CHENNAI SAHODAYA SCHOOLS COMPLEX

(General Instructions)

- Please check that this question paper contains ____6__ printed pages.
- Please check that this question paper contains __38___ questions.
- Please write down the serial number of the question before attempting it.
- Reading time of 15 minutes is given to read the question paper alone. No writing during this time.
- **This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.**
- **Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.**
- **Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.**
- **Section C has 6 Short Answer (SA)-type questions of 3 marks each.**
- **Section D has 4 Long Answer (LA)-type questions of 5 marks each.**
- **Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.**

COMMON EXAMINATION
Class-12
(MATHEMATICS – 041)

Roll No.: ***set - 3*** ***Maximum Marks:80***

Date: DD/MM/YYYY ***Time allowed: 3 hours***

Section –A

(Each question carries 1 mark)

(Multiple Choice Questions)

1. **Which of the following functions f: Z → Z is bijective?**

   (a) f(x)= x^3 (b) f(x) = x + 2 (c) f(x) = 2x + 1 (d) f(x) = x^2 +1

2. **If $[2x \quad 4]\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, then positive value of x is:**

   (a) 1 (b) 16 (c) 4 (d) 2

3. **Let A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and 10B = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & a \\ 1 & -2 & 3 \end{bmatrix}$. If B is inverse of A, then a =?**

   (a) 5 (b) 2 (c) -1 (d) -2

4. **If A = $\begin{bmatrix} cos\,x & -sin\,x \\ sin\,x & cos\,x \end{bmatrix}$ and A + A^T = I. Find x, when x ∈ $\left[0, \frac{\pi}{2}\right]$.**

   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

5. **If $\int e^x\,(tan\,x + 1)sec\,x\,dx = e^x$ f(x) + C, then f(x) is:**

   (a) tan x (b) tan x . sec x (c) sec x (d)sec^2 x

6. **The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx}$ - xy = 1 is:**

   (a) $\frac{-x}{1-x^2}$ (b) $\frac{x}{1-x^2}$ (c) - $\sqrt{1-x^2}$ (d) $\sqrt{1-x^2}$

7. The order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is:

(a) 1, 2 (b) 2, 1 (c) ½ , 2 (d) 2, ½

8. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -1\hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ and $\vec{a} + x\,\vec{b}$ *is perpendicular to* $\vec{c}$, then x is:

(a) 8 (b) 4 (c) 16 (d) 21

9. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} - \hat{\jmath}$ then a vector $\vec{b}$ satisfying the equation $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$ is:

(a) $\frac{2\hat{\imath}+2\hat{\jmath}+5\hat{k}}{3}$ (b) $\frac{-5\hat{\imath}+2\hat{\jmath}+2\hat{k}}{3}$ (c) $\frac{5\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$ (*d*) $\frac{5\hat{\imath}+2\hat{\jmath}+2\hat{k}}{3}$

10. If a line makes angles α, β, γ with the co-ordinate axes in space, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ = ?

(a) -2 (b) -1 (c) 1 (d) 2

11. The angle between the lines $\frac{x+1}{2} = \frac{y+3}{2}\ \frac{z-4}{-1}$ *and* $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$ is:

(a) $\cos^{-1}(1/9)$ (b) $\cos^{-1}(2/9)$ (c) $\cos^{-1}(3/9)$ (d) $\cos^{-1}(4/9)$

12. The line passing through the points (5, 1, a) and (3, b, 1) crosses yz-plane at the point (0, 17/2 , -13/2), then ;
(a) a = 8; b = 2 (b) a = 2; b = 8 (c) a = 4; b = 6 (d) a = 6; b = 4

13. The solution set of the inequation x + 2y > 3 is:
(a) Half plane containing the origin
(b) Half plane not containing the origin
(c) First quadrant
(d) Third quadrant

14. The maximum value of Z = 3x + 4y subjected to constraints $x + y \le 4$, $x \ge 0$ and $y \ge 0$ is:

(a) 12 (b) 14 (c) 16 (d) 10

15. Let X denote the number of colleges, where you will apply after class XII. And P(X = x) denotes your probability of getting admission in x number of colleges.

