CHENNAI SAHODAYA SCHOOLS COMPLEX

Marking scheme set - 1

COMMON EXAMINATION (2023- 24)
Class-12
(MATHEMATICS – 041)

Section – A

1. b	2. d	3. a	4. a	5. c	6. a	7. b	8. d	9. a	10.b
11. b	12 a	13. c	14. d	15. a	16.c	17. b	18.b	19. c	20. a

Section – B

21. (a) $\tan^{-1}\left(2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right)$

$\tan^{-1}(2\sin(2\pi/6))$. (1 mark)

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ (½ + ½ mark)

OR

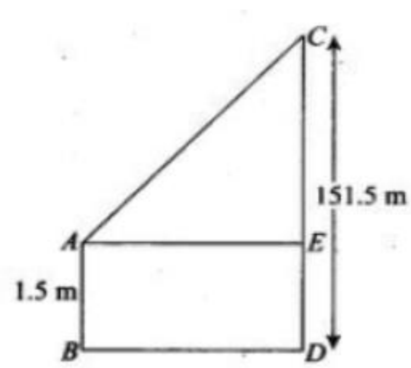
(b) Yes. Because $\frac{5\pi}{3}$ not belongs to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (½ mark)

So, $\sin^{-1}(\sin\frac{5\pi}{3}) = \sin^{-1}[\sin(2\pi - \frac{\pi}{3})]$ (½ mark)

$= \sin^{-1}[-\sin\frac{\pi}{3}]$ (1/2 mark)

$= -\frac{\pi}{3}$ (½ mark)

22.



Let L be the length of the string ,AC = L cm, AE = x cm .

Where , CE = 151.5 – 1.5 = 150 cm

In triangle ACE, $L^2 = x^2 + 150^2$ (1/ 2 mark)

By diff., 2L dL/dt = 2x dx/dt

dL/dt = 10x/L m/sec (½ mark)

when L = 250; $L^2 = x^2 + 150^2$

$250^2 = x^2 + 150^2$

x= 200m

therefore, dL/dt = 10(200)/250 = 8 m/sec (1/2 mark)

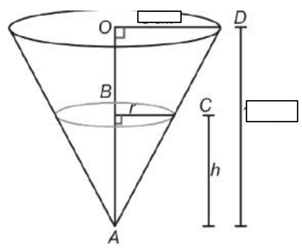
∴ the string is being let out at the 8m/sec, when kite is 250 m away from the boy.
((1/2 mark)

OR

Given ; In the fig. CD = 10cm; DA = 20 cm

Let r be the radius and h be the height of surface of water at time t.

Let V be the volume of water in funnel.



$$V = \frac{1}{3}\pi r^2 h \qquad \text{...(i)}$$

$$\frac{r}{h} = \frac{10}{20} \text{(from the fig, we have)}$$

$$r = \frac{1}{2}h \qquad \text{...(ii)}$$

Substituting equation (ii) in equation (i) we have

Water running out of funnel at 5 cm^3/sec

$$\frac{dv}{dt} = -5$$

$$\frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = -5$$

$$\frac{3\pi h^2}{12} \cdot \frac{dh}{dt} = -5$$

$$\frac{dh}{dt} = -\frac{-20}{\pi h^2}$$

$\therefore$ Rate of change of level of water (h) w.r.t time, $t = -\frac{20}{\pi h^2}$ **(1 mark)**

When water level is 5 cm from the top, h = 10 cm – 5 cm = 15 cm

$\therefore$ **Rate of change of water level at h = 15 cm is** $-\frac{20}{\pi\,(15)^2} = -\frac{4}{45\pi}$ **cm/sec.** (1 mark)

23. **To prove ; the function $f(x) = 5 - 3x + 3x^2 - x^3$ is decreasing on R.**

$f'(x) = -3x^2 + 6x - 3 = -3(x-1)^2$ **(½ mark)**

for $f'(x) = 0$, we get $x = 1$ **(½ mark)**

In (∞ , 1] , f ' (x) < 0 and In [1, ∞], f ' (x) < 0. **(½ + ½ mark)**

Therefore, f(x) is decreasing on R.

