\frac{3}{2}} |x\sin\pi x|\,dx$

$$|x\sin\pi x| = \begin{cases} x\sin\pi x & 0 \le x \le 1 \\ -x\sin\pi x & 1 \le x \le \frac{3}{2} \end{cases} \qquad - 1$$

$$I = \int_0^1 |x\sin\pi x|\,dx + \int_1^{3/2} |x\sin\pi x|\,dx$$

$$= \int_0^1 x\sin\pi x\,dx - \int_0^{\frac{3}{2}} x\sin\pi x\,dx \qquad - 1$$

$$= \left[-\frac{x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]_0^1 - \left[-\frac{x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]_1^{\frac{3}{2}} \qquad - 1$$

$$= \frac{2}{\pi}+\frac{1}{\pi^2}$$

(28) $\int \frac{x^2}{x^4+x^2+1} dx$

$= \frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1} dx + \frac{1}{2}\int \frac{x^2-1}{x^4+x^2+1} dx$ } — 1

$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+\sqrt{3}^2} dx = \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{(x+\frac{1}{x})^2-1} dx$ } — 1

$= \frac{1}{2}\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right) + \frac{1}{2}\cdot\frac{1}{2} \log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + c$

$= \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + \frac{1}{4}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + c$ } — 1

(29) $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$ } — 1

$\frac{dy}{dx} + Py = Q$

$P = \frac{2x}{1+x^2}$ $\quad Q = \frac{4x^2}{1+x^2}$

$IF = e^{\int P dx}$

$= e^{\int \frac{2x}{1+x^2} dx}$

$= 1+x^2$ } — 1

General solution

$$y\,(IF) = \int Q\,(IF)\,dx + C$$

$$y(1+x^2) = \int \frac{4x^2}{\cancel{1+x^2}}\,\cancel{(1+x^2)} + C$$

$$y(1+x^2) = \frac{4x^3}{3} + C.$$

— 1

OR

$$\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$$

$$\frac{dy}{dx} = -\frac{y^2+y+1}{x^2+x+1}$$

$$\int \frac{dy}{y^2+y+1} = -\int \frac{1}{x^2+x+1}\,dx$$

— 1

$$\int \frac{1}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dy = -\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$ — 1.5