If P(X =x) = $\begin{cases} kx\ , & if\ x = 0\ or\ x = 1 \\ 2kx\ , & if\ x = 2 \\ k(5-x), & if\ x = 3\ or\ x = 4. \end{cases}$, then the value of k is:

(a) 8 (b) 1/8 (c) -8 (d) -1/8

16. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ *at* $\left(\frac{1}{4}, \frac{1}{4}\right)$ *is*:

(a) ½ (b) 1 (c) -1 (d) 2

17. For what value of k, the function f(x) = $\begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2} & , \ x \neq 2 \\ k & , \ x = 2 \end{cases}$ is continuous for all real x.

(a) 7 (b) -7 (c) 1 (d) 1/7

18. If A = $\begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, then $|A \cdot adj\ A|$= ?

(a) 3 (b) 9 (c) 27 (d) 81

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

19. Assertion (A): The minimum value of the function y = cos x in $[0, \pi]$ is at x = π.
Reasoning(R): The first derivative of the function is zero at x = π and second derivative is negative at x = π.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

20. Assertion(A): The matrix A = $\begin{bmatrix} 2 & 3 & -\frac{1}{2} \\ 7 & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ is singular.
Reason(R) : the determinant value of singular matrix is 0.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Section –B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

21. Find the value of $\tan^{-1}\left(2 \sin\left(2 \cos^{-1}\frac{\sqrt{3}}{2}\right)\right)$.

22. Show that the function f defined by $f(x) = (x - 1)e^x + 1$ is an increasing function for all x > 0.

OR

(b) Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at x = $\frac{\pi}{6}$.

23. The volume of a sphere is increasing at the rate of 3 cubic centimetre per second. Find the rate of increase of its surface area, when the radius is 2 cm.

24. Evaluate: $\int \frac{1}{\sqrt{x^2+2x+2}}\,dx$.

OR

Evaluate : $\int e^x \frac{(1+\sin x)}{(1+\cos x)}\,dx$

25. Using integration, find the area under the curve $y = 2\sqrt{x}$ included between x= 0 and x = 1.

Section – C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$.

27. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\, dx$

OR

Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$

28. Evaluate: $\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\, dx$.

29. Find the general solution of the homogeneous differential equation:

$(x - y)\frac{dy}{dx} = x + 2y$.

30. Solve the linear programming problem graphically:

Minimise $Z = 300x + 400y$

Subject to constraints: $6x + 10y \geq 60$; $4x + 4y \geq 32$; $x \geq 0$, $y \geq 0$.

31. A bag I contains 5 red and 4 white balls and bag II contains 3 red and 3 white balls. Two balls are transferred from the bagI to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red and one white ball is transferred from the bag I to the bagII.

OR

Find the probability distribution of the random variable X, which denotes number of sixes in two tosses of a die.

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}.

OR

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & 3 \end{bmatrix}$ find A^{-1}. Using A^{-1}, solve the system of linear equations:

$x + y + z = 3$; $2x - y + z = 2$; $x - 2y + 3z = 2$.

33. Let $\vec{a}, \vec{b}, and\ \vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of of $\vec{c}$ along $\vec{a}$; and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

34. Sketch the graph $y = |x + 1|$. Evaluate $\int_{-3}^{1} |x + 1| dx$. What does this value represent of the graph?

OR

Using integration, find the area of the region bounded by the line $x = 2y + 3$ and the lines $y = 1$ and $y = -1$.

35. **Let f : R - $\left\{-\frac{4}{3}\right\} \to R$ be a function defined as f(x) = $\frac{4x}{3x+4}$. Show that the function f : R - $\left\{-\frac{4}{3}\right\} \to range\ of\ f$, is one – one and onto. Hence find range of f.**

Section –E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively].

36. Two friends Arpan and Sachin play a game. As per rules, in this game a person wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. Arpan decided to throw the die thrice but to quit as and when he gets a six.

Based on the given information, answer the following questions.

Let X be the amount won / lost by Arpan.

(i) **What are the possible values of X?**

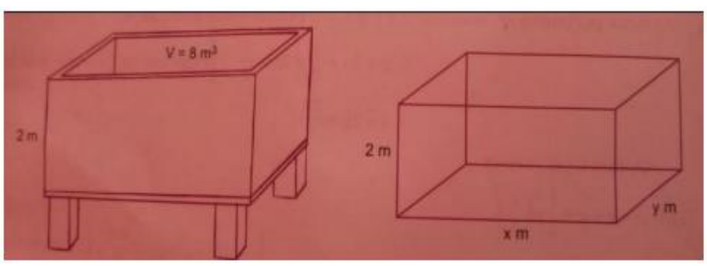
(ii) **What is the probability of getting rupees 3 loss?**

