CHENNAI SAHODAYA SCHOOLS COMPLEX

Marking scheme- set 2

COMMON EXAMINATION (2023- 24)
Class-12
(MATHEMATICS – 041)

Set - 2

Section – A

1. c	2. b	3. c	4. d	5. b	6. c	7. b	8. d	9. c	10. a
11. a	12. d	13. a	14. b	15. a	16. b	17. c	18.b	19. d	20. c

Section – B

21. (a) (1,1), (2,2), (3,3),(4,4) $\in R$, so R is reflexive. **(1 mark)**
$(1,3) \in R$ and $(3,2) \in R \Rightarrow (1,2) \in R$
So, R is transitive **(1 mark)**

22.

Let V be volume of cube of side x.

$\therefore \quad V = x^3$

From given condition,

$\frac{dV}{dt} = 7\ cm^3/s$

$\therefore \frac{d}{dt}(x^3) = 7 \quad \Rightarrow 3x^2\frac{dx}{dt} = 7 \quad \Rightarrow \frac{dx}{dt} = \frac{7}{3x^2} \quad ...(1)$ **(1 mark)**

Let S be surface area of cube

$\therefore \quad S = 6x^2$

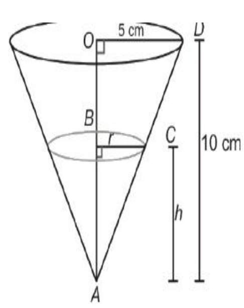
Rate of increase of surface area $= \frac{dS}{dt} = \frac{d}{dt}(6x^2)$

$= 12x\frac{dx}{dt} = 12x \times \frac{7}{3x^2} \quad [\because \text{ of } (1)]$

$= \frac{28}{x}$ **(1/2 mark)**

When x = 12, rate of increase of surface area $= \frac{28}{12} = \frac{7}{3} cm^2/s$. **(1/2 mark)**

OR



Let *r* be the radius and *h* be the height of surface of water at time *t*.

Let *V* be the volume of water in funnel.

$V = \frac{1}{3}\pi r^2 h \quad ...(i)$

$\frac{r}{h} = \frac{5}{10}$ (from the fig,we have)

$r = \frac{1}{2}h \quad ...(ii)$

Substituting equation (ii) in equation (i) we have

$V = \frac{\pi h^3}{12}$ **1 mark**

Water running out of funnel at 5 cm^3/sec

$\frac{dv}{dt} = -5$

$\frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = -5$

$\frac{3\pi h^2}{12}\cdot\frac{dh}{dt} = -5$

$\frac{dh}{dt} = -\frac{-20}{\pi h^2}$

$\therefore$ Rate of change of level of water (h) w.r.t time, $t = -\frac{20}{\pi h^2}$ **½ mark**

When water level is 2.5 cm form top, $h = 10 - 2.5 = 7.5$ cm

$\therefore$ Rate of change of water level at $h = 7.5$ cm

is $-\frac{20}{\pi(7.5)^2} = -\frac{20}{56.25\pi} = -\frac{16}{45\pi}\frac{cm}{sec}$ **1/2 mark**

23. Given profit function: p(x) = 41 + 24x – 18x².

∴ p'(x)=-72-36x

⇒x=-7236=-2 **(1 mark)**

Also,

p''(-2)=-36<0

By second derivative test, x=-2 is the point of local maxima of *p*.

∴ Maximum profit=p-2 **(1/2 mark)**

=41-72(-2)-18(-2)2=41+144-72=113 **(1/2 mark)**

Hence, the maximum profit that the company can make is 113 units.

24. $\int_0^{\frac{\pi}{2}} \sqrt{1 + \cos 2x}\, dx$.

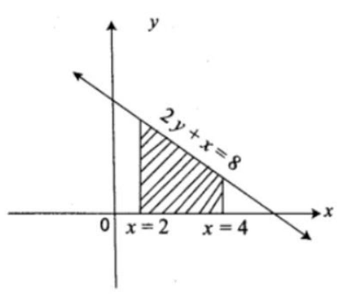
$$I = \int_0^{\pi/2} \sqrt{2\cos^2 x}dx = \sqrt{2}\int_0^{\pi/2} \cos x dx$$ **(1 mark)**

$$= \sqrt{2}[\sin x]_0^{\pi/2} = \sqrt{2}$$ **(1 mark)**

OR

$$\int_1^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}} = [sec^{-1} x]_1^{\sqrt{2}} = \frac{\pi}{4}$$ **(1 + 1)**

