CHENNAI SAHODAYA SCHOOLS COMPLEX

Marking scheme- set 3

COMMON EXAMINATION (2023- 24)
Class-12
(MATHEMATICS – 041)

Section – A

1. b	2. c	3. a	4. d	5. c	6. d	7. a	8. a	9. a	10. b
11. d	12. d	13. b	14. c	15. b	16. c	17. a	18.b	19. c	20. a

Section – B

21. the value of $\tan^{-1}\left(2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right)$

$$= \tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right)\right]$$ **½ m**

$$= \tan^{-1}\left[2\sin\left(2\left(\frac{\pi}{6}\right)\right)\right] = \tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$$ **½ m + ½ m**

$$\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$ **½ m**

22. Given as $f(x) = (x - 1)e^x + 1$

$\Rightarrow f'(x) = x e^x$ As given $x > 0$ 1 M

$\Rightarrow e^x > 0$

$\Rightarrow x e^x > 0$

$\Rightarrow f'(x) > 0$

Thus, condition for f(x) to be increasing

Hence f(x) is increasing on interval $x > 0$ 1M

OR

GIVEN: $f(x) = \sin x + \sqrt{3}\cos x$

$\Rightarrow f'(x) = \cos x - \sqrt{3}\sin x$ **½ m**

we must have $f'(x) = 0$

$\Rightarrow \cos x - \sqrt{3}\sin x = 0$

$\Rightarrow \cos x = \sqrt{3}\sin x$

$\Rightarrow \cot x = \sqrt{3}$

$\Rightarrow x = \frac{\pi}{6}$

Also , $f''(x) = -\sin x - \sqrt{3}\cos x$ **1/2m + 1/2m**

$\Rightarrow f''\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} - \sqrt{3}\cos\frac{\pi}{6} = -\frac{1}{2} - \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} - \frac{3}{2} = -2 < 0$

So, $x = \frac{\pi}{6}$ is point of maxima . **1/2 m**

23. Given:

The volume of a sphere is increasing at the rate of 3 cubic centimetre per second.

$dV/dt = 3\ cm^2\ /sec$

We have volume of sphere, $V = 4/3\ \pi r^3$

Differentiate w.r.t 't'

$dV/dt = 4/3 \times \pi \times 3 \times r^2\ dr/dt$

$3 = 4\pi\ (2)^2\ dr/dt$

$dr/dt = 3\ /16\pi\ cm/sec$ 1m

Surface area of sphere, $A = 4\pi r^2$

Differentiate w.r.t 't'

$dA/dt = 4\pi \times (2r)\ dr/dt$

$dA/dt = 4\pi \times (2\times2) \times (3/16\pi)$

$dA/dt = 3\ cm^2/sec$ ½ m

∴ the rate of increase of its surface area, when r = 2 cm is $3cm^2/sec$ ½ m

24. $\int \frac{1}{\sqrt{x^2+2x+2}}$ **dx.**

$$= \int \frac{1}{\sqrt{(x+1)^2 - 1 + 2}}\, dx$$

$$= \int \frac{1}{\sqrt{(x+1)^2 + 1}}\, dx$$ **(1 mark)**

$$= \log\left|x + 1 + \sqrt{(x+1)^2\ +\ (1)^2}\right| + C$$

$$= \boldsymbol{\log\left|x + 1 + \sqrt{x^2\ + 2x + 2}\right| + C}$$ **(1 mark)**

OR

$\int e^x \frac{(1+sinx)}{(1+cos\,x)}$ **dx**

Simplifying function $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

$$= e^x\left(\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{2\,cos^2\left(\frac{x}{2}\right)}\right)$$

$$= \frac{e^x}{2}\left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{cos\ \frac{x}{2}}\right)^2$$

$$= \frac{e^x}{2}\left(\frac{\sin\frac{x}{2}}{cos\ \frac{x}{2}} + \frac{\cos\frac{x}{2}}{cos\ \frac{x}{2}}\right)^2$$

$$= \frac{e^x}{2}\left(\tan\frac{x}{2} + 1\right)^2 \quad = \frac{e^x}{2}\left(\boldsymbol{tan^2\ \frac{x}{2} + 1} + 2\tan\frac{x}{2}\right)$$