24. maximum value is 3 and no minimum value. **(1 + 1 Mark)**

25. $f(x) = \sin x + \sqrt{3}\cos x$

$f'(x) = \cos x - \sqrt{3}\sin x$

if $f'(x) = 0$, we get $x = \frac{\pi}{6}$ **(½ mark)**

then $f(0) = 0 + \sqrt{3} = +\sqrt{3}$

$f(\frac{\pi}{6}) = 2$ and $f(\pi) = -\sqrt{3}$. **(1/2 mark)**

therefore, point of absolute maximum is $x = \frac{\pi}{6}$

and absolute maximum value is 2. (1/2 +1/2 mark)

SECTION – C

26. $\frac{dy}{dt}$ = a (- sin t + $\frac{1}{tan\ t/2}$. sec t/2 . ½) **(1/2 mark)**

= a (- sin t + $\frac{1}{sin\ t}$)

= a ($\frac{cos^2 t}{sin\ t}$) **(½ Mark)**

And d x/dt = a cost **(½ mark)**

Now, $\frac{dy}{dx} = \frac{a\left(\frac{cos^2 t}{sin\ t}\right)}{a \cos t}$ = sin t / cos t = tan t. **(1mark)**

$\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ = 1 **(1/2 mark)**

27. $I = \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x}\,dx$

$$= \int_0^{\frac{\pi}{2}} \frac{x}{1+\cos x}dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos x}dx$$ ½ marks

$$= \int_0^{\frac{\pi}{2}} \frac{x}{2\cos^2\frac{x}{2}}dx + \int_0^{\frac{\pi}{2}} \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$$ ½ mark

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} x\sec^2\frac{x}{2}dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}dx$$ ½ mark

$$= \frac{1}{2}[x.2\tan\frac{x}{2}]_0^{\frac{\pi}{2}} - \frac{1}{2}\int_0^{\frac{\pi}{2}} 1.2\tan\frac{x}{2}dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}dx$$ 1 mark

$$= \frac{1}{2}\left[\frac{\pi}{2}.2\tan\frac{\pi}{4} - 0\right]$$ ½ mark

$$= \frac{\pi}{2}$$

OR

$$\int_0^{\pi/2} \sin 2x \log\tan x\,dx \qquad \ldots.(i)$$

Then,

$$I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right)\log\tan\left(\frac{\pi}{2} - x\right)dx$$ 1 mark

$$\Rightarrow I = \int_0^{\pi/2} \sin 2x \log\cot x\,dx$$ ½ mark

.....(ii)

Adding (i) and (ii), we get ½ mark

$$2I = \int_0^{\pi/2} \sin 2x(\log\tan x + \log\cot x)dx$$ ½ mark

$$2I = \int_0^{\frac{\pi}{2}} \sin 2x\ \log(1)\,dx$$

2 I = 0 $\Rightarrow \therefore I = 0.$ ½ mark

28. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$

Suppose $e^x = t \quad \Rightarrow \quad e^x dx = dt$ — ½ mark

$\Rightarrow \quad I = \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{-(t^2+4t-5)}}$

$\Rightarrow \quad I = \int \frac{dt}{\sqrt{-(t^2+4t+4-9)}}$ — ½ mark

$\Rightarrow \quad I = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} = \sin^{-1}\frac{t+2}{3} + C$ — ½ + 1 mark

$\Rightarrow \quad I = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C$ — ½ mark

29. $\frac{dy}{dx} - 3y\cot x = \sin 2x$, **given that y = 2 when** $x = \frac{\pi}{2}$.

$$I.F. = e^{\int -3\cot x\,dx}$$
$$= e^{-3\log(\sin x)} = (\sin x)^{-3}$$
$$= \text{cosec}^3 x$$

1 mark

∴ Solution is :

$$y\cdot\text{cosec}^3 x = \int \sin 2x\cdot\text{cosec}^3 x\,dx$$
$$= \int 2\,\text{cosec}\,x\cot x\,dx$$
$$y\cdot\text{cosec}^3 x = -2\,\text{cosec}\,x + C$$