$$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = C$$ — 0.5

=

(30) $5x+2y \geq 30$

$5x+2y = 30$

Scale: on x axis: 1 cm = 2 unit

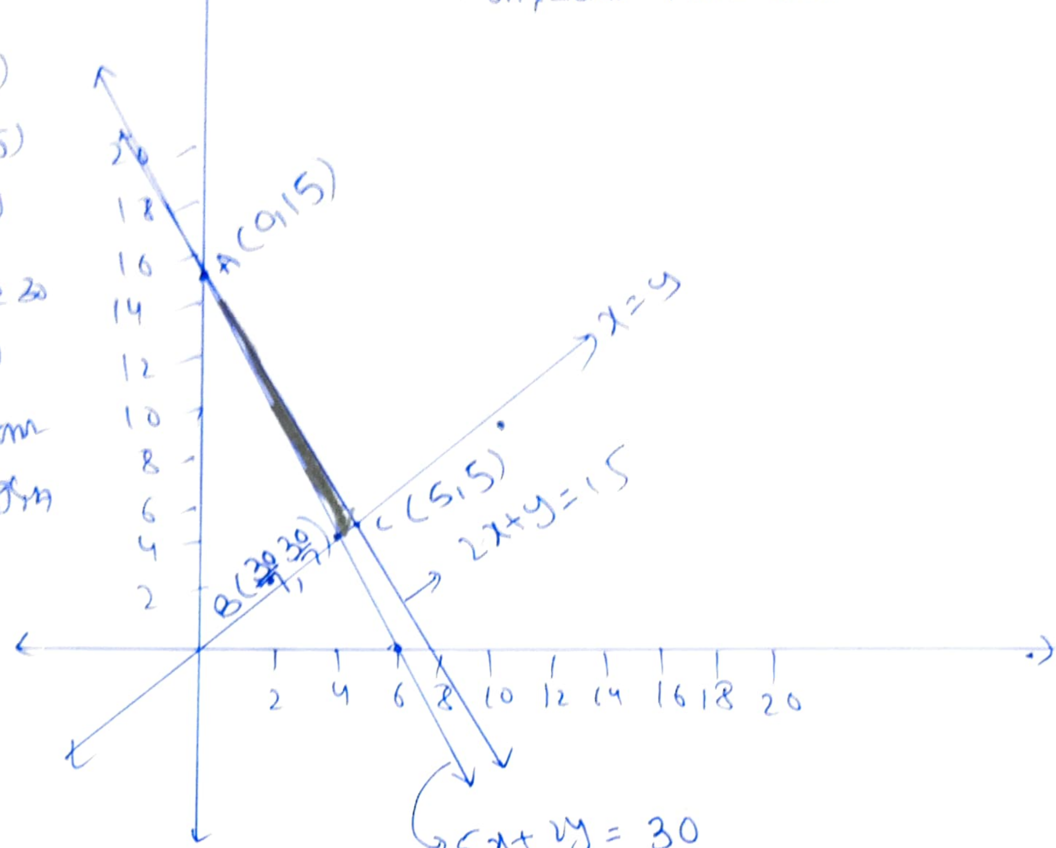
on y-axis 1 cm = 2 unit

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| 0 | 15 | (0, 15) |
| 6 | 0 | (6, 0) |

put (0,0) in $5x+2y \geq 30$

$0 \geq 30$ (F)

shade away from origin




$2x+y \leq 15$

| $x$ | $y$ | $(x, y)$ |
|---|---|---|
| 0 | 15 | (0, 15) |
| 7.5 | 0 | (7.5, 0) |

shade towards origin

Corner points | value of $Z = 400x+200y$

| Corner points | value of $Z = 400x+200y$ |
|---|---|
| (0, 15) | 3000 |
| (5, 5) | 3000 |
| $\left(\frac{30}{7}, \frac{30}{7}\right)$ | $\frac{18000}{7} = 2571.42$ |

$x \leq y$

$x = y$

(0 0) (5,5)

put (0,30) in $x \leq y$

$0 \leq 30$ (True)

shade toward (30, 0)