25. To find the area of the region bounded by the line 2y + x = 8, the x-axis and the lines x = 2 and x = 4.



Required area $= \int_2^4 y dx$

Now $2y + x = 8 \Rightarrow y = \frac{8-x}{2}$ Fig (½ mark)

So area $= \int_2^4 \left(\frac{8-x}{2}\right) dx$ **(1 mark)**

$= \frac{1}{2}\left[8x - \frac{x^2}{x}\right]_2^4$

= 5 square units **(1/2 mark)**

Section – C

26. If y = $(x)^{\cos x} + (\cos x)^{\sin x}$,

To find $\frac{dy}{dx}$:

Let $y = u + v$, where $u = (x)^{\cos x}$ and $v = (\cos x)^{\sin x}$.

Now, $u = (x)^{\cos x}$

$$\Rightarrow \log u = (\cos x)(\log x)$$

$$\Rightarrow \frac{1}{u}.\frac{du}{dx} = (\cos x).\frac{d}{dx}(\log x) + (\log x).\frac{d}{dx}(\cos x)$$

[on differentiating w.r.t. x]

$$= (\cos x).\frac{1}{x} + (\log x)(-\sin x)$$

$$\Rightarrow \frac{du}{dx} = u.\left[\frac{\cos x}{x} - (\log x)(\sin x)\right\}$$

$$\Rightarrow \frac{du}{dx} = (x)^{\cos x}\left\{\frac{\cos x}{x} - (\log x)(\sin x)\right\}$$

. ...(i) **1 ½ mark**

And, $v = (\cos x)^{\sin x}$

$\Rightarrow \log v = (\sin x)\log(\cos x)$

$\Rightarrow \frac{1}{v}.\frac{dv}{dx} = (\sin x).\frac{d}{dx}\{\log(\cos x)\} + \log(\cos x)$

$.\frac{d}{dx}(\sin x)$

[on differentiating w.r.t. x]

$\Rightarrow \frac{dv}{dx} = v.\left\{(\sin x).\frac{(-\sin x)}{\cos x} + \log(\cos x).\cos x\right\}$

$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}.\{-\sin x\tan x + \cos x$

$.\log(\cos x)\}.$ ...(ii) **1 mark**

$\therefore y = (u+v)$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = (x)^{\cos x}.\left\{\frac{\cos x}{x} - (\log x)\sin x\right\}$

$+ (\cos x)^{\sin x}.\{-\sin x\tan x + \cos x.\log(\cos x)\}.$

½ mark

27. $I = \int\left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right]$ **dx**

Consider log x = t

Where x = e^t

So we get

dx = e^t dt **½ mark**

So,

$$\int\left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right)dx = \int\left(\frac{1}{t} - \frac{1}{t^2}\right)e^t dt$$

Taking f (x) = 1/t and f' (x) = - 1/t² **1 mark**

$$= e^t \times \frac{1}{t} + c$$

1 mark

Now substitute the value of t as log x

$$= e^{\log x} \times \frac{1}{\log x} + c$$

We get

$$= \frac{x}{\log x} + c$$

½ mark

28. $I = \int \frac{\cos x}{\sin^2 x + 4\sin x + 5}$ **dx**

$= \int \frac{\cos x}{\sin^2 x + 4\sin x + 4 + 1}$ **dx**

$= \int \frac{\cos x}{(\sin x + 2)^2 + 1}$ **dx** **½ M**

take t= sin x, then dt = cos x dx **½ M**

$= \int \frac{dt}{(t+2)^2+1} = \tan^{-1}(t+2) + c$ **1 ½ M**

I = $\tan^{-1}$ (sin x + 2) + c **½ M**

29. $\frac{dy}{dx} - 3y\cot x = \sin 2x,$

Comparing with $\frac{dy}{dx}$ + Py = Q

P = −3 cot x & Q = sin 2x

IF = $e^{\int pdx}$

$= e^{-3\int \cot x\, dx}$

$= e^{-3 \log|\sin x|}$

$= e^{\log|\sin x|^{-3}}$

$= e^{\log\frac{1}{|\sin^3 x|}}$

$= e^{\log|cosec^3 x|}$ $= cosec^3 x$ **1M**

y × IF = $\int Q.IF\, dx$

y $cosec^3$x = $\int \frac{2\sin x \cos x}{\sin^3 x}$ dx **½ M**

y $cosec^3$x = 2 (−cosec x) + C **½ M**

given that y = 2 when x = $\frac{\pi}{2}$, we get C = 4 **½ M**

so, required particular solution is:

