ERODE SOHODAYA SCHOOLS COMPLEX

PRE-BOARD EXAMINATION 2023-24

CLASS XII | **MATHEMATICS** | **TIME: 3 hrs**

SET - A | **Max Marks: 80**

MARKING SCHEME.

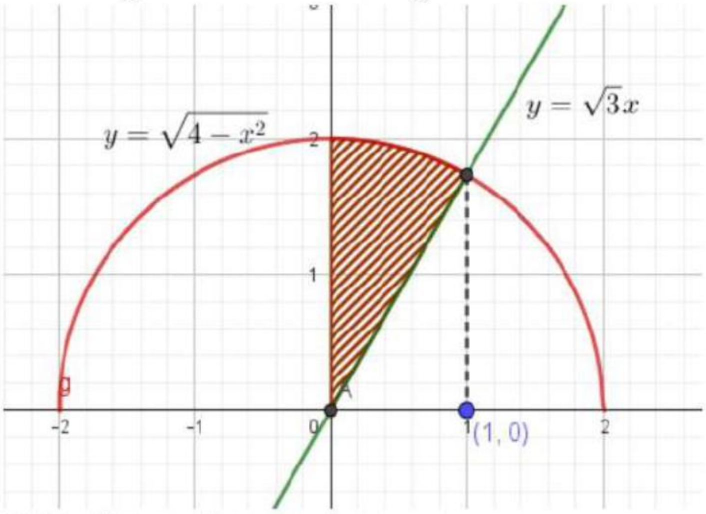
Q.NO	EXPECTED ANSWER	MARKS
1	(d) 2 and 4	1
2	(b) Unbounded	1
3	(a) 512	1
4	(b) x^2	1
5	(c) 0	1
6	(d) $\frac{1}{36}$	1
7	(b) I	1
8	(b) $A^2 = I$	1
9	(d) $\frac{1}{12}$	1
10	(a) $\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$	1
11	(b)- 1	1
12	(a) y	1
13	(d) Use midpoint formulae	1
14	(d) $\overrightarrow{PQ} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$; so unit vector $= \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$	1
15	(b) $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + p\hat{\jmath} + q\hat{k}) = \vec{0} \Rightarrow$ *vectors are parallel* $\Rightarrow \frac{2}{1} = \frac{6}{p} = \frac{27}{q}$	1
16	(b) 2	1
17	(c) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$	1
18	(b) $\tan^{-1} x^3 + c$	1
19	(c) Assertion is correct, reason is incorrect	1
20	(a) $f(x)$ has a minimum at $x = 2$ as $\frac{d}{dx}(f(x)) < 0, \forall x \in (2-h, 2)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (2, 2+h)$, where $'h'$ is an infinitesimally small positive quantity.	1
21	the volume of a cube with radius "x" is given by V = x^3 ans surface area = $6x^2$	½

	Hence, the rate of change of volume "V' with respect to the time "t" is given by: $\frac{dV}{dt} = 9$ cm³/s $9 = \frac{dV}{dt} = \frac{d}{dt}x^3 = 3x^2.\frac{dx}{dt}$, By using the chain rule $\frac{dx}{dt} = \frac{3}{x^2}$ $\frac{ds}{dt} = \frac{d(6x^2)}{dt} = 12\frac{dx}{dt} = 12 \times \frac{3}{x^2} = \frac{36}{x}$ At x= 10 , $\frac{ds}{dt} = 3.6$ cm²/s **OR** p(x) = 41-72x-18x² P'(x) = -72 -36 P'' = -36 For maximum or minina or critical points p'(x) =0 -72 -36x =0 , x=-2 x=-2, p'' (-2) = -36 <0 hence x= -2 is a point of local maxima, maximum profit = 113	1/2 1/2 1/2 1/2 1/2 1/2 1/2
22	We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. $= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$. $= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$ $= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\begin{array}{c}\because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0, \pi) \\ and \tan^{-1}(-x) = -\tan^{-1}x\end{array}\right]$ $= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$ $= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$ OR The domain of $\sin^{-1}$ x is [-1,1]. Therefore, f(x) = $\sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \le -x^2 \le 1$ $\Rightarrow \quad 1 \ge x^2 \ge -1$ $\Rightarrow 0 \le x^2 \le 1$ $\Rightarrow x^2 \le 1$ $\Rightarrow x^2 - 1 \le 0$ $\Rightarrow$ (x - 1)(x + 1) ≤ 0 $\Rightarrow -1 \le x \le 1$ Hence, the domain of f(x) = $\sin^{-1}(-x^2)$ is [-1, 1].	