Our Integration becomes

$$\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx\ = \int e^x\left(\tan\left(\frac{x}{2}\right) + \frac{1}{2}\sec^2\left(\frac{x}{2}\right)\right)dx$$ **1m**

It is of the form

$$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$$

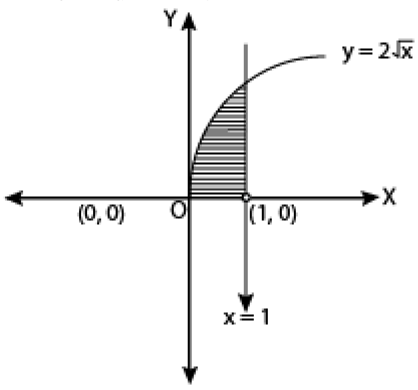
Where $f(x) = \tan^{-1} x$

$$f'(x) = \frac{1}{1+x^2}$$

So, our equation becomes

$$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = \boldsymbol{e^x \tan \frac{x}{2} + C}$$ **1m**

25. To find the area under the curve y = $2\sqrt{x}$ included between x= 0 and x = 1.



1m

From the figure, area of shaded region,

$$A = \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{2}{3}x^{3/2}\right]_0^1$$

$$= 2\left(\frac{2}{3}\right) = \frac{4}{3} \text{ sq. units}$$

1m

Section – C

26. x = ae^{θ}(sin $\theta - \cos\theta$) and y = ae^{θ}(sin $\theta + \cos\theta$).

$$\frac{dx}{d\theta} = ae^{\theta}(\sin\theta - \cos\theta) + ae^{\theta}(\cos\theta + \sin\theta)$$ 1m

$$= 2ae^{\theta}\sin\theta$$

$$\frac{dy}{d\theta} = ae^{\theta}(\sin\theta + \cos\theta) + ae^{\theta}(\cos\theta - \sin\theta)$$ 1m

$$= 2\,ae^{\theta}\cos\theta$$

$$\frac{dy}{dx} = \frac{2ae^{\theta}\cos\theta}{2ae^{\theta}\sin\theta} = \cot\theta$$ ½ m

$$\left.\frac{dy}{dx}\right|_{at\ \theta=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$$ ½ m

27. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{(1+e^x)} dx \ldots\ldots (i)$$

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(0-x)}{1+e^{(0-x)}} dx$$

$$= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^{-x}} dx$$

$$\Rightarrow I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^x \frac{(\cos x)}{(1+e^x)} dx \ldots\ldots (ii)$$

1+ ½ m

Adding (i) and (ii), we get

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x dx = [\sin x]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = 1 + 1 = 2$$

$\therefore I = 1$ **1 + ½ m**

OR

$$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx \quad \ldots.(1)$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx \quad \ldots(2)$$

add equation (1) and (2)

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx$$

1 m

Put, $\cos x = t \Rightarrow -\sin x dx = dt$

$$2I = -\int_1^{-1} \frac{\pi dt}{1+t^2}$$

$$2I = \pi \int_{-1}^{1} \frac{dt}{1+t^2}$$

1m

$$2I = \pi [\tan^{-1}(t)]_{-1}^{1}$$

$$2I = \pi \times \frac{\pi}{2}$$

$$I = \frac{\pi^2}{4}$$

1m

28. $\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}$ **dx.**

Let x = $\cos^2 2\theta$

$dx = -4 \cos 2\theta \sin 2\theta \, d\theta$

$$= \int \sqrt{\frac{1-\sqrt{(\cos^2 2\theta)}}{1+\sqrt{(\cos^2 2\theta)}}} \times -4 \cos 2\theta \sin 2\theta \; d\theta$$

1m

$$= -4 \int \sqrt{\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta - 1)}} \;\; \cos 2\theta \;\; (2\sin\theta\cos\theta) \;\; d\theta$$

$$= -8 \int \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} \;\; \cos 2\theta \cos\theta \;\; \sin\theta \; d\theta$$

