INFINITY

THINK BEYOND............

AN EDUCATIONAL INSTITUTE

X MATHEMATICS

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Volume 3

CBSE SOLVED SAMPLE PAPERS

AS PER LATEST Pattern 2024-25

#KEEP LEARNING

X - MATHEMATICS

SAMPLE PAPER - 1

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. What is the largest number that divides each one of 1152 and 1664 exactly ?

(a) 32 (b) 64 (c) 128 (d) 256

Q2. The roots of the equation $x^2 - 3x - m(m + 3) = 0$, where m is constant are

(a) m, m + 3 (b) 3 + 3, – m (c) m, –(m + 3) (d) – (m + 3), – m

Q3. The numberof zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is / are

(a) 2 (b) 1 (c) 0 (d) 3

Q4. The pair of equations $2x - 3y = 1$ and $3x - 2y - 4$ has ________ solution

(a) one (b) two (c) no (d) many

Q5. A triangle with vertices (4, 0), (–1, –1) and (3, 5) is a/an

(a) equilateral triangle (b) right-angled triangle

(c) isosceles right-angled triangle (d) none of these

Q6. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$ then $\Delta ABC \sim \Delta EDF$, if

(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$

Q7. If θ is an acuate angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Q8. The line segment joining the points P(–3, 2) and Q(5, 7) is divided by the y-axis in the ratio

(a) 3 : 1 (b) 3 : 4 (c) 3 : 2 (d) 3 : 5

Q9. In the given figure $\frac{AD}{BD} = \frac{AE}{EC}$ and $\angle ADE = 70°$, $\angle BAC = 50°$, then angle $\angle BCA =$

(a) 70° (b) 50° (c) 80° (d) 60°

Q10. In the given figure, AD = 1.28 cm, BD = 2.56 cm, AE = 0.65 cm, DE will be parallel to BC, if EC =

(a) 1.28 cm (b) 2.56 cm (c) 0.64 cm (d) 0.32 cm

Q11. How many tangents can a circle have

(a) 1 (b) 2 (c) Infinity many (d) None of these

Q12. If the circumference and thearea of a circle are numerically equal, then the radius of the circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

Q13. The surface area of a sphere is 616 cm^2, its radius is

(a) 19 cm (b) 7 cm (c) – 7 cm (d) 14 cm

Q14. d_i is the deviation of x_i from assumed mean a.

If mean = $x + \frac{\Sigma f_i d_i}{\Sigma f_i}$, then x is

(a) class size (b) number of observation

(c) assumed mean (d) none of these

Q15. A toothed wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. How many revolutions will the smaller wheel make when the larger one makes 15 revolutions ?

(a) 23 (b) 24 (c) 50 (d) 60

Q16. Mean of 100 items is 49. It was observed that three items which should have been 60, 70, 80 were wrongly need as 40, 20, 50 respectively. The correct mean is

(a) 48 (b) 49 (c) 50 (d) 60

Q17. 2000 tickets of a lottery were sold and there are 16 prizes on these tickets. Abhinav has purchased one lottery ticket. The probability that Abhinav wins a prize is

(a) 10.08 (b) 00.07 (c) 0.0008 (d) 0.080

Q18. At sometimes, the length of a shadow of a tower is $\sqrt{3}$ times its height, then the angle of elevation of the Sun, at that time is

(a) 15° (b) 30° (c) 45° (d) 60°

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** The HCF of two number is 9 and their LCM is 2016. If one of the number is 306, then the other is 54.

Statement R (Reason) : For any positive integers a and b, we have : Product two numbers = HCF × LCM.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** The value of $\sin\theta = \frac{4}{3}$ in not posible.

Statement R (Reason) : Hypotentuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

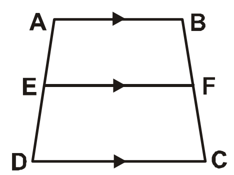
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

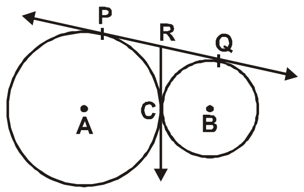
Section B consists of 5 questions of 2 marks each.

Q21. Find the sum of all multiples of 7 lying between 100 and 1000.

Q22. In the given figure, ABCD is a trapezium in which AB || DC || EF. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.



Q23. In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

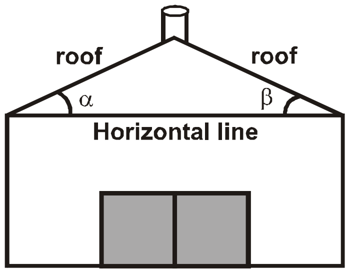


Q24. An arc of a circle of length 7π cm and the sector it bounds has an area 28π cm^3. Find the radius of the circle.

OR

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour ?

Q25. In some buildings especially in industries, the roof is inclined. This inclination of roof is the application of trigonometric functions. Here the roof of industry is inclined at angle α and β with horizontal line as shown. Determine the value of $\sin(\alpha+\beta)$, if $\operatorname{cosec}\alpha = \sqrt{2}$ and $\cot\beta = 1$, where both α and β are acute angles.



SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $3-2\sqrt{5}$ is irrational.

Q27. Solve for x : $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0;\ x \neq 3, \frac{-3}{2}$

Q28. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

OR

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Q29. If $\sec\theta = x+\frac{1}{4x}$, then prove that $\sec\theta-\tan\theta=\frac{1}{2x}$ or $2x$.

Q30. Prove that the line segment joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Q31. A game has 8 triangles of which 6 are blue and rest are green, 12 rectangles of which 3 are green and rest are blue, and 10 rhombuses of which 3 are blue and rest are green. One piece is lost at random. Find the probability that it is

(i) a rectangle (ii) a triangle of green colour

(iii) a rhombus of blue colour.

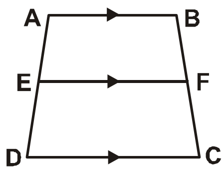
SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. If a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio, prove it.

Use the result to prove the following :

In the given figure, ABCD is a trapezium in which AB || DC || EF. Show that $\frac{AE}{ED}=\frac{BF}{FC}$.



Q33. At t minutes past 2 p.m. the time needed by the minutes hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

OR

At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age Nisha. Find the present age of btoh Asha and Nisha.

Q34. A circus tent is in the shape of a cylinder surmounted by a conical top of the same diameter. If their common diameter is 56m, the height of cylindrical part is 6m and the total height of the tent above the ground is 27m, find the area of canvas used in making the tent.

OR

A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid.

Q35. The marks of 80 students of class X in Mathematics test are given below. Find the mode of these marks obtained by the students in Mathematics test.

Marks	Frequency
0 – 10	2
10 – 20	6
20 – 30	12
30 – 40	16
40 – 50	13
50 – 60	20
60 – 70	5
70 – 80	1
80 – 90	4
90 – 100	1
Total	80

SECTION - E

Case study based questions are compoulsory.

Q36. Two friends Raj and Anuj have to travel to Shimla via Chandigarh from Gurgaon. When they reached the bus stand of Gurgaon, Raj got a call from his friend Ankit who was also on his way to bus stand. Ankit requested Raj to buy two tickets to Chandigarh and 3 tickets to Shimla also Anuj's friend Kamla asked Anuj to buy 3 tickets to Chandigarh and 4 tickets to Shimla. Raj purchased 2 tickets to Chandigarh and 3 tickets to Shimla for Rs. 3700, Anuj spent Rs. 5100 to buy 3 tickets to Chandigarh and 4 tickets to Shimla.

(i) If cost of one ticket to Chandigarh is Rs. x and cost of one ticket to Shimla is Rs. y then represent the situation algebraically.

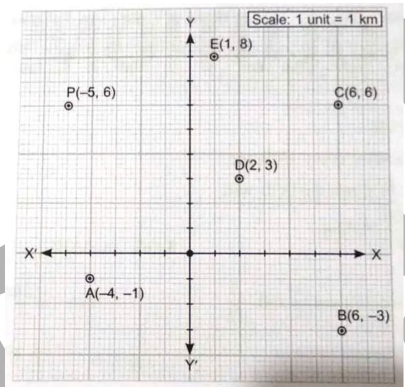
(ii) Find the cost of one ticket from Gurgaon to Chandigarh.

(iii) If Raj purchases 3 tickets to Chandigarh and 5 tickets to Shimla, how much amount he will pay ?

OR

If Anuj spends Rs. 5600 to buy tickets find how many total number of tickets he purchased ?