1 mark

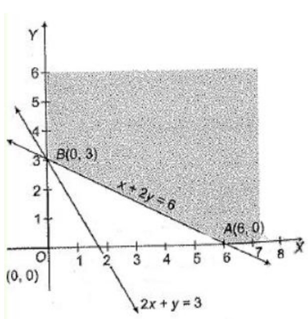
$$y = -2\sin^2 x + C\sin^3 x$$
$$x = \frac{\pi}{2},\ y = 2$$
$$C = 4$$
$$y = -2\sin^2 x + 4\sin^3 x$$

(1/2 + ½ mark)

30

(a) Minimise Z = x + 2y
subject to constraints: $2x + y \geq 3,\ x + 2y \geq 6,\ x \geq 0,\ y \geq 0.$

1 mark for graph, 1 mark for table,



The corner points are A and B.

A(6,0): Z = (6) + 2(0) = 6

B (0, 3) : Z = (0) + 2(3) = 6

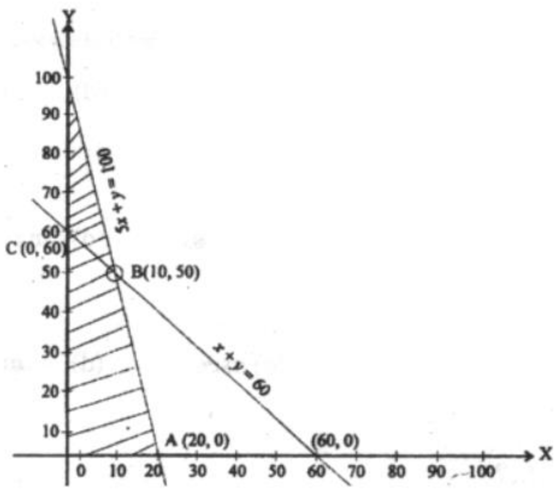
Draw z – line (dotted) x + 2y < 6 .

Therefore, the minimum value of z is 6, occurs at all points of the line segment A (6, 0) and B (0,6) . (1 mark)

OR

30. (b) $5x + y \le 100$; $x + y \le 60$, $x \ge 0, y \ge 0$ and objective function $Z = 50x + 15y$

2 mark for graph ; 1/2 mark for table



Corner points	Z = 50 x + 15 y
O(0, 0)	0
A(20,0)	1000
B(10, 50)	500 + 750 = 1250
C(0,60)	0 + 900 = 900

Therefore, Maximum value of Z is 1250 at the point (10, 50). 1/2mark

31. Let A and B be the event that the first and second horse is selected respectively. Obviously, A and B are independent events.

P(A) = ¼ , P(B) = 1/3 and P(A') = ¾ , P(B') = 2/3

(i) P (both of them selected) = P(A).P(B) $= \frac{1}{4} \cdot \frac{1}{3}$

$= \frac{1}{12}$ **(1 mark)**

(ii) P (only one of them is selected) = [P(A).P(B')] + [P(A').P(B)]

$= \frac{1}{4} \cdot \frac{2}{3} + \frac{3}{4} \cdot \frac{1}{3} = \frac{5}{12}$ **(1 mark)**

(iii) P (none of selected) = P(A') . P(B')

$= \frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12} =$ ½ **(1 mark)**

Section – D

32. Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by $f(x) = \frac{x-2}{x-3}$.

Let $x_1, x_2 \in A$.

Now, $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

Hence f is one-one function.

½ mark
½ mark
½ mark
½ mark

For Onto

Let $y = \frac{x-2}{x-3}$

$\Rightarrow \quad xy - 3y = x - 2 \quad \Rightarrow \quad xy - x = 3y - 2$

$\Rightarrow \quad x(y-1) = 3y - 2$

$\Rightarrow \quad x = \frac{3y-2}{y-1} \quad \ldots(i)$

1 ½ marks

From above it is obvious that ∀y except 1,

i.e., $\forall\, y \in B = R - \{1\}\ \exists x \in A$

(1 mark)

Here, range of f = codomain of f

Hence f is onto function.
Thus f is one-one onto function.

½ mark

OR

Given the relation n is R on Z defined by (a,b)∈R⇔|a−b|≤5.