$= -8 \int \left(\frac{1 - \cos 2\theta}{2}\right) \cos 2\theta \, d\theta$

$= -4 \int (\cos 2\theta - \cos^2 2\theta)\, d\theta$

$= 4 \int (\cos^2 2\theta - \cos 2\theta)\, d\theta$

$= 4 \int \cos^2 2\theta \, d\theta - 4 \int \cos 2\theta \, d\theta$

$= 4 \int \frac{\cos 4\theta + 1}{2} d\theta - 4 \int \cos 2\theta \, d\theta$ **1m**

$= \frac{\sin 4\theta}{2} + 2\theta - 2 \sin 2\theta + C$

$= \frac{2\sqrt{x}\sqrt{1-x}}{2} + 2\frac{\cos^{-1}\sqrt{x}}{2} - 2\sqrt{1-x} + C$ **1m**

$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x - x^2} + C$

29. (x – y) $\frac{dy}{dx}$ = x + 2y.

$\frac{dy}{dx} = \left(\frac{x+2y}{x-y}\right)$

Let $y = vx$

So, $\frac{dy}{dx} = \frac{d(vx)}{dx}$

$\frac{dy}{dx} = \frac{dv}{dx} . x + v \frac{dx}{dx}$

$\frac{dy}{dx} = \frac{dv}{dx} x + v$ **½ m**

$\frac{dv}{dx} x + v = \frac{x + 2vx}{x - vx}$

$\frac{dv}{dx} x + v = \frac{x(1 + 2v)}{x(1 - v)}$

$\frac{dv}{dx} x + v = \frac{1 + 2v}{1 - v}$

$\frac{dv}{dx} x = \frac{1 + 2v}{1 - v} - v$

$\frac{dv}{dx} . x = \frac{1 + 2v - v(1 - v)}{1 - v}$

$\frac{dv}{dx} . x = \frac{1 + 2v - v + v^2}{1 - v}$ **½ m**

Integrating Both Sides

$\int \frac{v - 1}{v^2 + v + 1} dv = \int \frac{-dx}{x}$

$\int \frac{v - 1}{v^2 + v + 1} dv = -\int \frac{dx}{x}$

$\int \frac{(v - 1)\, dv}{v^2 + v + 1} dv = -\log|x| + c$

$$\int \frac{v+\frac{1}{2}}{\left(v+\frac{1}{2}\right)^2+\frac{3}{4}}dv - \frac{3}{2}\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\frac{3}{4}}dv = -\log|x| + c$$ **1m**

Therefore the general solution is;

$$\frac{1}{2}\log\ |v^2+v+1| - \frac{3}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = -\log|x| + c$$ **½ m**

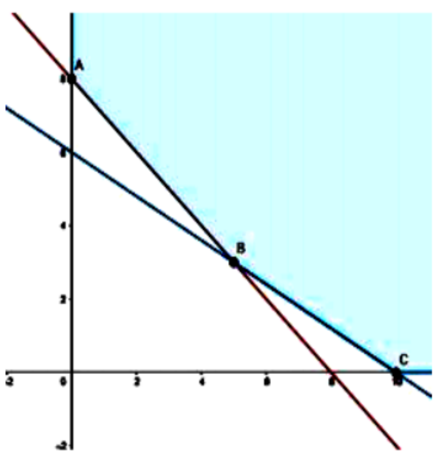
$$log\left|\frac{y^2}{x^2}+\frac{y}{x}+1\right| + log|x|^2 = 2\sqrt{3}\ \tan^{-1}\left(\frac{2y+x}{\sqrt{3}\,x}\right) + 2c$$

$$\boldsymbol{log|x^2+xy+y^2| = 2\sqrt{3}\ tan^{-1}\left(\frac{x+2y}{\sqrt{3}\,x}\right) + c}$$ **1/2 m**

30. **Minimise Z = 300x + 400y**

Subject to constraints: 6x +10 y ≥ 60 ; 4x + 4y ≥ 32; $x \geq 0,\ y \geq 0$.