Q37. Five ships are positioned in the Indian Ocean. Their positions were plotted on a graph paper in reference to a rectangular coordinate axes.



An enemy ship is spotted at P(–5 6).

(i) What is the distance between P and E ?

(ii) Find the coordinate of mid-point of BD.

(iii) Ship D is moved to a position which is mid-point of AE. Find the distance moved by D.

OR

We find a rock at new position G such that B, G and C are in a straight line and BG : GC = 3 : 1 then fin dthe coordinates of G.

Q38. Group of friends playing with cards bearing numbers 5 to 50. All cards placed in a box and are mixed thoroughly one friend withdrawns the card from box at random and then replace it. Answer the questions based on above.

(i) What is the probability that the card withdrawn from the box bears a prime number less than 10 ?

(ii) What is the probability that the card withdrawn from the box bears a number which is a perfect square ?

(iii) What is the probability that the card withdrawn from the box bears a number which is multiple of 7 between 40 and 50 ?

OR

Find the probabilit of drawing a card bearing number from 5 and 50.

X - MATHEMATICS
SOLUTIONS : SAMPLE PAPER - 1

A-1. (c) 128

A-2. (b) $x^2 - 3x - m(m + 3) = 0$

$\Rightarrow x^2 + mx - (m + 3)x - m(m + 3) = 0$

$\Rightarrow x(x + m) - (m + 3)(x + m) = 0$

$\Rightarrow (x + m)[x - (m + 3)] = 0$

$\Rightarrow x + m = 0$ or $x - (m + 3) = 0$

$\Rightarrow x = -m$ or $x = m + 3$

A-3. (c) The given polynomial is

$f(x) = (x - 2)^2 + 4$

for zeroes,

$f(x) = 0$

$\Rightarrow (x - 2)^2 + 4 = 0$

$\Rightarrow (x - 2)^2 = -4$

Which is not possible.

Hence the polynomial has no zeroes.

A-4. (a) The given equations are $2x - 3y = 1$ and $3x - 2y = 4$

Here $\frac{a_1}{a_2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-2} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations has unique solution.

A-5. (c) Let coordinates of vertices be A(4, 0), B(–1, –1) and C(3, 5).

$AB = \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{36}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$AC = \sqrt{(3-4)^2 + (5-0)^2} = \sqrt{26}$

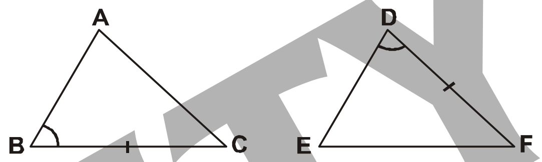
$\Rightarrow AB^2 + AC^2 = BC^2$

and AB = AC

Hence, triangle is an isosceles right-angled triangle.

A-6. (c) In ΔABC and ΔDEF

$$\frac{AB}{DE} = \frac{BC}{DF}$$



Also $\Delta ABC \sim \Delta EDF$

This is possible when $\angle D = \angle D$.

A-7. (c)

A-8. (d)

A-9. (d) $\because$ DE || BC

$\therefore \angle ABC = 70°$ (Corresponding $\angle$s)

Using angle sum property of triangle

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\Rightarrow \angle BCA = 180° - 70° - 50° = 60°$

A-10. (a) DE || BC, if $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{1.28}{2.56} = \frac{0.64}{EC}$

$\Rightarrow$ EC = 1.28 cm

A-11. (c)

A-12. (a) Let radius of the circle be r units

Circumference of the circle = $2\pi r$

Area of the circle = πr^2

A.T.Q

Circumference of the circle = Area of the circle

$\Rightarrow 2\pi r = \pi r^2$

$\Rightarrow r = 2$ units

A-13. (b) Let radius of the sphere be a cm

$\therefore$ Surface area of sphere = $4\pi a^2$

$\therefore\ 4 \times \pi a^2 = 616$

$\therefore\ 4 \times \frac{2}{7} \times a^2 = 616$

$\Rightarrow\ a^2 = \frac{616 \times 7}{22 \times 4} = 49$

$\Rightarrow\ a = 7$ cm

A-14. (c) $\because$ Mean = assumed mean + $\frac{\Sigma f_i d_i}{\Sigma f_i}$

$\therefore\ x =$ assumed mean.

A-15. (c) Circumference of smaller wheel

$= 30\pi$ cm

Circumference of bigger wheel

$= 50\pi$ cm

Now, $15 \times 50\pi$ = number of revolution $\times 30\pi$

$\Rightarrow$ number of revolutions = 25

A-16. (c) Sum of 100 observations

$= 100 \times 49 = 4900$

Correct sum $= 4900 - [40 + 20 + 50] + [60 + 70 + 80] = 5000$

$\therefore$ Correct mean $= \frac{5000}{100} = 50$

A-17. (c) Number of lottery ticket = 2000

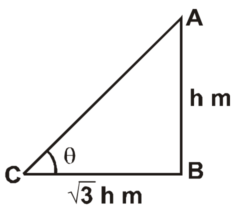
Total number of prizes = 16

$\therefore$ Probability that Abhinav wins a prize $= \frac{16}{2000} = \frac{1}{125} = 0.008$

A-18. (b) Here AB is tower of height h m.

Its shadow BC = $\sqrt{3}$ h m

Let θ be the angle of elevation



$\therefore$ In ΔABC,

$\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30°$

$\Rightarrow\ \theta = 30°$

A-19. (d) Assertion (A) is false but Reason (R) is true.

A-20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-21. All multiples of 7 lying between 100 and 1000 are 105, 112, 119,..., 994

These numbers from an A.P.

Here a = 105, d = 112 – 105 = 7

Let $a_n = 994$

$\Rightarrow\ a + (n - 1)d = 994$

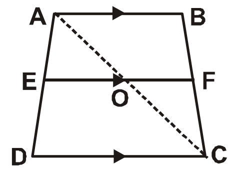
$\Rightarrow\ n = 128$

Now, $S_{128} = \frac{128}{2}(105 + 994)$

$= 70336$

A-22. **Given :** In trapezium ABCD, AB || DC || EF

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$



Construction : Join AC, where point O is intersection of AC and EF.

Proof : In ΔADC and ΔAEO, EO || DC

$\Rightarrow\ \frac{AE}{AD} = \frac{AO}{AC}$

$\Rightarrow\ \frac{AD}{AE} = \frac{AC}{AO}$

$\Rightarrow\ \frac{ED}{AE} = \frac{CO}{AO}$ ...(i)

In ΔCFO and ΔCBA, FO || BA

$$\Rightarrow \quad \frac{CF}{BC} = \frac{CO}{AC}$$

$$\Rightarrow \quad \frac{BC}{CF} = \frac{AC}{CO}$$

$$\Rightarrow \quad \frac{BF}{CF} = \frac{AO}{CO}$$

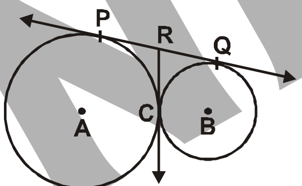
$$\Rightarrow \quad \frac{CF}{BF} = \frac{CO}{AO} \qquad ...(ii)$$

From (i) and (ii), we get

$$\frac{ED}{AE} = \frac{CF}{BF}$$

$$\Rightarrow \quad \frac{AE}{ED} = \frac{BF}{CF} \qquad \text{Hence proved.}$$

A-23. **Given :** PQ and RC are common tangents to the two circles.



To prove : RC bisects PQ or R bisects PQ.

Proof : PR and RC are tangents to a circle with centre A.

$\therefore$ PR = RC [$\because$ Length of tangents drawn from an external point R to a circle are equal] ...(i)

Similarly, RQ and RC are tangents to a circle with centre B.

$\therefore \quad RQ = RC \qquad ...(ii)$

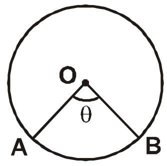
From (i) and (ii), we get

$$PR = RQ$$

$\therefore$ R bisects PQ. Hence proved.

A-24. Length of are AB = 7π cm,

Let $\angle AOB = \theta$



Now, length of an arc of a sector of angle

$$\theta = \frac{\theta}{360°} \times 2\pi r$$

$$\Rightarrow \quad 7\pi = \frac{\theta}{180°} \times \pi r$$

$$\Rightarrow \quad \frac{1260°}{r} = \theta$$

Now, area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$$\Rightarrow \quad 28\pi = \frac{\frac{1260°}{r}}{360°} \times \pi r^2$$

$$\Rightarrow \quad r = 8 \text{ cm}$$

Radius of the circle 8 cm.