∴ The minimum value of Z is 2571.42

Data – 1

graph – 1.5

Finding minimum – $\frac{1}{2}$

ORE

OR

$2x - y \geq -5$

$2x - y = -5$

$(0, 5)\ (1, 7)$

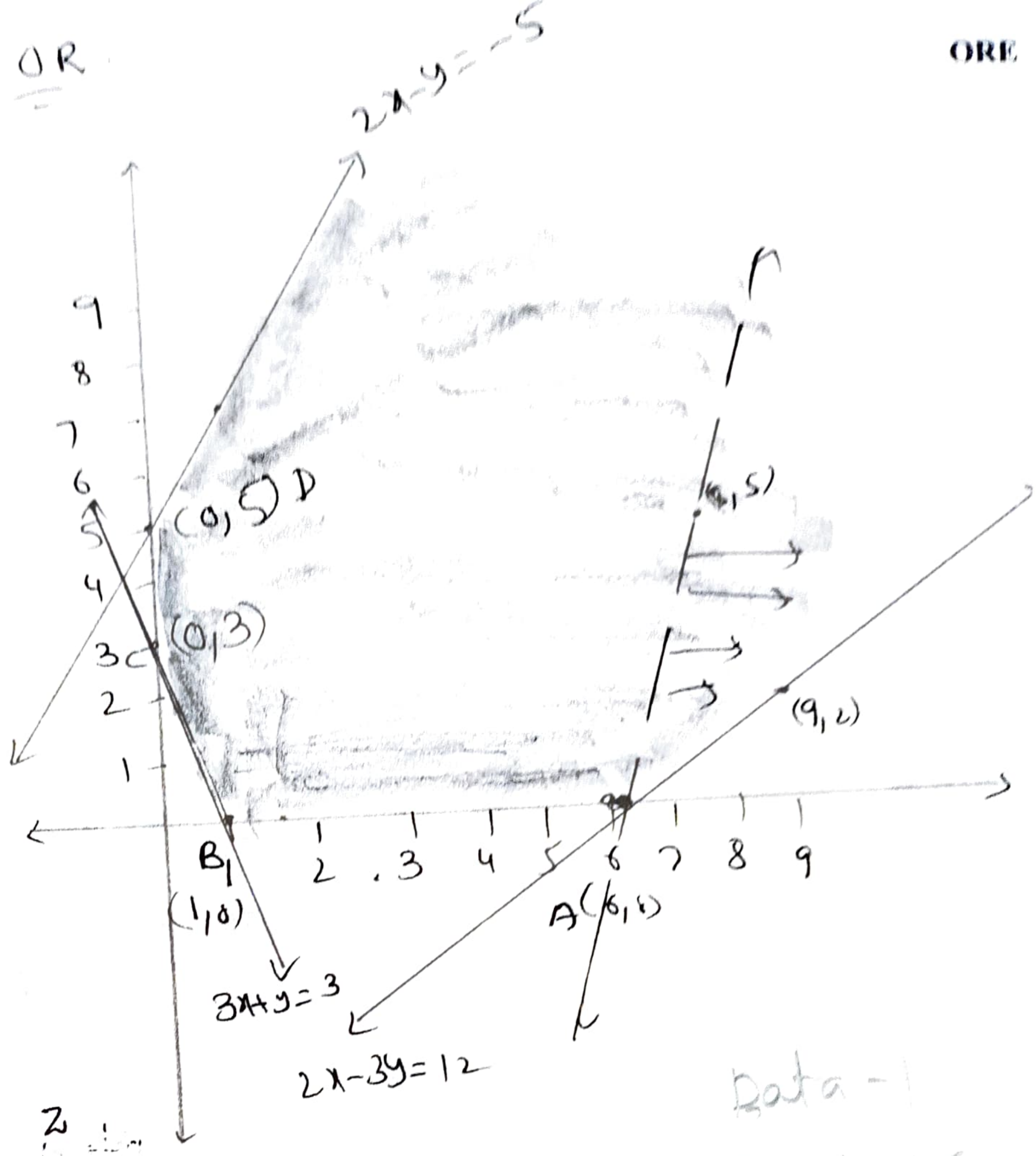
$3x + y \geq 3$

$3x + y = 3$

$(1, 0)\ (0, 3)$

$2x - 3y = 12$

$(6, 0)\ (9, 2)$




| Corner point | Z |
|---|---|
| (0, 5) | 100 |
| (0, 3) | 60 |
| (1, 0) | -50 |
| (6, 0) | -300 → minimum |

Data - 1
graph - 1.5
Finding minima - $\frac{1}{2}$

$-5x + 2y < -30$

feasiable region and $-5x + 2y < -30$ have common points

$\therefore$ Z has no minimum

31)

possible outcome

(1, 2) (1,3) (1,4) (1,5) (2,1) (2,3) (2,4) (2,5) (1,6) (2,6)

(3,1) (3,2) (3,4) (3,5) (3,6) (4,1) (4,2) (4,3) (4,5)

(4,6) (5,1) (5,2) (5,3) (5,4) (5,6) (6,1) (6,2) (6,3) (6,4) (6,5)

$\}$ — 1

$n(S) = 30$

$P(X=2) = \frac{2}{30} = \frac{1}{15}$

$P(X=3) = \frac{4}{30} = \frac{2}{15}$

$P(X=4) = \frac{6}{30} = \frac{1}{5}$

$P(X=5) = \frac{8}{30} = \frac{4}{15}$

$P(X=6) = \frac{10}{30} = \frac{1}{3}$

$\}$ — 1

| $\cancel{X}_i$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $P(x_i)$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{1}{5}$ | $\frac{4}{15}$ | $\frac{1}{3}$ |

$E(X) = \Sigma x_i P(x_i) = \frac{70}{15}$

$= \frac{14}{3}$

$\}$ — 1

Section-D

32)

$R \subset A \times A$ and $S \subseteq A \times A$

$R \cap S \subseteq A \times A$

$R \cap S$ is also a relation on $A$

$\}$ — 1

Section-D

Reflexive: $a \in A$ $\quad a \in A$ $\quad$ ($\because$ S and R are reflexive)