The feasible region determined by 6x + 10 y ≥ 60, 4x + 4y ≥ 32, x ≥ 0, y ≥ 0is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8),B(5,3),C(10,0) . **1m**

The value of Z at corner point is

Corner Point	Z = 300x + 400y
A(0, 8)	3200
B(5, 3)	2700
C(10, 0)	3000

1m

Z< 2700 (draw dotted line on the graph) **½ m**

The minimum value of Z is 2700 at point (5,3). **½ m**

31. A bag I contains 5 red and 4 white balls and bag II contains 3 red and 3 white balls. Two balls are transferred from the bagI to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red and one white ball is transferred from the bag I to the bagII.

Let, E_1 : Two white balls are transferred

E_2 : Two red balls are transferred

E_3 : One red and one white ball are transferred.

A : The ball drawn from the bag II is red.

$$P(E_1) = \frac{^4C_2}{^9C_2} = \frac{4\times3}{9\times8} = \frac{1}{6}$$

$$P(E_2) = \frac{^5C_2}{^9C_2} = \frac{4\times5}{9\times8} = \frac{5}{18}$$

$$P(E_3) = \frac{^5C_1 \times {^4C_1}}{^9C_2} = \frac{4\times5\times2}{9\times8} = \frac{5}{9}$$

$$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{5}{8},$$

$$P(A/E_3) = \frac{4}{8}$$

2 m

The required probability, $P(E_3/A)$, by Bayes' Theorem

$$= \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{5}{9}\times\frac{4}{8}}{\frac{1}{6}\times\frac{3}{8} + \frac{5}{18}\times\frac{5}{8} + \frac{5}{9}\times\frac{4}{8}}$$

$$= \frac{20}{37}$$

1m

OR

the probability distribution of the random variable X, which denotes number of sixes in two tosses of a die.

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

½ m

∴ P (X = 0) = P (not getting six on any of the dice)

$= \frac{5\times5}{6\times6}$

$= \frac{25}{36}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$= 2\left(\frac{1}{6}\times\frac{5}{6}\right)$

$= \frac{10}{36}$

P (X = 2) = P (six on both the dice) $= \frac{1}{36}$

1 ½ m

∴ The required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

1m

Section – D

32. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

$$A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -22 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Now **calculating A^{-1}** using

$A^3 - 6A^2 + 9A - 4I = 0$

Post multiplying by A^{-1} both side

$(A^3 - 6A^2 + 9A - 4I) A^{-1} = 0.A^{-1}$

$A^3 . A^{-1} - 6A^2 . A + 9AA^{-1} - 4IA^{-1} = 0$

$A^2 . \mathbf{AA^{-1}} - 6A. \mathbf{A^{-1} A} + 9\mathbf{AA^{-1}} - 4IA^{-1} = 0$

$A^2 I - 6AI + 9I - 4IA^{-1} = 0$ *($AA^{-1} = I$)*

$A^2 - 6A + 9I - 4A^{-1} = 0$ *($AI = A$)*

$4A^{-1} = A^2 - 6A + 9I$

$A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$

Putting value

$$A^{-1} = \frac{1}{4}\left(\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$\text{Hence, } \mathbf{A}^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

33. If A= $\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & 3 \end{bmatrix}$; $|A|$ = -9

$\therefore A^{-1} = \frac{1}{|A|} adj\ A$

A^{-1} = -1/9 $\begin{bmatrix} -1 & -5 & -3 \\ -5 & 2 & 3 \\ 2 & 1 & -3 \end{bmatrix}$ **2m**

Write given equations in matrix form;

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

We take as; BX= C

X= B^{-1} C

$X = (A^T)^{-1}$ C **1m**

$$X = \frac{1}{-9} \begin{bmatrix} -1 & -5 & 2 \\ -5 & 2 & 1 \\ -3 & 3 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{-1}{9} \begin{bmatrix} -3-10+4 \\ -15+4+2 \\ -9+6-6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-9}{-9} \\ \frac{-9}{-9} \\ \frac{-9}{-9} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 1, z = 1$

2m

33.Let $\vec{a}, \vec{b}, and\ \vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$.

the projection of $\vec{b}$ along $\vec{a} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

the projection of $\vec{c}$ along $\vec{a} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$