OR

Given, diameter of the wheels of car = 80cm.

$\Rightarrow$ Radius = 40 cm

Circumference of the wheel

$$= 2\pi r = 2 \times \frac{22}{7} \times 40 \text{ cm}$$

Speed of the car = 66 km/h

Distance covere in 10 minutes

$$= \frac{66 \times 10}{60} = 11 \text{ km}$$

$$= 1100000 \text{ cm}$$

$\therefore$ Number of revolutions

$$= \frac{\text{Total distance in 10 minutes}}{\text{Circumference of the wheel}}$$

$$= \frac{1100000 \times 7}{2 \times 22 \times 40} = 4375$$

A-25. Given, $\operatorname{cosec}\alpha = \sqrt{2}$

$\Rightarrow \quad \sin\alpha = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \alpha = 45°$ and $\cot\beta = 1$

$\Rightarrow \quad \tan\beta = 1 \Rightarrow \beta = 45°$

$\therefore \quad \sin(\alpha + \beta) = \sin(45° + 45°)$

$= \sin 90° = 1$

OR

$\sin^6\theta - \cos^6\theta$

$= (\sin^3\theta)^2 - (\cos^3\theta)^2$

$= (\sin^3\theta - \cos^3\theta)(\sin^3\theta + \cos^3\theta)$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta) \times (\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$

$= (\sin^2\theta - \cos^2\theta)(1 + \sin\theta\cos\theta)(1 - \sin\theta\cos\theta)$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

A-26. Let us suppose that $3 - 2\sqrt{5}$ is irrational.

$\therefore \quad 3 - 2\sqrt{5}$ can be written in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

$\Rightarrow \quad 3 - 2\sqrt{5} = \dfrac{p}{q} \qquad \Rightarrow \quad 3 - \dfrac{p}{q} = 2\sqrt{5}$

$\Rightarrow \quad \dfrac{3q - p}{q} = 2\sqrt{5} \qquad \Rightarrow \quad \dfrac{3q - p}{2q} = \sqrt{5}$

Since p and q are integers, we get $\dfrac{3q - p}{2q}$ irrational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

$\therefore \quad \dfrac{3q - p}{2q} \neq \sqrt{5}$

So, our supposition is wrong.

Hence, $3 - 2\sqrt{5}$ is irrational.

A-27. $\dfrac{2x}{x-3} + \dfrac{1}{2x+3} + \dfrac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \quad \dfrac{2x(2x+3) + x - 3 + 3x + 9}{(x-3)(2x+3)} = 0$

$\Rightarrow \quad 4x^2 + 10 + 6 = 0$

$\Rightarrow \quad 2x^2 + 5x + 3 = 0$

$\Rightarrow \quad (x + 1)(2x + 3) = 0$

$\Rightarrow \quad x = -1$ or $x = \dfrac{-3}{2}$

When $x = \dfrac{-3}{2}$, given equation is not defined.

$\therefore \quad x = -1$

A-28. Let total number of pottery articles produced in a particular day be x.

Cost of production per article = Rs. $\dfrac{90}{x}$

ATQ $\qquad 2x + 3 = \dfrac{90}{x}$

$\Rightarrow \quad x(2x + 3) = 90$

$\Rightarrow \quad 2x^2 + 3x = 90$

$\Rightarrow \quad 2x^2 + 3x - 90 = 0$

$\Rightarrow \quad (2x+15)(x-6) = 0$

$\Rightarrow \quad 2x = -15$ or $x - 6 = 0$

$\Rightarrow \quad x = -\dfrac{15}{2}$ (rejected) or $x = 6$

$\therefore$ Number of articles produced in a particular day = 6

Cost of production per article

$$= \frac{90}{6} = \text{Rs. } 15$$

OR

Given $a_{11} = 38$ and $a_{16} = 73$

$\Rightarrow$ $a + 10d = 38$

and $a + 15d = 73$

$\Rightarrow$ $a + 15d - a - 10d = 73 - 38$

$\Rightarrow$ $5d = 35$

$\Rightarrow$ $d = 7$

$\therefore$ $a_{11} = a + 10 \times 7 = 38$

$\Rightarrow$ $a = 38 - 70 = -32$

$\therefore$ $a_{31} = a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210 = 178$

A-29. Given $\sec\theta = x + \frac{1}{x}$

squaring both sides, we get

$$\sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \sec^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right) \text{ or } -\left(x - \frac{1}{4x}\right)$$

Consider LHS = $\sec\theta - \tan\theta$

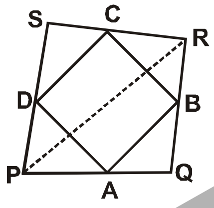
$$= x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$\text{or} \quad x + \frac{1}{4x} + \left(x - \frac{1}{4x}\right) = \frac{1}{2x} \text{ or } 2x$$

= RHS

$\therefore$ LHS = RHS

A-30. **Given :** In a quadrilateral PQRS, A, B, C and D are the mid-points of sides PQ, QR, RS and SP respectively.



To prove : ABCD is a parallelogram.

Construction : Join PR.

Proof : In ΔPQR, A and B are mid-points of sides PQ and QR respectively.

$\therefore$ AB || PR (Using mid-point theorem) ...(i)

In ΔPSR, D and C are mid-pionts of sides PS and SR respectively.

$\therefore$ DC || PR (Using mid-point theorem) ...(ii)

From (i) and (ii), we get

AB || DC

Similarly, we have AD || BC

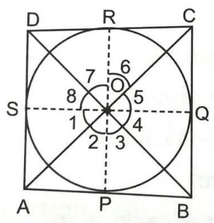
$\therefore$ In quadrilateral ABCD, AB || CD and AD || BC.

$\therefore$ ABCD is a parallelogram, because both pairs of opposite sides ofa quadri-lateral ABCD are parallel.

OR

AB touches at P and BC, CD and DA touch the circle at Q, R and S.

Construction : Join OA, OB, OC, OD and OP, OQ, OR, OS.



$\therefore$ $\angle 1 = \angle 2$ [OA bisects $\angle$POS]

Similarly, $\angle 4 = \angle 3$;

$\angle 5 = \angle 6;$

$\angle 8 = \angle 7$

$2[\angle 1 + \angle 4 + \angle 5 + \angle 8] = 360°$

$(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 180°$

$\angle AOD + \angle BOC = 180°$

Similarly $\angle AOB + \angle COD = 180°$

Hence, opposite sides of quadrilateral circumscribing a circle subtend supplementary angles at the centre of a circle.

A-31. Total number of objects = 12+8+10 = 30

Number of blue triangles = 6

Number of green triangles = 8 – 6 = 2

Number of green rectangles = 3

Number of blue rectangles = 12 – 3 = 9

Number of blue rhombus = 3

Number of green rhombuses = 10 – 3 = 7

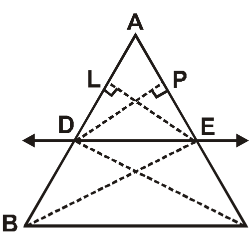
(i) Probability that one piece lost is a rectangle = $\frac{12}{30} = \frac{2}{5}$

(ii) Probability that one piece lost is a tiangle of green colour = $\frac{2}{30} = \frac{1}{15}$

(iii) Probability that one piece lost is a rhombus of blue colour = $\frac{3}{30} = \frac{1}{10}$

A-32. First part :

Given : A triangle ABC



DE || BC, meeting AB at D and AC at E

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE, CD and draw EL ⊥ AD.

Proof : ΔBDE and ΔCDE are on the same base and between the same parallel BC and DE, hence equal in area, i.e,

$ar(\Delta BDE) = ar(\Delta CDE)$ ...(i)

Now, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}.AD.EL}{\frac{1}{2}.BD.EL} = \frac{AD}{BD}$ ...(ii)

Similarly, $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2}.AE.DP}{\frac{1}{2}.EC.DP} = \frac{AE}{EC}$ ...(iii)

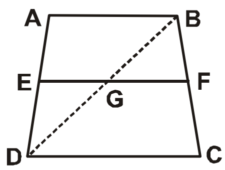
Also, $\frac{ar(\Delta ADE)}{\Delta(BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$ [Using (i)]

$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$ [From (ii) and (iii)]

Second Part :

Join intersecting EF at G.