(iii) **(a)Find the probability distribution of X.**

Or

(b) Find the expected amount Arpan gets

37. At the request of villagers, a construction agency designs a tank with the help of an architect. The tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below:



Based on the above information answer the following:

(i) **If x and y represent the length and breadth of its rectangular base, find the relation between the x and y.**

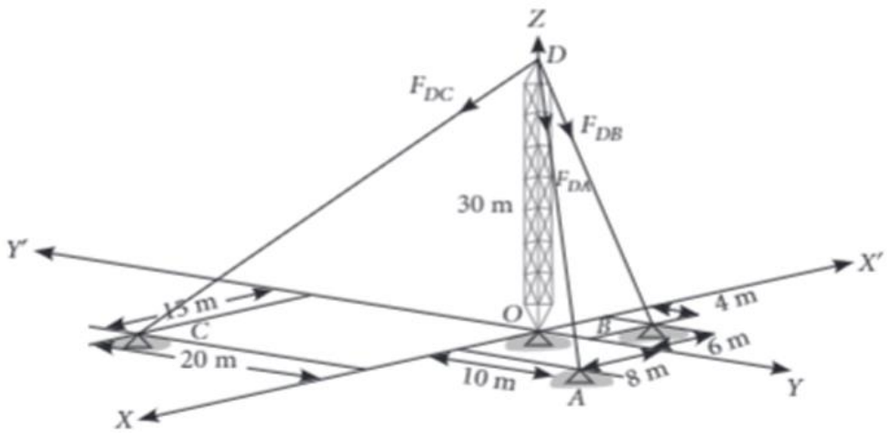
(ii) **If the construction of the tank cost Rs.70 per sq. meter for the base and Rs.45 per sq.meter for sides, find the making cost 'C' expressed as a function of x.**

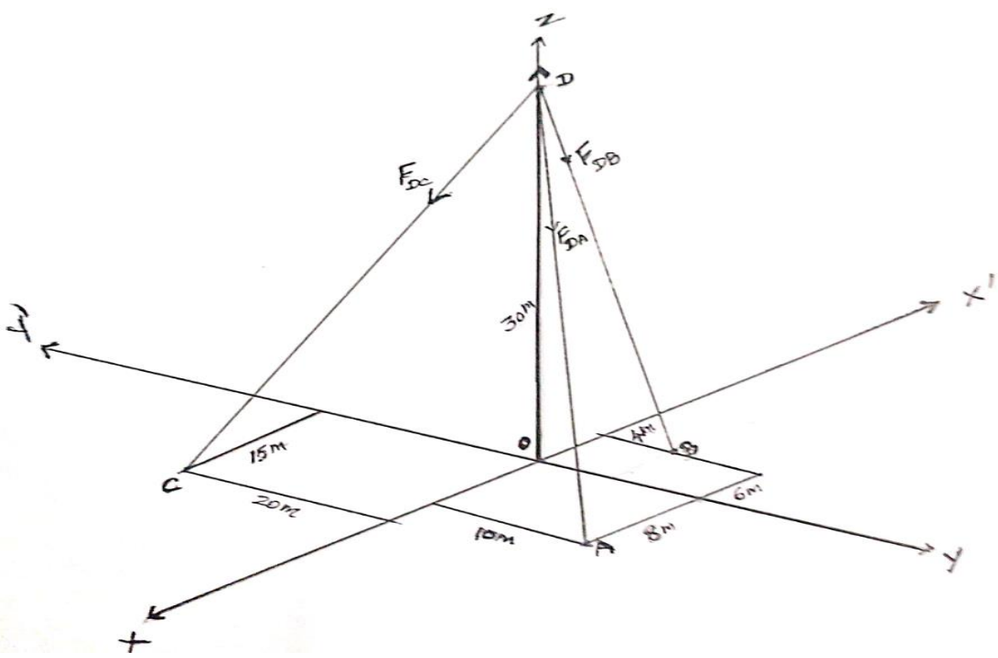
(iii) **(a)The owner of a construction agency is interested in minimizing the cost 'C' of the whole tank. So, for what value of x , C is minimum?**

OR

(b)Find the minimum construction cost of the tank .

38. Observe the diagram to answer the questions given below:





(i) Find the equation of line along the cable AD.
(ii) Find the length of cable DC.
(iii) (a) Find the vector BD.
OR
(b)Find the sum of distances of points A, B and C from the origin.

********** END OF THE PAPER **********