According to the question,

Projection of $\vec{b}$ along $\vec{a}$= Projection of $\vec{c}$along $\vec{a}$

$$\Rightarrow \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$$

$$\Rightarrow \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$$

since $\vec{b}$and $\vec{c}$ are perpendicular to each other, we have

$\vec{b}\cdot\vec{c} = 0$

1m

$$\left(3\vec{a} - 2\vec{b} + 2\vec{c}\right)\cdot\left(3\vec{a} - 2\vec{b} + 2\vec{c}\right)$$

$$= 9|\vec{a}|^2 - 6\vec{a}\cdot\vec{b} + 6\vec{a}\cdot\vec{c} - 6\vec{b}\cdot\vec{a} + 4\left|\vec{b}\right|^2 - 4\vec{b}$$

$$\cdot\vec{c} + 6\vec{c}\cdot\vec{a} - 4\vec{c}\cdot\vec{b} + 4|\vec{c}|^2$$

$$\left|3\vec{a} - 2\vec{b} + 2\vec{c}\right|^2 = 9|\vec{a}|^2 + 4\left|\vec{b}\right|^2 + 4|\vec{c}|^2 - 12\vec{a}\cdot\vec{b}$$

$$+ 12\vec{a}\cdot\vec{c} - 8\vec{b}\cdot\vec{c}$$

2m

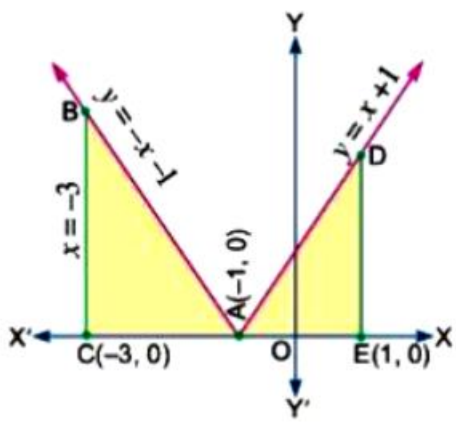
$$\left|3\vec{a} - 2\vec{b} + 2\vec{c}\right|^2 = 9|\vec{a}|^2 + 4\left|\vec{b}\right|^2 + 4|\vec{c}|^2$$

$$\Rightarrow \left|3\vec{a} - 2\vec{b} + 2\vec{c}\right|^2 = 9\times1 + 4\times4 + 4\times9 = 61$$

$$\Rightarrow \left|3\vec{a} - 2\vec{b} + 2\vec{c}\right| = \sqrt{61}$$

2m

34. Sketch the graph y = |x + 1|. Evaluate $\int_{-3}^{1}|x+1|dx$. What does this value represent of the graph?



1 ½ m

Now,

Now, $\int_{3}^{1}\left|x+1\right|dx$

$= \int_{3}^{-1} -(x+1)\,dx + \int_{1}^{1}(x+1)\,dx$ **1m**

$= -\left[\frac{(x+1)^2}{2}\right]_{-3}^{-1} + \left[\frac{(x+1)^2}{2}\right]_{-1}^{1}$

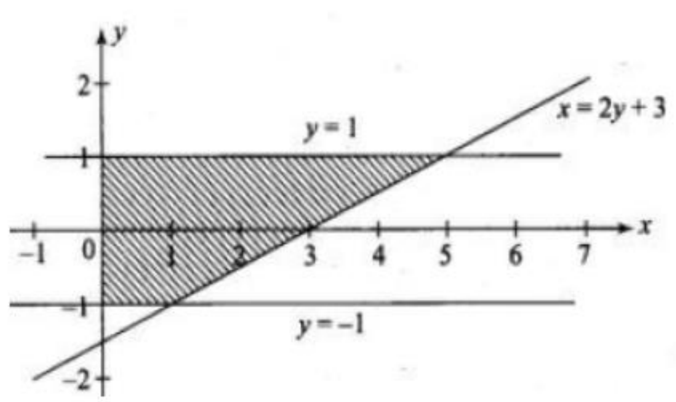
$= -\left[0 - \frac{4}{2}\right] + \left[\frac{4}{2} - 0\right] = 4$ sq units

This value represents the area of the shaded portion shown in figure.