In ΔDAB, EG || AB



$\therefore \quad \frac{AE}{DE} = \frac{BG}{GD}$ [Using B.P.T.) ...(i)

In ΔDBC, GF || DC

$\therefore \quad \frac{BG}{GD} = \frac{BF}{FC}$ ...(ii)

From (i) and (ii)

$\frac{AE}{DE} = \frac{BF}{FC}$

A-33. ATQ $(60 - t) = \frac{t^2}{4} - 3$

$\Rightarrow 240 - 4t = t^2 - 12$

$\Rightarrow$ $t^2 + 4t - 252 = 0$

$\Rightarrow$ $t^2 + 18t - 14t - 252 = 0$

$\Rightarrow$ $(t + 18)(t - 14) = 0$

$\Rightarrow$ $t = 14, -18$ [rejected]

$\Rightarrow$ $t = 14$ minutes.

OR

Let present age of Asha be x years

and present age of Nisha be ye years

ATQ $x = y^2 + 2$

Difference in ages = (x – y) years

Mother's age after (x – y) yers is

$x + (x - y) = 10y - 1$

$\Rightarrow$ $2x - y - 10y + 1 = 0$

$\Rightarrow$ $2(y^2 + 2) - 11y + 1 = 0$

$\Rightarrow$ $2y^2 + 4 - 11y + 1 = 0$

$\Rightarrow$ $2y^2 - 11y + 5 = 0$

$\Rightarrow$ $2y^2 - 10y - y + 5 = 0$

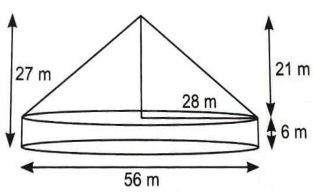
$\Rightarrow$ $(y - 5)(2y - 1) = 0$

$\Rightarrow$ $y = 5$ or $y = \frac{1}{2}$ (rejecting)

Neha's present age = 5 years

Asha's present age = $5^2 + 2 = 27$ years.

A-34.



Let l be the slant height of conical part of tent.

Radius of conical part (r) = 28 m

Height of conical part (h) = 21 m

Now, $l = \sqrt{(28)^2 + (21)^2}$

$= \sqrt{784 + 441}$

$= \sqrt{1225} = 35$ m

Curved surface area of conical part

$= \pi r l = \pi(28)35$

$m^2 = 980\pi\ m^2$

Radius of cylindrical part = 28 m

Height of cylindrical part = 6m

Curved surface area of cylindrical part

$= 2\pi rh = 2\pi(28)6$

$= 336\pi\ m^2$

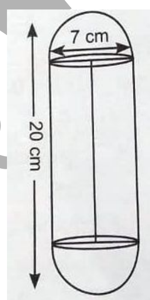
Total curved surface area = $980\pi + 336\pi$

$= 1316\pi\ m^2 = \frac{1316 \times 22}{7}$

$= 4136\ m^2$

$\therefore$ Area of canvas used = $4136\ m^2$

OR



Diameter of cylinder = diameter of the hemisphere

= 7 cm

$\therefore$ Radius of cylinder = $\frac{7}{2}$ cm

Total height of the solid = 20 cm

Height of the cylinder = $20 - \left(\frac{7}{2} + \frac{7}{2}\right)$

= 13 cm

Volume of the solid = Volume of the cylinder + 2 × vol. of one hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2\left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(13 + \frac{4}{3} \times \frac{7}{2}\right) cm^3$

$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\left(13+\frac{14}{3}\right)$

= 680.167 cm³.

A-35.

Marks	Frequency
0 – 10	2
10 – 20	6
20 – 30	12
30 – 40	16
40 – 50	13
50 – 60	20
60 – 70	5
70 – 80	1
80 – 90	4
90 – 100	1
Total	80

Here, frequency of the class 50 – 60 is maximum.

∴ Modal class is 50 – 60

Also, $l = 50$, $f_0 = 13$, $f_1 = 20$, $f_2 = 5$, $h = 10$

∴ Mode $= l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

$= 50+\left(\frac{20-13}{2\times 20-13-5}\right)\times 10$

$= 50+\frac{7}{22}\times 10$

= 50 + 3.18 = 53.18

So, the mode marks are 53.18.

A-36. (i) $\sqrt{40}$ km

(ii) (5, 1)

(iii) $\frac{\sqrt{50}}{2}$ km or $\left(6, \frac{15}{4}\right)$

A-37. (i) 2x + 3y = 3700, 3x + 4y = 5100

(ii) Rs. 500

(iii) Rs. 6000 **OR** 8

A-38. (i) Prime number from 5 to 10 are 5 and 7 only.

∴ number of favourable cases = 2

Total possible outcomes = 46

P(prime number less than 10)

$= \frac{2}{46} = \frac{1}{23}$

(ii) Perfect squares from 5 to 50 are 9, 16, 25, 36, 49

Number of favourable cases = 5

Total possible outcomes = 46

P(a perfect square number from 5 to 50) $= \frac{5}{46}$

(iii) Multiple of 7 between 40 and 50 are 42 and 49

Number of favourable outcomes = 2

Total possible outcomes = 46

P(multiple of 7 between 40 and 50)

$= \frac{2}{46} = \frac{1}{23}$

OR

P(from 5 and 50) $= \frac{2}{46} = \frac{1}{23}$

X - MATHEMATICS
SAMPLE PAPER - 2

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$, then HCF (a, b) =

(a) 90 (b) 180 (c) 360 (d) 540

Q2. $(x + 2)^3 = 2x(x^2 - 1)$ is

(a) linear equation (b) not quadratic equation
(c) quadratic equation (d) not defined

Q3. In an AP, 18, 13, 8, 3,... S_{35} =

(a) 2345 (b) 2435 (c) –2345 (d) – 2435

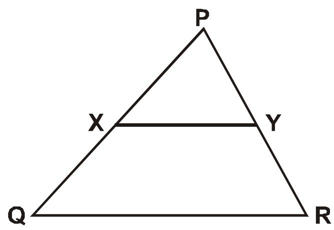
Q4. x = a and y = b is the solution of the linear equation x – y = 2 and x + y = 4, then values of a and b are

(a) 2, 1 (b) 3, 1 (c) 4, 6 (d) 1, 2

Q5. Three vertices of a parallelogram taken in order are (–1, –6), (2, –5) and (7, 2). The fourth vertex is

(a) (1, 4) (b) (1, 1) (c) (4, 4) (d) (4, 1)

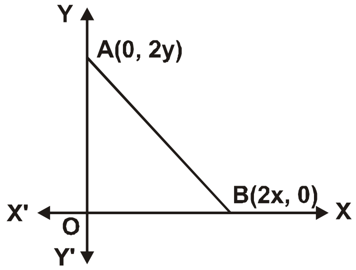
Q6. In figure XY || QR, $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



(a) $XY = \frac{1}{3}QR$ (b) XY = QR

(c) $XY^2 = QR^2$ (d) $XY = \frac{1}{2}QR$

Q7. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Q8. If $\sin\theta = \sqrt{3}\cos\theta,\ 0° < \theta < 90°,$ then θ is equal to

(a) 30° (b) 45° (c) 60° (d) 90°

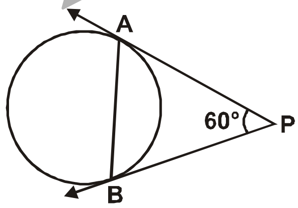
Q9. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(a) $\Delta PQR \sim \Delta CAB$ (b) $\Delta PQR \sim \Delta ABC$ (c) $\Delta CBA \sim \Delta PQR$ (d) $\Delta BCA \sim \Delta PQR$

Q10. The LCM of 2.5, 0.5 and 0.175 is

(a) 2.5 (b) 5 (c) 7.5 (d) 17.5

Q11. In figure PA and PB are tangents to a circle, PA = 9 cm and $\angle APB = 60°$, then chord AB =



(a) 4 cm (b) 7 cm (c) 6 cm (d) 9 cm

Q12. The arc of a circle of radius 30 cm having length 19 cm, then angle subtended by this acr at the centre O of the circle is

(a) 36.27° (b) 36° (c) 30.99° (d) 34°

Q13. If two solid hemispheres of the same base radius r are joined together along their bases, then curved surface area circle is

(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

Q14. While computing mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes
(b) centred at the class marks of the classes
(c) centred at the upper limits of the classes
(d) centred at the lower limits of the classes

Q15. In making 1000 revolutions, a wheel covers 88 km, then the diameter of the wheel is

(a) 7 m (b) 14 m (c) 36 m (d) 28 m

Q16. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the card will not be an ace is

(a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) $\frac{12}{13}$ (d) $\frac{3}{4}$

Q17. The probability of getting a red face card from a pack of cards is

(a) $\frac{3}{26}$ (b) $\frac{1}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{4}$

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** If in a ΔABC, a line DE || BC, intersects AB in D and AC in E, then $\frac{AB}{AD} = \frac{AC}{AE}$.