y = 4 $\sin^3$x − 2 $\sin^2$ x **½ M**

OR

(x – 1) $\frac{dy}{dx}$ = 2 x^3 y

By separating the variables

$$\frac{dy}{y} = 2x^3 \frac{dx}{(x-1)}$$ **½ M**

$$\frac{dy}{y} = 2\left(x^2 + x + 1 + \frac{1}{x-1}\right)dx$$

By integrating both sides

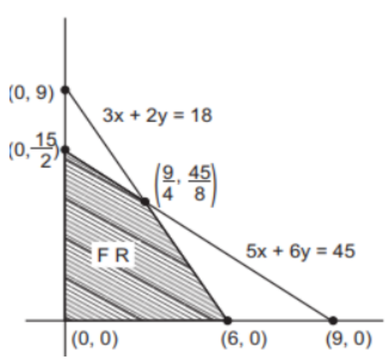
$$\int \frac{dy}{y} = \int 2\left(x^2 + x + 1 + \frac{1}{x-1}\right)dx + c$$ **1M**

$$\log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$$ **1 ½ M**

30. Maximise Z = 60 x + 40 y

Subject to constraints: 5x + 6y ≤ 45 ; $3x + 2y \le 18$; $x \ge 0$, $y \ge 0$.

Let the owner buys x machines of type A and y machines of type B.



2 M

The shaded region in the figure represents the feasible region which is bounded.

Let us now evaluate Z at each corner point.

at (0, 0) Z is $60 \times 0 + 40 \times 0 = 0$

Z at $\left(0, \frac{15}{2}\right)$ is $60 \times 0 + 40 \times \frac{15}{2} = 300$

Z at (6, 0) is $60 \times 6 + 40 \times 0 = 360$

Z at $\left(\frac{9}{4}, \frac{45}{8}\right)$ is $60 \times \frac{9}{4} + 40 \times \frac{45}{8} = 135 + 225 = 360.$

$\Rightarrow$ max. Z = 360

Maximum at all points of the line segment (6, 0) and (9/4, 45/8),

maximum value is 360. **1 M**

31. Three critics review a book. For the three critics, the odds in favour of the book are (5:2), (4:3) and (3:4) respectively.

A, B, C denote the evets that the book be favoured by the first, second and third critic. Then,

$$P(A)=\frac{5}{7}, P(B)=\frac{4}{7}, P(C)=\frac{3}{7},$$
$$P(\overline{A})=\left(1-\frac{5}{7}\right)=\frac{2}{7}, P(\overline{B})=\left(1-\frac{4}{7}\right)$$
$$=\frac{3}{7} \text{ and } P(\overline{C})=\left(1-\frac{3}{7}\right)=\frac{4}{7}.$$

1M

Required probability = P (2 critics favour the book 0r 3 critics favour the book)

$$=P(A\cap B\cap \overline{C})+P(A\cap \overline{B}\cap C)$$
$$+P(\overline{A}\cap B\cap C)+P(A\cap B\cap C)$$
$$=\left(\frac{5}{7}\times\frac{4}{7}\times\frac{4}{7}\right)+\left(\frac{5}{7}\times\frac{3}{7}\times\frac{3}{7}\right)$$
$$+\left(\frac{2}{7}\times\frac{4}{7}\times\frac{3}{7}\right)+\left(\frac{5}{7}\times\frac{4}{7}\times\frac{3}{7}\right)$$

1M

(since A, B, C are independent events)

$$=\left(\frac{80}{343}+\frac{45}{343}+\frac{24}{343}+\frac{60}{343}\right)$$

1M

Hence, the required probability is $\frac{209}{343}$.

OR

Let E_1 = event of selecting a bag from the first group,

E_2 = event of selecting a bag from the second group, and

E = event of drawing a white ball.

Then, $P(E_1)=\frac{5}{11}$ and $P(E_2)=\frac{6}{11}$.

1M

$P(E/E_1)$= probability of getting a white ball, given that it is from a bag of the first group

$=\frac{5}{8}$.

