ERODE SOHODAYA SCHOOLS COMPLEX

PRE-BOARD EXAMINATION 2023-24

CLASS XII | **MATHEMATICS** | **TIME: 3 hrs**

SET - A | **Max Marks : 80**

General Instructions:

1. This Question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4marks each) with sub-parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1mark

1). Order and degree of differential equation $\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$

(a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4

2). The area of the feasible region for the following constraints $3y + x \geq 3$, $x \geq 0$, $y \geq 0$ will be

(a) Bounded (b) Unbounded (c) Convex (d) Concave

3). The number of all possible matrices of order 3x3 with each entry -1 or 1 is

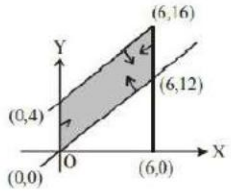
(a) 512 (b) 81 (c) 27 (d) 18

4). Integrating factor of differential equation $x\frac{dy}{dx} + 2y = x^2$

(a) $\frac{1}{x^2}$ (b) x^2 (c) x (d) $\frac{1}{x}$

5). The feasible region for LPP is shown shaded in the figure.
Let Z = 3 x – 4 y be the objective function, then maximum value of Z is

(a) 12
(b) 8
(c) 0
(d) –18



6). The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$

7). If $A=\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$ then the value of the product AA^T is

A) null matrix B) I C) A^2 D) A

8). Let $A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then the only correct statement about the matrix A is

A) A^{-1} does not exist. B) $A^2=I$

C) A is a zero matrix D) A = (-1) I, where I is identity matrix

9). If A and B are invertible matrices of order 3, $|A|=2$ and $|(AB)^{-1}|=6$. Find $|B|$

A) 3 B) $\frac{1}{3}$ C) 12 D) $\frac{1}{12}$

10). If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$ = P + Q , where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to

(a) $\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & \frac{5}{2} \\ \frac{-5}{2} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & \frac{-5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{5}{2} & 4 \end{bmatrix}$

11). If f(x) is continuous at x=1 and $f(x) = \begin{cases} 4x^2 + 3bx, & x \neq 1 \\ 5x - 4, & x = 1 \end{cases}$ then the value of 'b' is

(a) 3 (b) - 1 (c) 1 (d) - 3

12). If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is equal to

(a) y (b) - y (c) x (d) – x.

13). In ΔABC, $\overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$. If D is the midpoint of BC, then $\overrightarrow{AD}$ is equal to

(a) $4\hat{\imath} + 6\hat{k}$ **(b)** $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ **(c)** $\hat{\imath} - \hat{\jmath} + \hat{k}$ **(d)** $2\hat{\imath} + 3\hat{k}$

14). Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are $(2, 1, -1)$ and $(4, 4, -7)$ is

(a) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ **(b)** $-2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

(c) $\frac{-2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$ **(d)** $\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$

15). If $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + p\hat{\jmath} + q\hat{k}) = \vec{0}$, then the value of p and q are:

(a) $p = 6, q = 27$ **(b)** $p = 3, q = \frac{27}{2}$

(c) $p = 6, q = \frac{27}{2}$ **(d)** $p = 3, q = 27$

16). If a line makes an angle α,β,γ with x, y z axis, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

A) 1 B) 2 C) 0 D) 3

17). Direction cosines of the line joining the points (3, 7, -2) & (1,4,4) are given by

A) (2,3,-6) B) (-2,-3,6) C) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$ D) $\left(\frac{1}{7}, \frac{2}{7}, \frac{3}{7}\right)$

18). $\int \frac{3x^2}{x^6 + 1}\, dx$ *is equal to*

(a) $\log(x^6 + 1) + c$ (b) $\tan^{-1} x^3 + c$ (c) $3 \tan^{-1} x^3 + c$ (d) $\log x^2 + c$

ASSERTION-REASON BASED QUESTIONS:

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19). **Assertion**: A relation R = {(a,b) : |a-b|<2} defined on the set A = {1, 2, 3, 4, 5} is reflexive.

Reason : A relation R on the set A is said to be reflexive if $(a,b) \in R$ *and* $(b,c) \in R$ *for all* $a, b \in A$.

20). **Assertion (A):** Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-2)^3(x-3)^2$, then $f(x)$ has a minimum at $x = 2$.

Reason (R): When $\frac{d}{dx}(f(x)) < 0, \forall\, x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall\, x \in (a, a+h)$, where $'h'$ is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2marks each

21). The volume of the cube is increasing at the rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters?

OR

Find the maximum profit that a company can make, if the point function is given by

$p(x) = 41-72x-18x^2$

22). Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

OR

Find the domain of $f(x) = \sin^{-1}(-x^2)$.

23). Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$

(a) Strictly increasing (b) strictly decreasing

24). Evaluate $\int_0^{\pi} \frac{1}{1+e^{\cos x}} dx$

25). Check whether the function $f: R \to R$ defined by $f(x) = x^3 + x$ has any critical point/s or not. If yes, then find the points.