Statement R (Reason) : If a line a drawn parallel to one side of a triangle intersecting the other two sides, then the other two sides are divided in the same ratio.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** In a right-angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

Statement R (Reason) : (greatest side)^2 i.e. (hypotenuse)^2 = (perpendicular)^2 + (base)^2

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

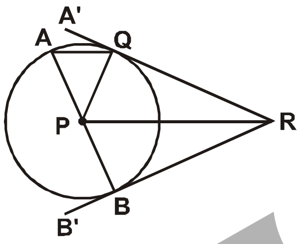
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. Aftab tells is daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Represent this situation algebraically.

Q22. In the given figure, QR is a tangent at Q. P is centre of the circle and PR || AQ, where AQ is a chord through A, an end point of the diameter AB. Prove that BR is tangent at B.

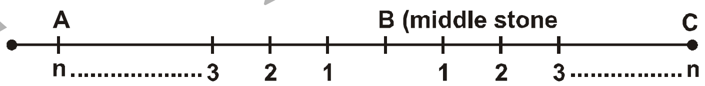


Q23. If a hexagon ABCDEF circumscribes a circle prove that AB + CD + EF = BC + DE + FA.

Q24. If 5th term of an AP is zero, show that 33rd term is two times is 9th term.

OR

Along a road lies an odd number of stones of weight 10 kg each, placed at intervals of 10 metres. These stones have to assembled around the middle stone. Nirvah, a stone loader can carry only one stone of 10 kg at a time. He started the job with one of the end stones by carrying them in succession. In carrying all the stones, he covered a distance of 3 km. Find the number of stones.



Q25. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, prove that $(m^2 + n^2)\cos^2\beta = n^2$.

OR

Solve the equation for θ : $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.

Q27. Solve the following system of equations graphically :

$x + 2y = 4, \; 4x + 3y = 10$

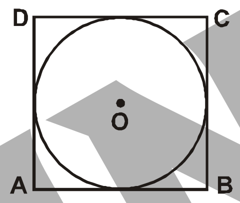
Q28. Solve for x and y : $\frac{15}{x+y}-\frac{2}{x-y}=1$ and $\frac{15}{x+y}+\frac{7}{x-y}=10$ $(x+y\neq 0,\ x-y\neq 0)$

OR

Determine by drawing graph, whether the following pair of linear equations has infinite number of solutions or not : y = 5 nd y + 3 = 0.

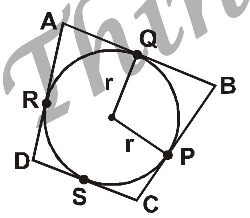
Q29. If $\cot\theta=\sqrt{7}$, show that $\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}=\frac{3}{4}$

Q30. A quadrilateral ABCD is a drawn to circumscribes a circle. Prove that : AB + CD = AD + BC.

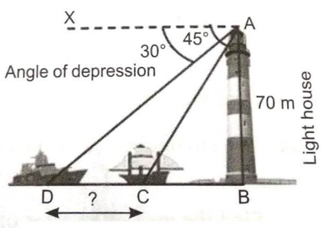


OR

In the given figure, a circle is inscribed in a quadrilateral ABCD in which ∠B = 90°. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle.



Q31. Light house is a tower with a bright light at the top. Light house serve as a navigational aid and to warn boats or ships about dangerous area.
Study the diagram and answer the question based on it.



If one ship is exactly behind the other on the same side of the light-house, find the distance between the two ships. (Use $\sqrt{3}=1.73$)

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. The sum of three numbers of an AL is 3 and the product of the first and the third number is –35. Find the three numbers.

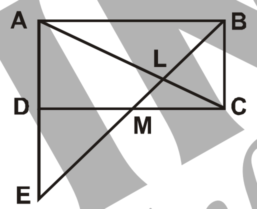
OR

Shalini gets pocket money from her father every day. Out of the pocket money, she saves Rs. 30 on the first day and on each succeeding day, she increases her saving by 500 paise. At the end of every month, Shalini purchases some biscuits packs, toffees and nuts from the amount that she saved and distribute these items to the needy children in her school.

(i) Find the amount saved by Shalini on 10th day.

(ii) Find the total amount saved by Shalini in 30 days.

Q33. In the given figure, M is mid-point of the side CD of a rectangle ABCD. BM when joined meets AC at L and AD produced at E. Prove that EL = 2BL.



Q34. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 24m long, 20 m high and 0.5 m thick while the cement and sand mixture occupies $\frac{1}{20}$th ofthe volume of the wall.

OR

Irrigation canals are used to move water from a source (whether it is a stream, reservoir or holding tank). A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10m in diameter and 2 m deep. If water flows through the pipe at the rate of 6 km/h, in how much time will the tank be filled ?

Q35. The marks obtained by 100 students in a mathematics test consisting of 100 marks are given in the following table :

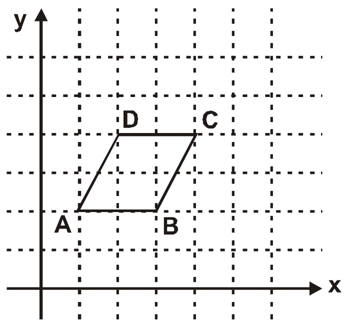
Marks obtained	0 – 14	14 – 28	28 – 42	42 – 56	56 – 70
No. of students	8	20	28	18	26

Find the mean marks obtained by the students.

SECTION - E

Case study based questions are compoulsory.

Q36. We can determine whether a quadrilateral placed on coordinate plane is a parallelogram or not. In coordinate geometry, distance formula and mid point formula are enough to show that quadrilateral placed on coordinate axes is a parallelogram or not. If vertices of triangle are given then using distance formula we can find length of sides of triangle, e.g., ABCD is a quadrilateral place on coordinates axes as shown.



(i) If A(2, 3), B(4, 6), C(7, 4) and D(a, b) are the vertices of a quadrilateral, such that diagonals AC and BD intersects each other at O. If O is mid point of AC and BD then find the value of a and b.

(ii) Three vertices of a parallelogram taken in order are (0, 3), (0, 0) and (5, 0), then find the fourth vertex.

(iii) If P(5, 2), Q(2, – 2) and R(2, y) are vertices of right-angled triangle where ∠Q = 90° then find y.

OR

Three vertices of rectangle AOBC are A(0, 3), O(0, 0), B(5, 0), then find the length of diagonals AB and OC.

Q37. India is competitive manufacturing location due to the low cost of manpower and strong technical and enginnering capabilities contributing to higher quality production runs. The production of TV set in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, answer the following questions :

(i) Find the production in first year.

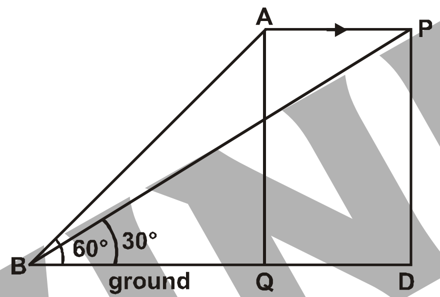
(ii) In which year, the production will be 29200 ?

(iii) Find the difference of the production during 7th year and 4th year.

OR

Find the difference between 12th year and first year.

Q38. Point A is the position of jet fighter flying in the sky. The angle of elevation of point A from ground is shown. After 15 seconds, the jet figher moves in direction AP and reaches at point P. The angle of elevation of point P on the ground is shown (Assume that figher is flying at the constant height above the ground).



Based on the above information, answer the following questions :

(i) What is the distance of AP, if jet is flying with speed 720 km/h in 15 seconds ?

(ii) If the jet is flying at the speed of 360 km/h then find the distance covered in 15 seconds.

(iii) If the jet in flying at a speed of 720 km/h then find the constant height at which it is flying.

OR

If the jet is flying at the speed of 360 km/h then find the constant height at which it is flying.

SOLUTIONS : SAMPLE PAPER - 2

A-1. (b) HCF (a, b) = $2^2 \times 3^2 \times 5 = 180$

A-2. (b) We have $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 8 + 3(x)(2)(x+2) = 2x^3 - 2x$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

It is not a quadratic equation.