This relation is reflexive as ∀a∈Z, (a,a)∈R since |a−a|=0≤5. (1 mark)

Also the relation is symmetric as |b−a|=|a−b|≤5 so (a,b)∈R⇒(b,a)∈R,∀a,b∈Z (2 marks)

But the relation is not transitive as (1,2)∈R,(2,7)∈R but (1,7)∉R (2 marks)

33. The given system of equations can be written as A X = B

Where $A = \begin{bmatrix} 4 & 3 & 2 \\ 9 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$ **½ mark**

$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 9 & 2 & 3 \\ 6 & 2 & 3 \end{vmatrix} = 25 \neq 0$**.so A^{-1} exist. 1 mark**

Adj A = $\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$ **and ½ mark**

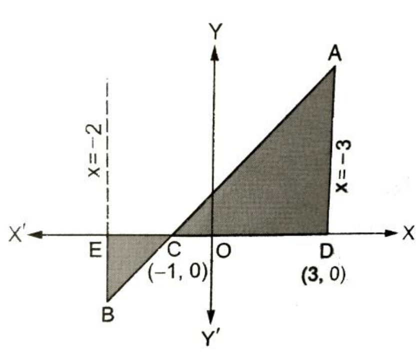
$A^{-1} = \frac{1}{|A|}$ **adj A** $= 1/25 \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$ **1 mark**

Since A X = B , then X = A^{-1} B

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1/25 \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$ **1 ½ mark**

Therefore x = 5; y = 8; z = 8 ½ mark

34. The line y – 1 = x intersect x axis at x = -1. And x = -2 and x = 3 are parallel to x-axis.



(1 mark for fig)

Requried =(area CDAC + area CBEC)

$$= \int_{-1}^{3} y\,dx + \int_{-2}^{-1} (-y)dx$$

$$= \int_{-1}^{3} (x+1)dx + \int_{-2}^{-1} -(x+1)dx.$$

(1 mark)

$$= [\frac{x^2}{2} + x]_{-1}^{3} - [\frac{x^2}{2} + x]_{-2}^{-1}$$

Substituting the value of x

$$= [(\frac{9}{2} + 3) - (\frac{1}{2} - 1)] - [(\frac{1}{2} - 1) - (2 - 2)]$$

On further calculation

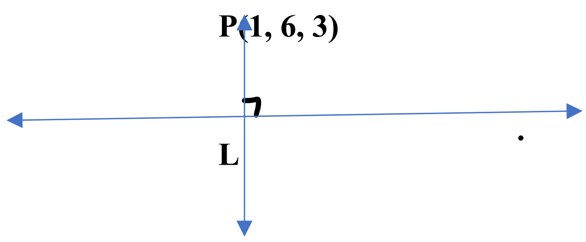
$$= [\frac{15}{2} + \frac{1}{2}] - (-\frac{1}{2}) = 8 + \frac{1}{2}$$

$$= 17/2 = 8.5 \text{ sq units}$$

2marks

1 mark

35. Let L be the foot of the perpendicular from the point P(1,6,3) to the given line.



$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

The coordinates of a general point on the given line are $\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

i.e., x = λ, y = 2λ + 1, z = 3λ + 2.

If the coordinates of L are (λ, 2λ + 1, 3λ + 2) **(1 mark)**

Then the direction ratios of PL are λ – 1, 2λ – 5, 3λ – 1.

But the direction ratios of given line which is perpendicular to PL are 1, 2, 3. **(1 mark)**

Since, PL perpendicular to given line; $a_1a_2 + b_1b_2 + c_1c_2 = 0$. **(1/2 mark)**

Therefore, (λ – 1) 1 + (2λ – 5) 2 + (3λ – 1) 3 = 0, which gives λ = 1. **(1 mark)**

Hence the coordinate of the foot of perpendicular is (1, 3, 5). **(1 mark)**

OR

Find the shortest distance between the following pair of lines and determine whether they intersect or not:

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \; and \; \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}.$$

$(\vec{a_2} - \vec{a_1}) = 3\hat{\imath} + 0\hat{\jmath} + 8\hat{k}$ **(1 mark)**