23	f(x) = $4x^3 - 6x^2 - 72x + 30$ $f'(x) = 12x^2 - 12x - 72$ For critical points $f'(x) = 0$, $12x^2 - 12x - 72 = 0$ 12(x-3)(x+2) = 0 X= 2, x= -3 $(-\infty, -2)$ and $(3, \infty)$ the function is increasing (-2,3) the function is decreasing	 1/2 1/2 1/2 1/2
24	$I = \int_0^{\pi} \frac{1}{1+e^{\cos x}} dx ----(1)$ Applying $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ $I = \int_0^{\pi} \frac{1}{1+e^{\cos(\pi - x)}} dx = \int_0^{\pi} \frac{1}{1+e^{-\cos x}} dx$ $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+1} dx ----(2)$ Adding (1) and (2) $2I = \int_0^{\pi} \frac{e^{\cos x}+1}{e^{\cos x}+1} dx = \int_0^{\pi} dx$ $\therefore 2I = \pi \Rightarrow I = \frac{\pi}{2}$	 1/2 1/2 1/2 1/2
25	$f(x) = x^3 + x, x \in R$ $f'(x) = 3x^2 + 1, x \in R$ But for any $x \in R, 3x^2 + 1 \neq 0$ $\therefore f(x)$ has no critical points.	 1/2 1 1/2
26.	$\frac{dx}{dy} + \frac{2xy}{(y^2+1)} = \frac{1}{(1+y^2)(y^2+1)}$ $IF = e^{\int \frac{2y dy}{1+y^2}} = 1 + y^2$ Solution x(IF) = $\int Q(IF)dy + c$ $x(y^2+1) = \tan^{-1} y + C$ OR Let y= Vx = > dy/dx = V + x(dV/dx) $y + x\sin\left(\frac{y}{x}\right) = x\frac{dy}{dx} \Rightarrow \frac{dx}{x} = \frac{dV}{\sin V}$ Solving => cx = (cosecx – cotx) Particular solution x (√2 – 1)/2= (cosecx – cotx)	1/2 1 1/2 1 1/2 1 1 1/2
27	$I = \int_{-5}^{5} \frac{x^2}{1+e^x} dx$ ..... (i), using property $\int_{-5}^{5} f(x)dx = \int_{-5}^{5} f(-x)dx$ $I = \int_{-5}^{5} \frac{x^2}{1+e^{-x}} dx$ .....(ii)	1/2 1/2