$\Rightarrow (a,a) \in R \quad (a,a) \in S$

$\Rightarrow (a,a) \in R \cap S$ — 1

Thus $(a,a) \in R \cap S$ for all $a \in A$

$\therefore R \cap S$ is reflexive

Symmetric: let $a, b \in A$,

$(a,b) \in R \cap S$

$\Rightarrow (a,b) \in R$ and $(a,b) \in S$ [$\because$ R, S are symmetric)

$\Rightarrow (b,a) \in R$ and $(b,a) \in S$

$\Rightarrow (b,a) \in R \cap S$ — 1

$\therefore R \cap S$ is symmetric

Transitive: $a, b, c \in A$

$(a,b) \in R \cap S$ and $(b,c) \in R \cap S$

$\Rightarrow \{(a,b) \in R$ and $(a,b) \in S\}$ and $\{(b,c) \in R$ and $(b,c) \in S\}$

$\Rightarrow (a,c) \in S$ and $(b,c) \in S$ [$\because$ R, S are transitive)

$\Rightarrow (a,c) \cap (b,c) \in R \cap S$ — 2

$\therefore$ R is transitive

$\therefore$ R is equivalence relation

OR

one - one: $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \qquad (x_1 \neq 3,\ x_2 \neq 3)$$

$$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

— 2

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$\Rightarrow x_1 = x_2$$

$\therefore$ f is one-one

onto: $f(x) = \frac{x-2}{x-3}$

$$\frac{x-2}{x-3} = y$$

$$x - \cancel{2} = xy - 3y$$

$$x - xy = 2 - 3y$$

$$x = \frac{2-3y}{1-y}$$

$$f(x) = \frac{x-2}{x-3}$$

— 2

$$= \frac{\frac{2-3y}{1-y} - 2}{\frac{2-3y}{1-y} - 3}$$

$$= \frac{2-3y-2+2y}{2-3y-3+3y}$$

$$= \frac{-y}{-1} = y$$

1dn ∴ for every $y \in R-\{1\}$ there exist a $x \in R-\{3\}$ such that $f(x)=y$

∴ $f$ is onto

$$f(a) = 3$$

$$\frac{a-2}{a-3} = 3$$

$$a-2 = 3a-9$$

$$2a = 7$$

$$a = \frac{7}{2}$$

1

(33) ~~A~~ BA

BA

$$= \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

2

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$BA = 6I$

$B(\text{adj}B) = 6I$

$|A| = 6,$

$\text{adj}\, B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ — 1

$x - y = 3$

$2x + 3y + 4z = 17$

$y + 2z = 7$

By writing the given equation in matrix equation form

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$BX = C$

— 1

$X = B^{-1} C$

$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix}$

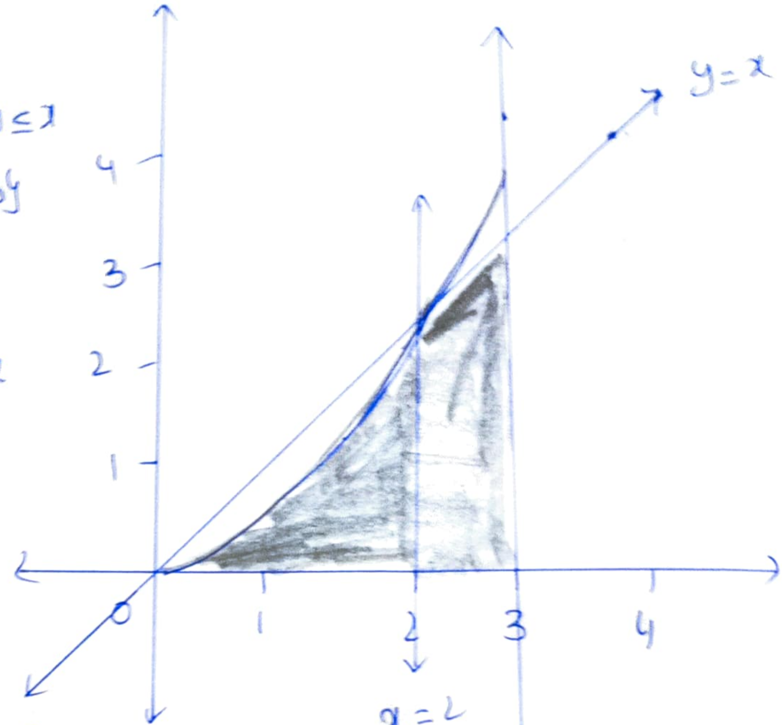
$= \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$ — 1