A-3. (c) The given AP is 18, 13, 8, 3,...

Here a = 18, d = 13 – 18 = – 5

$$S_{35} = \frac{35}{2}[2\times18+34(-5)] = -2345$$

A-4. (b) We have x – y = 2 and x + y = 4

Also x = a and y = b

$\therefore$ Equaton become

a – b = 3 ...(i)

and a + b = 4 ...(ii_

On adding (j) and (ii), we get a = 3, b = 1.

A-5. (d) Let fourth vertex be (x, y)

Mid-points of diagonals

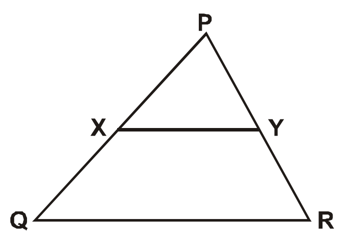
$$\Rightarrow \left(\frac{-1+7}{2}, \frac{-6+2}{2}\right) = \left(\frac{2+x}{2}, \frac{-5+y}{2}\right)$$

$\Rightarrow$ x = 4 y = 1

A-6. (a) In ΔPQR

XY || QR

$\Rightarrow \Delta PXY \sim \Delta PQR$ (AA similarity)



$$\therefore \frac{PX}{PQ} = \frac{PY}{PR} = \frac{XY}{QR}$$

[As $\Delta PXY \sim \Delta PQR$] ...(i)

Also $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$ (Given)

$$\Rightarrow \frac{PX}{PQ} = \frac{PY}{PR} = \frac{1}{3}$$

$$\Rightarrow \frac{XY}{QR} = \frac{1}{3} \Rightarrow XY = \frac{1}{3}QR$$

A-7. (a) $\because$ ΔAOB is a right triangle

$\therefore$ Mid-point of AB is equidistant from A, O and B.

Mid-piont of AB

$$= \left(\frac{0+2x}{2}, \frac{2y+0}{2}\right) = (x, y)$$

A-8. (c) $\sin\theta = \sqrt{3}\cos\theta = \frac{\sin\theta}{\cos\theta} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$

A-9. (a) $\because \frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

Then $\Delta CAB \sim \Delta PQR$

A-10. (d)

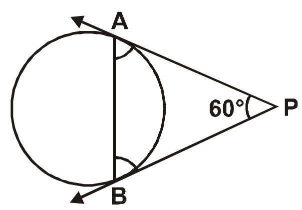
A-11. **(d)** PA and PB are tangent to a circle

Also PA = PB

(Equal tangents from common external point)

$\therefore$ $\angle PAB = \angle PBA$

(Angles opposite to equal side of triangle are equal)



In ΔAPB

$\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow$ $\angle PAB + \angle PAB + 60° = 180°$

$\Rightarrow$ $\angle PAB = 60°$

$\therefore$ ΔAPB is an equilateral triangle

$\Rightarrow$ PA = PB = AB

(All sides are equal)

$\Rightarrow$ AB = 9 cm

A-12. (a) Right of circle = 30 cm

Length of an arc of a circle = $\frac{\pi r\theta}{180°}$

θ is the angle subtended by arc at the centre of circle.

$\therefore$ $19 = \frac{22}{7} \times \frac{30 \times \theta}{180°}$

$\Rightarrow$ $36.27° = \theta$

A-13. (a) $4\pi r^2$

A-14. (b) centred at the class marks of the classes.

A-15. (d) Let radius of wheel be 4 m

$\therefore$ Distance of travelled during one revolution = $2\pi r$

Distance travelled during 1000 revolutons

$= 1000\ (2\pi r)$

$\Rightarrow$ $88 \times 1000 = 1000\ (2\pi r)$

$\Rightarrow$ $r = \frac{88 \times 7}{2 \times 22} = 14$

Diameter = 2r = 2 × 14 m

= 28 m

A-16. (c) Total number of cards = 52

Number of ace = 4

P(not be an ace) = $\frac{48}{52} = \frac{12}{13}$

A-17. (a) $\frac{3}{26}$

A-18. (b) In ΔABC, $\angle B = 90°$

$\therefore$ $\tan A = 1 - \tan 45°$

$\Rightarrow$ $A = 45°$

$\Rightarrow$ $2 \sin A \cos A = 2 \sin 45° \cos 45°$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

A-19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-21. Let the present age of A flab be x years and present age of his daughter be y years.

According to question,

$x - 7 = 7(y - 7)$

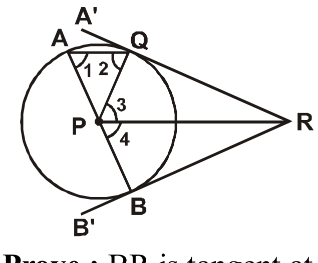
$\Rightarrow$ $x - 7y = -42$ ...(i)

and $x + 3 = 3(y + 3)$

$\Rightarrow$ $x - 3y = 6$ ...(ii)

Thus , the algebraic representation is given by (i) and (ii).

A-22. **Given :** QR is tangent at Q to a circle havin centre at P and chord AQ || PR.



To Prove : BR is tangent at B.

Proof : We have AQ || PR

$\therefore$ $\angle 1 = \angle 4$ (Correspoinding angles) ...(i)

and $\angle 2 = \angle 3$ (Alternative interior angles) ...(ii)

Also $\angle 1 = \angle 2$ ...(iii)

($\because$ PA = PQ,radii of the same circle)

From (i), (ii) and (iii), we get

$\angle 3 = \angle 4$ ...(iv)

In ΔPQR and ΔPBR,

$PR = PR$ (Common)

$PQ = PB$ (Radii of the same circle)

$\angle 3 = 4$ (From iv)

$\therefore \Delta PQR \cong \Delta PBR$ (SAS congruence rule)

$\Rightarrow \angle PBR = \angle PQR$ (CPCT)

Now, $\angle PQR = 90°$ [QR is tangent and PQ is radius]

$\therefore \angle PBR = 90°$

$\Rightarrow$ BR is tangent at B. Hence Proved.

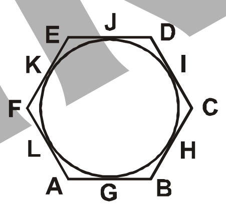
A-23. **Given :** ABCDEF hexagon circumscribe a circle and touches at G, H, I, J, K, L

To prove :

$AB + CD + EF = BC + DE + FA$

Proof : Hexagon ABCDEF touches a circle at G, H, I, J, K, L. So, from the external point, tangents drawn on the circle are equal in length.

If A is external point and AG and AL are tangents, so



$AG = AL$ ...(i)

Similarly for B,

$BG = BH$ ...(ii)

Similarly for C,

$CI = CH$ ...(iii)

Similarly for D,

$DI = DJ$ ...(iv)

Similarly for E,

$EK = EJ$ ...(v)

and similarly for F,

$FK = FL$ ...(vi)

Adding (i), (ii), (iii), (iv), (v), (vi), we get

$AG + BG + CI + DI + EK + FK$

$= AL + BH + CH + DJ + EJ + FL$

$\Rightarrow (AG + BG) + (CI + DI) + (EK + FK) = (BH + CH) + (JD + EJ) + (FL + AL)$

$\Rightarrow AB + CD + EF = BC + DE + FA$

Hence proved.

A-24. Let the term of AP be a and the common difference be d.

A.T.Q. $a_5 = 0$

$\Rightarrow a + 4d = 0 \Rightarrow a = -4d$ ...(i)

Now, $a_{33} = a + 32d$

$\Rightarrow a_{33} = -4d + 32d$ [using (i)]

$\Rightarrow a_{33} = 28d$ ...(ii)

Also $a_{19} = a + 18d$

$\Rightarrow a_{19} = -4d + 18d$ ...[using (i)]

$\Rightarrow a_{19} = 14d$

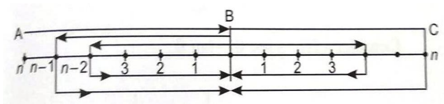
On multiplying with 2 on both sides, we get

$\Rightarrow 2 \times a_{19} = 2 \times 14d = 28d$ ...(iii)

From (ii) and (iii)

$a_{33} = 2 \times a_{19}$ Hence proved.

OR



Let there are $(2n + 1)$ stones. The middle stone is at B. The middle stone is at B. Let n stones are on one side of B and n stones on other side of B.

Let man started from A.