SECTION C

This section comprises of Short Answer type questions (SA) of 3marks each

26). Find the general solution of the differential equation $\left(y^2+1\right)\frac{dx}{dy} + 2xy = \frac{1}{1+y^2}$

OR

Find the particular solution of the differential equation when x = 2, y = π/2 $y + x\sin\left(\frac{y}{x}\right) = x\frac{dy}{dx}$

27). Evaluate $\int_{-5}^{5} \frac{x^2}{1+e^x} dx$

OR

Evaluate $\int_0^4 \left(|x-1| + |x-2|\right) dx$

28). Evaluate $\int \frac{2x}{\left(x^2+1\right)\left(x^2+2\right)} dx$

29). If $y\sqrt{1+x^2} = \log\left[\sqrt{1+x^2} - x\right]$, then show that $\left(1+x^2\right)\frac{dy}{dx} + xy + 1 = 0$

30) Solve the following linear programming problem graphically.

Minimize $Z = 3x + 9y$

Subject to

$x + 3y \leq 60, \quad x + y \geq 10, \quad x \leq y, x, y \geq 0$

(OR)

Solve the following linear programming problem graphically.

Minimize $Z = 50x + 70y$

Subject to

$2x + y \geq 8, \quad x + 2y \geq 10, \quad x, y \geq 0$

31). A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find, the probability of one of them being red and another black.

SECTION D

This section comprises of Long Answer-type questions (LA) of 5 marks each

32). Two factories decided to award their employee for three values of (a) adaptable to new situation, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with total prize money of ₹ 29000. The second factory decided to honour respectively 5, 2 and 3 employees with a total prize money of ₹ 30500. If three prizes per person together cost ₹ 9500 then

(i) Represents the above situation by a matrix equation and form linear equations using matrix multiplication,

(ii) Solve these equation using matrices.

33). By computing the shortest distance determine whether the lines intersect or not. If not then find the shortest distance between the lines.

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad and \quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

OR

Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad and \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

34). Using integration, find the area of region bounded by the line $y = \sqrt{3}x$, the curve $y = \sqrt{4 - x^2}$ and $y - axis$ in first quadrant.

35). Let N be the set of all-natural numbers and R be a relation defined on $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$. Also find the equivalence class of $(2,6)$.

OR

A function $f: [-5,5] \rightarrow [0,5]$ is given by $f(x) = \sqrt{25 - x^2}$. Show that f is an onto function, but not a one-one function. Further, find all possible values of $'a'$ for which $f(a) = \sqrt{21}$.

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each. First two questions have three sub-parts (1),(2),(3) of marks 1,1,2 respectively. The third case study question has two sub-parts of 2 marks each.

36). Read the following passage and answer the questions given below.

A house is being constructed and a lot of planning is put into it. Now a person is confused about the window. He wants the window in the form of a rectangle surmounted by a semicircle such that the perimeter of the window is to be 10 metres. If radius of the semicircular portion is 'r' metres and height of the rectangular portion is 'x' metres, then

(1) Write a relation between x and r.
(2) Represent the area in terms of 'r'.
(3) Find the critical point, with respect to area, in terms of 'r'.

(OR)

(3) What are dimensions so that maximum light may enter the room?

(Note: Internal choice is for option 3)

37). A magician has three bags. 1st bag contains 4 red, 5 green and 6 yellow balls, 2nd bag contains 5 red, 5 green and 5 yellow balls and 3rd bag contains 6 red, 4 green and 5 yellow balls. The magician is showing a trick to randomly draw a ball from a bag. The probability of drawing a ball from 1st bag, 2nd bag and 3rd bag is 25% ,35% *and* 40% respectively.

Based on above information answer the following:

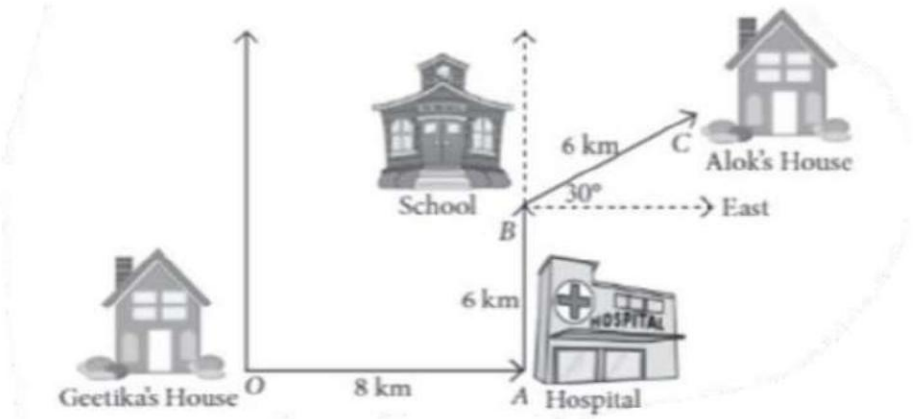
(i) What is the probability that he drew a red ball? [2]

(ii) a) The magician drew a red ball. What is the probability that he drew from 1st bag? [2]

OR

(ii) b) if E_1, E_2, E_3 are the three mutually exclusive and exhaustive events and E is an event associated with them, then find $\sum_{i=1}^{3} P(E_i / E)$ [2]

38).



Gitika house is situated at Shalimar Bag at O, going to Aloke's house she first travel 8 km in the east, here at point A a hospital is situated. From the hospital she takes auto and goes 6 km in the north. Here at point B a school is situated. From school she travels by bus to reach Aloke's house which is 30^0 of east and 6 km from point B.

(i) What is vector distance from Gitika's house to school? 1

(ii) What is vector distance from school to Aloke's house? 1

(iii) What is vector distance from Gitika's house to Aloke's house? 2

OR

What is the total distance travel by Gitika from her house to Aloke's house?

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