$\Rightarrow x = 2, \quad y = -1, \quad z = 4$

(34)

$\{(x,y) : 0 \le 2y \le x^2,\ 0 \le y \le x$
$0 \le x \le 3\}$

Corresponding inequations are




$x \ge 0,\ y \ge 0,\quad 2y \le x^2$
$y \le x\quad x \le 1$

Draw the graph

$2y = x^2$ and $y = x$

Sketch — 1.5

$2x - x^2 = 0$

$x = 0,\ 2$

1

$\text{Area} = \int_0^2 \frac{x^2}{2}\,dx + \int_2^3 x\,dx$

1.5

$= \frac{1}{6}\left[x^3\right]_0^2 + \frac{1}{2}\left(x^2\right]_2^3$

$= \frac{4}{3} + \frac{5}{2}$

1

$= \frac{23}{6}$ sq units

(35)

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \quad \text{—} ①$$

$x = 2\lambda+1,\ y = 3\lambda+2,\ z = 4\lambda+3$

$$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu \quad \text{—} ②$$

$x = 5\mu+4,\ y = 2\mu+1,\ z = \mu$

1.5

The general point (1) is

$P(2\lambda+1, 3\lambda+2, 4\lambda+3)$

The general point on (2) is

$Q(5\mu+4, 2\mu+1, \mu)$

The given lines will intersect only when they have a common point. This will happen when 'P' and 'Q' coincide for some particular values of '$\lambda$' and '$\mu$'

-2.5

$2\lambda - 5\mu = 3$ —(i) | $4\lambda - \mu = -3$

$3\lambda - 2\mu = -1$ (ii)

By solving (i) and (ii)

$\lambda = -1, \mu = -1$

Clearly these values of $\lambda$ and $\mu$ satisfies (iii)

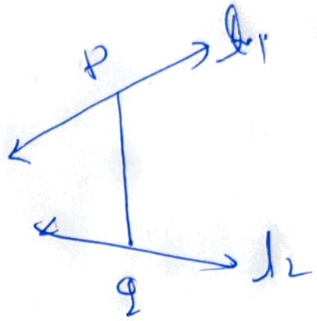
$\therefore$ The given lines intersect

point of intersection is

$(-1,-1,-1)$ — 1

OR

$$\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}=\lambda$$




$P(3\lambda+7,\ 2\lambda+5,\ \lambda+3)$ — 1

$$\cancel{Q(}\ \frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}=\mu$$

$Q(2\mu+1,\ 4\mu-1,\ 3\mu-1)$

D r's PQ = $3\lambda-2\mu+6,\ 2\lambda-4\mu+6,\ \lambda-3\mu+4$ — 1

According equation

$$\frac{3\lambda-2\mu+6}{2}=\frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$$ — 2

$\lambda+2\mu=0$ $\quad 2\mu=2$

$\mu=1$

$\lambda=-2$

$\therefore P(1,1,1)\quad Q(3,3,2)$

$PQ=3.$

Equation of PQ $\quad \frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$ — 1

(36) lost card is diamond ($E_1$) lost card not a diamond ($E_2$)

$P(E_1) = \frac{1}{4}$ $P(E_2) = \frac{3}{4}$

'A' be the event that the two cards ~~are~~ drawn are both diamond.

(i) $P(A|E_1) = \frac{{}^{12}C_2}{{}^{51}C_2}$.