Distance covered from A to B

$= 10 \times nm = 10n$ metres

Distance covered to carry IInd stone

$= 2 \times (n - 1) \times 10$ metres

Distance covered to carry IIIrd stone

$= 2 \times (n - 2) \times 10$ metres

and so on.

$\therefore$ Total distance covered to carry n stones from this side of B.

$= 10n + 2 \times 2(n-1) \times 10 + 2(n-2) + 10 + ... + 2 \times 10$

$= 10[n + 2(n-1) + 2(n-2) + 2... + 2]$

$= 10\ \{n + 2[(n-1) + (n-2) + ... + 1]\}$

$$= 10\left\{n + 2\times\frac{n-1}{2}\times[(n-1)+1\right\}$$

$= 10[n + (n-1)n]$

$= 10[n + n^2 - n] = 10n^2$

Now, distance covered to collect n stones from other side of B will be 10n metres more than this distance as the person has to move from B to C to pick the stone at other end and come back.

$\therefore$ Distance covered to collect n stones from other side $= 10n^2 + 10n$

Total distance covered

$= 10n^2 + 10n^2 + 10n$

$= 20n^2 + 10n$

$\Rightarrow\ 20n^2 + 10n = 3000$

$\Rightarrow\ 2n^2 + 25n - 24n - 300 = 0$

$\Rightarrow\ n(2n + 25) - 12(2n + 25) = 0$

$\Rightarrow\ (n - 12)(2n + 25) = 0$

$\Rightarrow\ n - 12 = 0$ or $2n + 25 = 0$

$\Rightarrow\ n = 12$ or $n = -\frac{25}{2}$ (rejecting)

$\therefore$ Total number of stones

$= 2n + 1 = 2 \times 12 + 1 = 25$

A-25. We have $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$

Consider LHS $= (m^2 + n^2)\cos^2\beta$

$$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha(\sin^2\beta + \cos^2\beta)}{\sin^2\beta}$$

$$= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = n^2 = \text{RHS}$$

$\therefore$ LHS = RHS Hence Proved

OR

$$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$$

$$\Rightarrow \frac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 3$$

$$\Rightarrow \frac{\cos^2\theta\sin^2\theta}{\cos^2\theta(1 - \sin^2\theta)} = 3$$

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = 3 \Rightarrow \tan^2\theta = 3$$

$\Rightarrow \tan\theta = \sqrt{3} = 60° \Rightarrow \theta = 60°$

A-26. Let $\sqrt{p}$ be rational so that it can be written in the form of $\frac{a}{b}$

$\sqrt{p} = \frac{a}{b}$ (where a and b are coprime)

Squaring both sides, $p = \frac{a^2}{b^2}$

a^2 has a factor p. $pb^2 = a^2$ ...(i)

so, a also has a factor p.

so, $a = pc; a^2 = p^2c^2$

Put the value of a^2 in equation (i)

$$pb^2 = p^2c^2; b^2 = pc^2$$

b^2 has a factor p. $\therefore$ b has a factor p

But a and b are common factor p. But as stated earlier a, b are coprimes.

So, our supposition is wrong, $\sqrt{p}$ must be an irrational number. (where p is a prime number)

We can prove $\sqrt{q}$ is also an irrational number (where q is a prime number).

Sum of two irrational number is irrational if both are prime numbers.

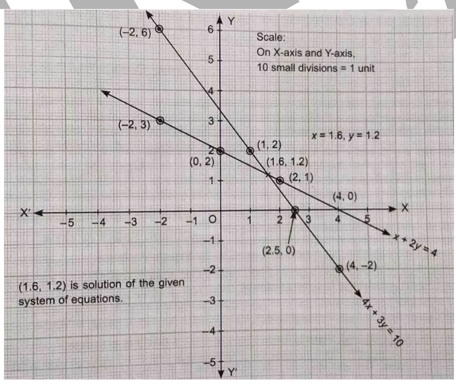
So, $\sqrt{p}+\sqrt{q}$ is irrational number.

A-27. The solution table for x + 2y = 4

x	0	4	2	–2
y	2	0	1	3

The solution table for 4x + 3y = 10 is

x	1	4	–2	2.5
y	2	–2	6	0



A-28. Let $\frac{1}{x+y} = A$ and $\frac{1}{x-y} = B$

$\therefore$ Given equation becomes

$15A - 2B = 1$ ...(i)

$15A + 7B = 10$ ...(ii)

Subtracting (i) from (ii), we get

$B = 1$

Putting B = 1 in (i), we get $A = \frac{1}{5}$

Now, $\frac{1}{x+y} = \frac{1}{5}$

$\Rightarrow x + y = 5$ ...(iv)

and $B = 1$

$\Rightarrow \frac{1}{x-y} = 1$

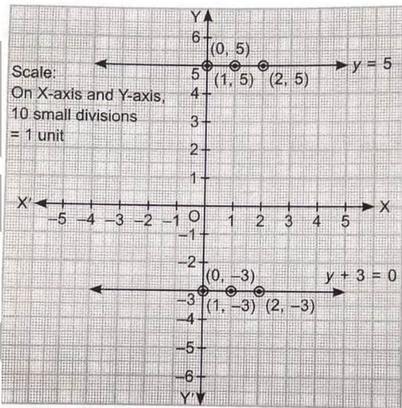
$\Rightarrow x - y = 1$ ...(v)

Adding (iv) and (v), we get

Putting x = 3 in (iv), we get y = 2

$\therefore$ x = 3, y = 2

OR



Given equation are y = 5 and y + 3 = 0

Table for y = 5 is

x	0	1	2
y	5	5	5

Table for y + 3 = 0 $\Rightarrow$ y = 3 is

x	0	1	2
y	–3	3	–3

$\because$ Graph represent two parallel lines.

$\therefore$ Given pair of linear equations has no common solution.

A-29. Given, $\cot\theta = \sqrt{7}$

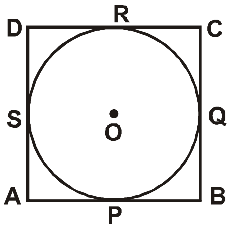
$$LHS = \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

Dividing the numerator and denominator by $\sec^2\theta$,

$$\frac{\cot^2\theta - 1}{\cot^2\theta + 1} = \frac{(\sqrt{7})^2 - 1}{(\sqrt{7})^2 + 1} = \frac{7-1}{7+1}$$

$$= \frac{6}{8} = \frac{3}{4} = RHS$$

A-30. Given : A quadrilateral ABCD circumscribes a circle with centre O.



To prove : AB + CD = AD + BC

Proof : Here, AP = AS ...(i)

(Lengths of tangents drawn from an external point to a circle are equal)

Similarly, BP = BQ ...(ii)

CR = CQ ...(iii)

and DR = DS ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

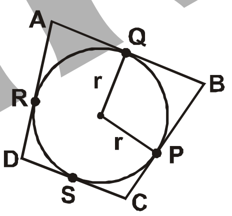
$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

Hence proved.

OR

Given : Quadrilateral ABCD circumscribed a circle.



$\angle B = 90°$, AD = 23 cm, AB = 29 cm, DS = 5 cm.

To find : Radius of circle

OQ $\perp$ AB and OP $\perp$ BC

(Radius is perpendicular to the tangent)

OQ = OP (Radii of a circle)

$\therefore$ OPBQ is a square.

$\Rightarrow$ BQ = BP = OP = r cm

Now, RD = DS $\Rightarrow$ RD = 5 cm

$\therefore$ AR = AD – RD

= 23 – 5 = 18cm

Also, AR = AQ $\Rightarrow$ AQ = 18 cm

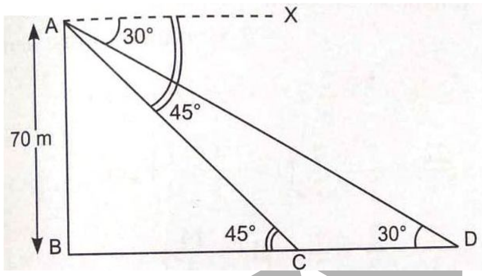
Now, AB = AQ + BQ

$\Rightarrow$ 29 = 18 + r

$\Rightarrow$ r = 11 cm

Hence, the radius (r) of the circle = 11cm.

A-31. Here AB be the lighthouse and C and D be the two ships.