½ M

$P(E/E_2)$ = probability of getting a white ball, given that it is from a bag of the second group

$=\frac{2}{6}=\frac{1}{3}$

½ M

Probability of getting the ball from a bag of the first group, given that it is white

$P(E_1/E)$

$$=\frac{P(E/E_1).P(E_1)}{P(E/E_1).P(E_1)+P(E/E_2).P(E_2)} \text{[by Bayes's theorem]}$$

$$=\frac{\left(\frac{5}{8}\times\frac{5}{11}\right)}{\left(\frac{5}{8}\times\frac{5}{11}\right)+\left(\frac{1}{3}\times\frac{6}{11}\right)}=\frac{75}{123}.$$

½ + ½ M

Section – D

32. Given that, R be the relation in N × N defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in N × N.

<u>R is Reflexive if (a, b) R (a, b) for (a, b) in N × N</u>

Let (a, b) R (a, b)

⇒ a + b = b + a

which is true since addition is commutative on N.

⇒ R is reflexive. **1 ½ M**

R is Symmetric if (a, b) R (c, d) ⇒ (c, d) R (a, b) for (a, b), (c, d) in N × N

Let (a, b) R (c, d)

⇒ a + d = b + c

⇒ b + c = a + d

⇒ c + b = d + a [since addition is commutative on N]

⇒ (c, d) R (a, b)

⇒ R is symmetric. **1 ½ M**

R is Transitive if (a, b) R (c, d) and (c, d) R (e, f) ⇒ (a, b) R (e, f) for (a, b), (c, d),(e, f) in N × N

Let (a, b) R (c, d) and (c, d) R (e, f)

⇒ a + d = b + c and c + f = d + e

⇒ (a + d) – (d + e) = (b + c) – (c + f)

⇒ a - e = b - f

⇒ a + f = b + e

⇒ (a, b) R (e, f)

⇒ R is transitive. **1 ½ M**

Hence, R is an equivalence relation. **1/2 M**

Or

The given function f:N→N is defined by,

f(n)={ n+1 /2 , if n is odd

n/ 2 , if n is even

Assume x_1 =5 and x_2 =6.

f(5)= 6 / 2 =3

f(6)= 6/ 2 =3

So, f(x_1)=f(x_2) but $x_1 \neq x_2$. **2M**

Therefore, f is not one-one.

A function f:X→Y is onto or surjective,

if for every y∈Y, there exists an element in x such that f(x)=y.

Let, n∈N be any element.

If n is odd, then n=2r+1 for some r∈N there exists 4r+1∈N such that,

f(4r+1) =2r+1 **1M**

If n is even, then n=2r for some r∈N there exists 4r∈N such that,

f(4r)= 4r /2 =2r **1M**

So, f is onto.

Thus, f is onto but not one-one. **1M**

33. Given System of equations:
x – y + z = 4 ; x – 2y – 2z = 9; 2x = y = 3z = 1.

$$Let A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

½ M

Then the given system of equations is AX=C.

$$Now,\ AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4+7+5 & 4-1-3 & 4-3-1 \\ -4+14-10 & 4-2+6 & 4-6+2 \\ -8-7+15 & 8+1-9 & 8+3-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$\Rightarrow A.\left(\frac{1}{8}B\right) = I$$ **1 M**

$$\Rightarrow A^{-1} = \frac{1}{8}B = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$ **1 M**

Now , AX=C

$$\Rightarrow X = A^{-1}C$$ **½ M**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}.\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

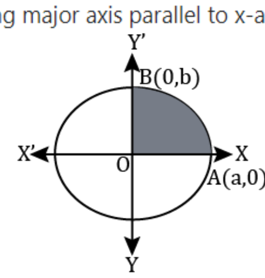
$$\frac{1}{8}.\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}.\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$ **1 + ½ M**

Hence , x=3, y=-2 and z=-1 **½ M**

34. To find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Let A be the area of ellipse

We start with the given equation of ellipse in standard form having major axis parallel to x-axis.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = b\sqrt{1-\left(\frac{x}{a}\right)^2} \implies y = \frac{b}{a}\sqrt{a^2-x^2}$$ **1M**

Ellipse has four quadrants. Area of the ellipse is four times the area of single quadrant.

$$A = 4\int_0^a y dx$$

$$A = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}dx$$

$$A = \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}dx$$ **1 M**

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$ **1 M**

$$= \frac{4b}{a} \times \frac{a^2}{2}\sin^{-1}(1)$$ **1M**

$$= 2ab \times \frac{\pi}{2}$$

∴ Required Area = πab **square units** **1M**

35. To find check the lines intersect or not;

Line 1: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t$ ½ M

Line 2: $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$

Let (x, y, z) = (2t + 1, 3t + 2, 4t + 3) be a point on line 1. **1 ½ M**

For this point of to be point of intersection of line 1 & line 2 it must satisfy line2.