$= \frac{66}{1275}$ — 1

(ii) $E_1$ – heart is missing $E_2$ 'heart not missing

$P(A|E_2) = \frac{{}^{13}C_2}{{}^{51}C_2}$

$= \frac{78}{1275}$. — 1

(iii) $P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$

$= \frac{1}{4} \times \frac{66}{1275} + \frac{3}{4} \times \frac{78}{1275}$

$= \frac{300}{4 \times 1275}$ — 2

$= \frac{1}{17}$

or

(iv) $P(E_1|A) = \frac{\frac{66}{1275} \times \frac{1}{4}}{\frac{66}{1275} \times \frac{1}{4} + \frac{78}{1275} \times \frac{3}{4}} = \frac{11}{50}$

(37) (i) $\vec{a}+\vec{b} = 4\hat{i} + \hat{j} + 0\hat{k}$

$\hat{c} = \frac{4}{\sqrt{17}}\hat{i} + \frac{1}{\sqrt{17}}\hat{j}$ — 1

(ii) projection of $\vec{c}$ on $\vec{b}$

$= \frac{\vec{c}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{-6-6-12}{\sqrt{2\cancel{1}}}$ — 1

$= \frac{-\cancel{12}\,24}{\sqrt{\cancel{8}2}}$

(iii) Area $= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|-36\hat{i}+4\hat{j}-25\hat{k}|$

$= \frac{1}{2}\sqrt{1937}$ sq units — 2

or

$\vec{AD} = \frac{\sqrt{82}}{2}$ units — 2

Reason

$\vec{AD} = \frac{\vec{AC}+\vec{AB}}{2} = \frac{|\hat{i}-9\hat{j}|}{2}$

$= \frac{\sqrt{1+81}}{2} = \frac{\sqrt{82}}{2}$ units

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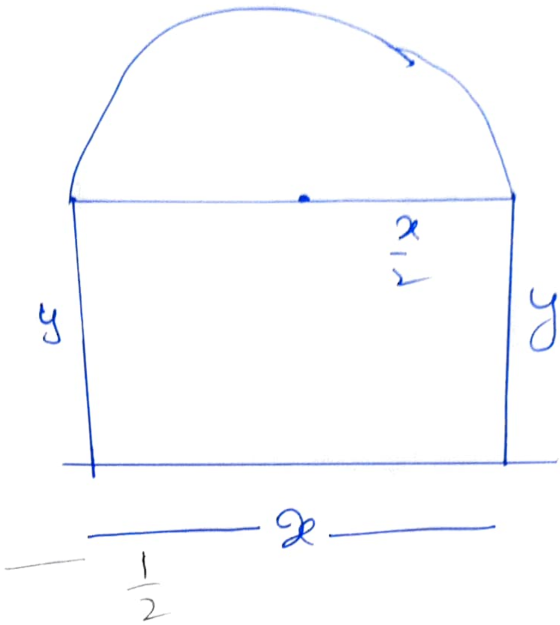
(1)

$P = 10$




$x + 2y + \pi \frac{x}{2} = 10$

$2x + 4y + \pi x = 20$

$y = \frac{20 - \pi x - 2x}{4}$ — $\frac{1}{2}$

$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$= x\left(\frac{20 - \pi x - 2x}{4}\right) + \frac{1}{2}\frac{\pi x^2}{4}$ — $\frac{1}{2}$

$= \frac{20x - \pi x^2 - 2x^2}{4} + \frac{\pi x^2}{8}$

$= \frac{40x - 2\pi x^2 - 4x^2 + \pi x^2}{8}$ — $\frac{1}{2}$

$= \frac{40x - 4x^2 - \pi x^2}{8}$ — $\frac{1}{2}$

$f'(x) = \frac{40 - 8x - 2\pi x}{8}$ — 1

$f'(x) = 0$

$40 - 8x - 2\pi x = 0$

$x = \frac{40}{8 + 2\pi} = \frac{20}{\pi + 4}$ — $\frac{1}{2}$

$x = \frac{20}{4+\pi}$

$\frac{1}{2}$

$f''(x) = -8-4\pi$

$f''\left(\frac{10}{4+\pi}\right) = -8-4\pi < 0$

$\therefore$ Area is maximum when $x = \frac{20}{4+\pi}$ m

$\therefore y = \frac{10}{\pi+4}$ m