2 + ½ m

OR

The line x = 2y + 3 and the lines y =1 and y = -1.



1 ½ m

From the fig; to find area

$A = \int_{-1}^{1}(2y+3)dy = [y^2 + 3y]_{-1}^{1}$ **2+ 1**

$= [1 + 3 - 1 + 3] = 6$ sq. units. **½ m**

35. Let f : R - $\left\{-\frac{4}{3}\right\} \to R$ be a function defined as f(x) = $\frac{4x}{3x+4}$;

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ such that $f(x_1) = f(x_2)$

or $\frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

or $12\not{x}_1 x_2 + 16x_1 = 12\not{x}_1 x_2 + 16x_2$

or $x_1 = x_2$

$\therefore$ f is one-one.

2m

Let $f(x) = y$

i.e. $\frac{4x}{3x+4} = y$

$3xy + 4y = 4x$

or $x = \frac{4y}{4-3y}$

1m

Function f is onto , when range of f = R – { 4/3} 1+1 m

Section – E

36. (i) The possible values of X are 1, 0, -1, -3

(ii) P(X = -3) = 125/216

(iii)

X	1	0	-1	-3
P(X = x)	1/6	5/36	25/216	125/216

OR

(iv) expected amount : loss of Rs 91/54 .

37. (i) If x and y represent the length and breadth of its rectangular base, then relation between the x and y is:

Depth of tank = **h = 2 m**

& Volume of tank = **V = 8 m³** Volume of tank = Length × Breadth × Height

$$8 = 2 \times x \times y$$
$$4 = xy$$
$$y = \frac{4}{x} \quad ...(1)$$ **1m**

(ii) If the construction of the tank cost Rs.70 per sq. meter for the base and Rs.45 per sq. meter for sides,
to find the making cost 'C' expressed as a function of x;

Let **C be the total cost** of tank

$$\mathbf{C}\,(x) = \text{Cost of Base} + \text{Cost of Sides}$$
$$C\,(x) = 70xy + 180\,(x + y)$$
$$C\,(x) = 70 \times x \times \frac{4}{x} + 180\left(x + \frac{4}{x}\right) \qquad \textit{(From (1): } y = \frac{4}{x}\textit{)}$$
$$C\,(x) = 280 + 180\left(x + \frac{4}{x}\right)$$

(iii) to find for what value of x , C is minimum:

$$C'(x) = 180\left(1 - \frac{4}{x^2}\right)$$

Putting $C'(x) = 0$

$$180\left(1 - \frac{4}{x^2}\right) = 0$$
$$\left(1 - \frac{4}{x^2}\right) = 0$$
$$\frac{(x^2 - 4)}{x^2} = 0$$
$$x^2 - 4 = 0$$
$$(x - 2)(x + 2) = 0$$

So, $x = 2$ or $x = -2$

Since length of ***side cannot be negative***

$\therefore x = 2$ **only**

C''(x)

$$= \frac{1440}{x^3}$$,

Since $\mathbf{C}'' > 0$ for x = 2

Thus, **C is minimum** at x = 2

OR

Least cost of construction = C(2)

$$= 280 + 180\left(2 + \frac{4}{2}\right)$$
$$= 280 + 180\,(2 + 2)$$

Hence, least cost of construction is **Rs 1,000**

38. (i) A(8, 10, 0) D (0, 0, 30)

$\therefore$ Equation of AD is given by

$$\frac{x-0}{8-0}=\frac{y-0}{10-0}=\frac{z-30}{-30}$$

$$\Rightarrow \quad \frac{x}{4}=\frac{y}{5}=\frac{30-z}{15}$$

(ii) C (15, -20, 0) , D (0, 0, 30); $|CD| = 5\sqrt{61}$ **.**

(iii) B(-6, 4, 0) D(0, 0, 30); vector DB is = $-6\hat{\imath}+4\hat{\jmath}-30\hat{k}$

OR

(iv) A (8, 10, 0) ; B(-6, 4, 0) ; C (15, -20, 0)

$|OA| + |OB| + |OC| = \sqrt{164} + \sqrt{52} + \sqrt{625}.$