	From i and ii , adding $2I = \int_{-5}^{5} x^2 dx \Rightarrow \quad I = \frac{125}{3}$ OR $\int_{0}^{4} (\lvert x-1 \rvert + \lvert x-2 \rvert) dx$ $= \int_{0}^{1} (3-2x)dx + \int_{1}^{2} dx + \int_{2}^{4} (2x-3)dx$ =2+1+6 = 9	1/2 1 +1/2 1/2+1/2 +1/2 1 1/2
28	$I = \int \frac{2x}{(x^2+1)(x^2+2)} dx$ let $x^2 = t \Rightarrow 2xdx = dt$ $\frac{1}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2}$ Solving A = 1, B = -1 Correct integration and solution $I = \log\left(\frac{x^2+1}{x^2+2}\right)$	1/2 1/2 1/2 + 1/2 1/2 + 1/2
29	For correct differentiation of LHS 1 marks Right hand side with simplification 1 marks Proving 1 marks	1 1 1
30	Draw the graph of 3 lines x+3y = 60 , x+y = 10 , x = y , correctly. Shading the feasible region correctly Corner points of the feasible solutions are (0,10) , (5,5) , (15,15) and (0,20) Minimum value is 60 at the point (5,5) (OR) Draw the graph of 2 lines 2x + y =8 , x + 2y = 10, correctly Shading the feasible region (unbounded) correctly Corner points are (0,8) , (2,4) and (10,0) Smallest value of Z is 380 at (2,4) Draw the graph of $50x + 70y < 380$ correctly Minimum value of Z is 380 at (2,4)	3X1/2 1/2 1 1 1/2 1/2 1/2 1/2
31	Given, bag A = 4 black and 6 red balls bag B = 7 black and 3 red balls. Let E_1 = The event that die show 1 or 2 E_2 = The event that die show 3 or 4 or 5 or 6 E = The event that among two drawn balls, one of them is red and other is black $P(E_1) = \frac{2}{6}$, $P(E_2) = \frac{4}{6}$ [$\because$ total number in a die is six] $\therefore P\left(\frac{E}{E_1}\right)$ = P (getting one red and one black from) bag A = $\frac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \frac{4\times6\times2}{10\times9}$ $\Rightarrow P\left(\frac{E}{E_2}\right)$ = P(getting one red and one black from bag B)	

	$= \frac{7C_1 \times {}^3C_1}{{}^{10}C_2}$ $= \frac{7\times3\times2}{10\times9}$ Now, by theorem of total probability P(E) = $P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right)$ $= \frac{2}{6}\cdot\left(\frac{4\times6\times2}{10\times9}\right) + \frac{4}{6}\cdot\left(\frac{7\times3\times2}{10\times9}\right)$ $= \frac{4\times6}{6\times10\times9}(4+7) = \frac{4\times6\times11}{6\times10\times9} = \frac{22}{45}$	
32	(i) 2x +4y +3z = 29000 5x +2x +3x = 30500 x +y + z = 9500 (ii) matrix method AX=B $X=A^{-1}B$ IAI = -1 adj A= $\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$ calculation X= 2500, y= 3000 and z =4000	1 ½ ½ 2 ½ ½
33	$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ *and* $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ $\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\overrightarrow{a_2} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ $\overrightarrow{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\overrightarrow{b_2} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ $\overrightarrow{a_2} - \overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ $\overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\lvert\overrightarrow{b_1} \times \overrightarrow{b_2}\rvert = \sqrt{6}$ $\overrightarrow{(b_1} \times \overrightarrow{b_2}).\overrightarrow{(a_2} - \overrightarrow{a_1}) = 1$ S.D. = d = $\left\lvert \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{\lvert\overrightarrow{b_1} \times \overrightarrow{b_2}\rvert} \right\rvert$ Shortest distance d = $\frac{1}{\sqrt{6}}$ The lines do not intersect **OR** Eq. of line $\vec{r} = \vec{a} + \lambda\vec{b}$ Line passes through (1,2,-4) and let (a,b,c) be the D,Ratio of line then	 1 ½ 1 ½ ½ ½ ½ 1/2 ½

	Eq of line is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$ Line is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ *and* $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ $\vec{a} \times \vec{b}$ *is perpendicular to* $\vec{a}$ *and* $\vec{b}$ *both* $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$ $= 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ Hence D' Ratio of line is (24,36,72) Eq. of line $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$ $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$ $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$	1/2 $2\frac{1}{2}$ ½ ½ ½
34	Correct figure and correct shading  Point of intersection at x=1 Required Area= $\int_0^1 \sqrt{4-x^2}dx - \int_0^1 \sqrt{3}x\, dx$ $= \left\{\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right\}_0^1 - \frac{\sqrt{3}}{2}[x^2]_0^1$ $= \frac{\sqrt{3}}{2} + 2 \times \frac{\pi}{6} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ *Sq. units*	$1 + \frac{1}{2}$ 1/2 $1\frac{1}{2}$ 1 1/2