$(\overrightarrow{b_2} \times \overrightarrow{b_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix} = -10\hat{i} - 47\hat{j} + 39\hat{k}$ **(1 marks)**

Shortest distance = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_2} \times \overrightarrow{b_1})}{|(\overrightarrow{b_2} \times \overrightarrow{b_1})|}\right|$ **(½ mark)**

S D = $\left|\frac{(3\hat{i} + 0\hat{j} + 8\hat{k}) \cdot (-10\hat{i} - 47\hat{j} + 39\hat{k})}{\sqrt{10^2 + 47^2 + 39^2}}\right|$ **(1 mark)**

S.D = $\left|\frac{282}{\sqrt{3830}}\right|$ **(1mark)**

As shortest distance between given lines not equal to zero,

So, the lines do not intersect. **(½ mark)**

Section – D

36. (i) $(\vec{a} + \vec{b} + \vec{c}) = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Unit vector along $(\vec{a} + \vec{b} + \vec{c}) = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{4+9+36}} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}.$ **(1 mark)**

(ii) direction ratios of BC vector < -5, 5, 8 >

And direction cosines of BC vector $< \frac{-5}{3\sqrt{10}}, \frac{5}{3\sqrt{10}}, \frac{8}{3\sqrt{10}} >$ **(1 mark)**

(iii) a. Area of triangle = ½ $|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= ½ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$

$= 1/2\ |-36\hat{i} + 4\hat{j} - 25\hat{k}|$

$= ½ \sqrt{1296 + 16 + 625}$

$= ½ \sqrt{1937}$ **sq . units.** **(2 mark)**

OR

(iii)b Let angle ACB = θ; $\vec{a} = \overrightarrow{CA} = -3\hat{i} - 2\hat{j} + 4\hat{k}$;

$\vec{b} = \overrightarrow{BA} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

$\text{Cos}\ \theta = \left|\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right| = \left|\frac{-3(2) - 2(-7) + 4(-4)}{\sqrt{9+4+16}\ \sqrt{4+49+16}}\right| = \frac{8}{\sqrt{29x69}} = \frac{8}{\sqrt{2001}}.$ **(2 mark)**

37. Let A, B and C be the events denoting the selection of A, B and C as managers, respectively.
And E' denote that profit not happened.

(i) Probability of selection of A = P(A) = 1/7 (1 mark)

(ii) P(E' /A) = probability profit not happened by the changes introduces by A
= 1 – 0.8
= 0.2 **(1 mark)**

(iii) **(a).P(E') = P (A). P(E'/A) + P (A). P(E'/A) + P (A). P(E'/A)**
= (1/7 . 2/10) + (2/7. 5/10) + (4/7 . 7/10)
= 40/70
= 4/7 **(2 marks)**

OR

(iii)(b) P(C/ E') = $\frac{P(C).P(\frac{E'}{C})}{P(A).P(E'/A) + P(A).P(E'/A) + P(A).P(E'/A)}$

$= \frac{\frac{4}{7} \cdot \frac{7}{10}}{\frac{4}{7}}$ **= 7/10** **(2 marks)**

38. (i) relation between x and y is : $2x + 4y + \pi r = 10$
$2x + 4y + \pi x = 10$ **(1 mark)**

(ii) Area (A) of the window as a function expressed in term of x is
$A = ½ \pi r^2 + 2x.2y$

$A = ½ \pi x^2 + 10x - \pi x^2 - 2x^2$

$A(x) = 10x - 2x^2 - ½ \pi x^2$ **(1 mark)**

(iii)a. $A'(x) = 10 - 4x - \pi x$

$A''(x) = -4 - \pi$; $A'(x) = 0$ **, we get** $x = \frac{10}{\pi+4}$ **m (1 + 1 mark)**

b. $A = 10\left(\frac{10}{\pi+4}\right) - \left(\frac{10}{\pi+4} \cdot \frac{10}{\pi+4}\right) \cdot \left(\frac{\pi+4}{2}\right)$

therefore, max area = $\frac{50}{\pi+4}$ m^2. **(1 + 1 marks)**

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