$\Rightarrow \frac{2t+1-4}{5} = \frac{3t+2-1}{2} = \frac{4t+3}{1}$

$\Rightarrow t = -1$

2M

Therefore the given lines intersect each other at the point (-1, -1, -1). 1M

Or

To find the shortest distance between the following pair of lines:

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}.$$

$(\vec{a_2} - \vec{a_1}) = 3\hat{i} + 0\hat{j} + 8\hat{k}$ **(1 mark)**

$(\vec{b_2} \times \vec{b_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix} = -10\hat{i} - 47\hat{j} + 39\hat{k}$ **(1 marks)**

Shortest distance $= \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_2} \times \vec{b_1})}{|(\vec{b_2} \times \vec{b_1})|} \right|$ **(1mark)**

S D $= \left| \frac{(3\hat{i} + 0\hat{j} + 8\hat{k}) \cdot (-10\hat{i} - 47\hat{j} + 39\hat{k})}{\sqrt{10^2 + 47^2 + 39^2}} \right|$ **(1 mark)**

S.D $= \left| \frac{282}{\sqrt{3830}} \right|$ **(1mark)**

Section – E

36. (i) F'(x) = $6(x+1)^2 (x-3)^2 (x-1)$

(ii) The critical points of the given function are -1,3,1

(iii) The interval in which the function is strictly increasing are $(1, 3) \cup (3, \infty)$.

OR

(iv) The interval in which the function is decreasing are $(\infty, 1]$.

37. (i) To Find the vector that represents the flight path of Airplane 1.

$\overrightarrow{OP} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{OQ} = 3\hat{i} + 4\hat{j} - \hat{k}$

the vector representing the flight path of Airplane 1 as:

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$
$$= (3\hat{i} + 4\hat{j} - \hat{k}) - (-2\hat{i} + \hat{j} + 3\hat{k})$$
$$= 5\hat{i} + 3\hat{j} - 4\hat{k}$$

1M

(ii) Uses vector subtraction to find the vector representing the flight path from R to Q as:

$$\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR}$$
$$= (5\hat{i} + 3\hat{j} - 4\hat{k}) - (5\hat{i} + \hat{j} - 2\hat{k})$$
$$= 2\hat{j} - 2\hat{k}$$

1 M

(iii) the angle between the flight paths of Airplane 1 and Airplane 2

just after take-off :

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PQ}| \cdot |\overrightarrow{PR}|}$$

$$= \frac{(5\hat{i} + 3\hat{j} - 4\hat{k}) \cdot (5\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{50} \cdot \sqrt{30}}$$

$$= \frac{18}{5\sqrt{15}}$$

$$\theta = \cos^{-1}\left(\frac{18}{5\sqrt{15}}\right)$$ **2M**

OR

Considers a point S which divides PQ internally in the ratio 1:2.
Finds the position vector of point S as:

$$\overrightarrow{OS} = \frac{1(\overrightarrow{OQ}) + 2(\overrightarrow{OP})}{1+2}$$

$$= \frac{1(3\hat{i} + 4\hat{j} - \hat{k}) + 2(-2\hat{i} + \hat{j} + 3\hat{k})}{3}$$

$$= -\frac{1}{3}\hat{i} + 2\hat{j} + \frac{5}{3}\hat{k}$$ **2M**

38. (i) Number of Blue balls = 12,
Number of red balls = 8,
Number of yellow balls = 10,
Number of green balls = 5
Total number of balls = 35.
Since the two balls are drawn without replacement:

P(first ball drawn is blue) $= P(B) = \frac{12}{35}$

P(second ball drawn is green) $= P\left(\frac{G}{B}\right) = \frac{5}{34}$

P (B ∩ G) = $P(B).\,P\left(\frac{G}{B}\right)$

$$= \frac{12}{35} \times \frac{5}{34}$$

$$= \frac{6}{119}$$ **1M**

(ii) P(both are red) = 8/35 . 7/34
= 4/85

(iii) P (Y' ∩ G) = P(G) . P(Y'/G) = 5/35. 24/34
= 12/119.

OR

(iv) P(F' ∩ E') = P(E'). P(F'/E') = 23/35 . 22/34
= 253/595