35	Let $(a, b) \in N \times N$. Then we have $ab = ba$ (by commutative property of multiplication of natural numbers) $\Rightarrow (a, b)R(a, b)$ Hence, R is reflexive. Let $(a, b), (c, d) \in N \times N$ such that (a, b) R (c, d). Then $ad = bc \Rightarrow cb = da$ (by commutative property of multiplication of natural numbers $\Rightarrow (c, d)R(a, b)$ Hence, R is symmetric. Let $(a, b), (c, d), (e, f) \in N \times N$ such that (a, b) R (c, d) and (c, d) R (e, f). Then $ad = bc, cf = de$ $\Rightarrow adcf = bcde$ $\Rightarrow af = be$ $\Rightarrow (a, b)R(e, f)$ Hence, R is transitive. Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$. $[(2,6)] = \{(x, y): y = 3x, x, y \in N\}$ $= \{(1,3), (2,6), (3,9) \ldots\}$ OR Onto: Let $y \in [0,5]$ *such that* $y = \sqrt{25 - x^2}$ $\Rightarrow y^2 = 25 - x^2$ $\Rightarrow x^2 = 25 - y^2$ $\Rightarrow x = \pm\sqrt{25 - y^2}$ $i.e, for$ $y \in [0,5]$ there exist an $x \in [-5,5]$ such that $f(x) = y$ $\therefore f$ *is onto* One-one: Giving an example like, $f(1) = \sqrt{25 - 1} = \sqrt{24}$ and $f(-1) = \sqrt{25 - 1} = \sqrt{24}$ But $1 \neq -1$ Hence f is not one-one. $f(a) = \sqrt{21} \Rightarrow \sqrt{25 - a^2} = \sqrt{21}$ $\Rightarrow a^2 = 4$ $\therefore a = 2$ *or* -2	1 1 1 1 1 2 2 1
36	(1) $2x + 2r + \pi r = 10$ $2x + (2 + \pi) r = 10$ (2) $A = x(2r) + \frac{1}{2}\pi r^2 = r[10 - (2 + \pi)r] + \frac{1}{2}\pi r^2 = 10r - (2 + \pi) r^2 + \frac{1}{2}\pi r^2$	1 1

	(3) $\frac{dA}{dr} = 10 - 2(2+\pi)r + \pi r$ $\frac{dA}{dr} = 0$ $r = \frac{10}{4+\pi}$ m (OR) $\frac{d^2A}{dr^2} = -2(2+\pi) + \pi < 0\ for\quad r = \frac{10}{4+\pi}$ $2x + (2+\pi).\frac{10}{4+\pi} = 10$ $x = \frac{10}{4+\pi}$ m	1 1 1 1
37	P(E1) = 0.25 P(E/E1) = 4/15 P(E2) = 0.35 P(E/E2) = 5/15 P(E3) = 0.40 P(E/E3) = 6/15 i) P(E) = ∑P(Ei)P(E/Ei) = 103/300 ii) a) P(E1/E) = P(E1) P(E/E1) / P(E) formula = 20/103 calculation + ans ii) b) $\sum_{i=1}^{3} P(E_i / E)$ = P(E1) P(E/E1) / P(E) + P(E2) P(E/E2) / P(E) + P(E3) P(E/E3) / P(E) = P(E) / P(E) = 1	All prob value ½ Formula ½ Cal + ans ½ + ½ 1 1 ½ 1 ½ + ½
38	(i)we have $\overrightarrow{OA} = 8\hat{\imath}\ km\ \overrightarrow{AB} = 6\hat{\jmath}$ vector distance from Gitika's house to school =$8\hat{\imath} + 6\hat{\jmath}$ (ii)vector distance from school to Aloke's house = $6\cos30^0\hat{\imath} + 6sin30^0\hat{\jmath}$ = $3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$ (iii)vector distance from Gitika's house to Aloke's house= $8\hat{\imath} + 6\hat{\jmath} + 3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$ =$(8 + 3\sqrt{3})$+$9\hat{\jmath}$ **OR** The total distance travel by Gitika from her house to Aloke's house= 8 + 6 + 6 = 20 km	1 1 2 2