$\therefore$ $\angle XAD = 30°$ and $\angle XAC = 45°$

$\because$ AX $\parallel$ BD

$\Rightarrow$ $\angle ADB = 30°$ and $\angle ACB = 45°$

(Alternate interior angles)

In right-angled ΔABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \frac{70}{BC} = 1$$

$\Rightarrow$ BC = 70 m

In right-angled ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\Rightarrow \frac{70}{BD} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ BD = $70\sqrt{3}$m

$\therefore$ Distance between the two ships,

CD = BD – BC

$= 70\sqrt{3} - 70$

$= 70(\sqrt{3} - 1)$

= 70(1.73 – 1) = 70 × 0.73

= 51.1 m

A-32. Let the three numbers of an AP are a – d, a and a + d

According to question,

a – d + a + d = 3

$\Rightarrow$ 3a = 3

$\Rightarrow$ a = 1 ...(i)

Also (a – d)(a + d) = – 35

$\Rightarrow \quad (1 - d)(1 + d) = -35$ [using (i)]

$\Rightarrow \quad 1 - d^2 = -35$

$\Rightarrow \quad d^2 = 36$

$\Rightarrow \quad d = 6$ or -6

when a = 1 and d = 6, the required three numbers are 1 – 6, 1, 1 + 6; i.e, –5, 1, 7

When a = 1 and d = – 6, the required numbers are {1 – (–6)}, 1, {1 + (–6)}; i.e, 7, 1, – 5.

OR

Money saved on 1st day = Rs. 30

Money saved on IInd day = Rs. 35

Money saved on IIIrd day = Rs. 40 and so on.

Amount of money saved on successive days is an AP with a = 30 and d = 5.

(i) Money saved on 10th day,

$$a_{10} = a + 9d = 30 + 9 \times 5$$
$$= \text{Rs. } 75$$

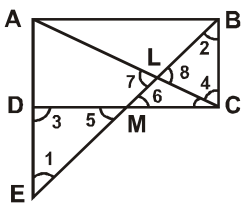
(ii) Money saved in 30 days

$$S_{30} = \frac{30}{2}[2 \times 30 + (30 - 1) \times 5]$$
$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d\right]$$
$$= 15(60 + 145)$$
$$= 15 \times 205 = \text{Rs. } 3075$$

A-33. **Given :** In a rectangle ABCD, M is mid-point of CD. BM intersects AC at L and meets AD on producing at E.

To Prove : EL = 2BL

Proof : In ΔEDM and ΔBCM.



$\angle 5 = \angle 6$ (vertically opposite angles)

$\angle 3 = \angle 4$ (Altrenate interiror angles)

DM = CM ($\because$ M is mid-point of CD)

$\therefore \quad \Delta EDM \cong \Delta BCM$ (ASA Congurnece rule)

$\Rightarrow \quad ED = BC$ (CPCT) ...(i)

Also, $\quad AD = BC$ ...(ii) (opposite sides of rectangles)

Adding (i) and (ii), we get

$ED + AD = 2BC$

$\Rightarrow \quad AE = 2BC$ ...(iii)

In ΔALE and ΔCLB,

$\angle 7 = \angle 8$ (Vertically opposite angels)

$\angle 1 = \angle 2$ (Alternate interior angels)

$\therefore \quad \Delta ALE \sim \Delta CLB$ (AA similarity)

$$\Rightarrow \quad \frac{AE}{BC} = \frac{EL}{BL}$$

$$\Rightarrow \quad \frac{2BC}{BC} = \frac{EL}{BL} \quad \text{[Using (iii)]}$$

$\Rightarrow \quad EL = 2BL$ Hence Proved

A-34. Dimensions of brick are l = 25 cm

$$= \frac{25}{100} \text{ m}$$

$$b = 12.5 \text{ m} = \frac{12.5}{100} \text{ m}$$

$$h = 7.5 \text{ cm} = \frac{7.5}{100} \text{ m}$$

Volume of one brick = $l \times b \times h$

$$= \frac{25}{100} \times \frac{12.5}{100} \times \frac{7.5}{100} \text{ m}^3$$

Volume of the wall = L × B × H

$$= 24 \times 20 \times 0.5 \text{ m}^3$$
$$= 240 \text{ m}^3$$

Volume of occupied by bricks

$$= (240 - 12) \text{ m}^3$$
$$= 228 \text{ m}^3$$

$\therefore$ Number of bricks required

$$= \frac{\text{Vol. occupied by bricks}}{\text{Vol. of one brick}}$$

$$= \frac{228}{\frac{25}{100} \times \frac{12.5}{100} \times \frac{75}{100}}$$

$$= \frac{228 \times 100 \times 100 \times 100}{25 \times 12.5 \times 7.5} = 9728$$

OR

Radius of cylindrical tank = r = 5m

Depth of cylindrical tank = h = 2m

$\therefore$ Volume of cylindrical tank = $\pi r^2 h$

$= \pi \times 5 \times 5 \times 2$

$= 50\pi \text{ m}^3$

Radius of the pipe = 10cm = 0.1m

Rate of flow of water = 6 km/h

= 6000 m/h

$\therefore$ Volume of water that flows in 1 hour

$= \pi \times 0.1 \times 0.1 \times 6000 \text{ m}^3$

$= 60\pi \text{ m}^3$

$\therefore$ Time required to fill the tank

$$= \frac{50\pi}{60\pi} = \frac{5}{6} \text{hours}$$

$$= \frac{5}{6} \times 60 = 50 \text{ minutes}$$

A-35.

Marks Obt.	No. of students (f_i)	Class mark (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0 – 14	8	7	–2	–16
14 – 28	20	21	–1	–20
28 – 42	28	35 = A	0	0
42 – 56	18	49	1	18
56 – 70	26	63	2	52
	$\Sigma f_i = 100$			$\Sigma f_i u_i = 34$

Using step-deviation method, we have

$$\overline{x} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

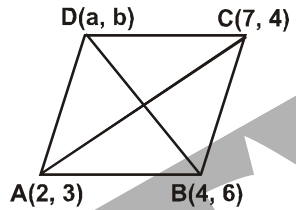
$$= 35 + \left(\frac{34}{100}\right) \times 14$$

$= 35 + 4.76$

$= 39.76$

So, the mean marks obtained is 39.76.

A-36. (i) Here O is mid piont of AC and BD



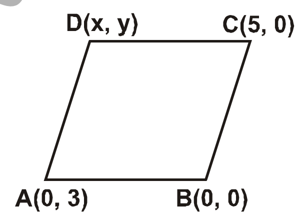
$$\therefore \quad \frac{2+7}{2} = \frac{a+4}{2}$$

$\Rightarrow \quad 9 = a + 4 \Rightarrow a = 5$

$$\text{and} \quad \frac{3+4}{2} = \frac{6+b}{2}$$

$\Rightarrow \quad 7 = 6 + b \Rightarrow b = 1$

(ii) As we know that diagonals of a parallelogram bisect each other.



$$\text{Now,} \quad \frac{0+x}{2} = \frac{0+5}{2}$$

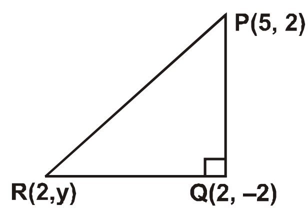
$\Rightarrow \quad x = 5$

$$\text{and} \quad \frac{0+y}{2} = \frac{3+0}{2}$$

$\Rightarrow \quad y = 3$

$\therefore$ Fourth vertex is (5, 3).

(iii) Using pythagoras theorem in right-angled ΔPQR, we get



$(PR)^2 = (PQ)^2 + (QR)^2$

$\Rightarrow \quad (5-2)^2 + (2-y)^2 = (5-2)^2 + (2+2)^2 + (2-2)^2 + (y+2)^2$

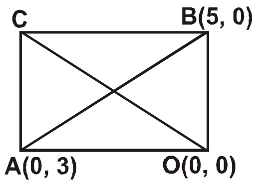
$\Rightarrow \quad 9 + 4 + y^2 - 4y = 9 + 16 + y^2 + 4y + 2$

$\Rightarrow \quad -4y = 16 + 4y$

$\Rightarrow \quad y = -2$

OR

As we know that diagonals of rectangle are equal.



$\therefore \quad AB = OC$

$\therefore \quad AB = \sqrt{(5-0)^2 + (0-3)^3}$

$= \sqrt{25+9} = \sqrt{34}$

Hence $\quad AB = OC = \sqrt{34}$ units

A-37. (i) $a_6 = 16000$

$a_9 = 22600$

$a + 5d = 16000 \quad$ ...(i)

$a + 8d = 22600 \quad$ ...(ii)

From (i) and (ii), we get

$d = 2200$

$\therefore \quad a + 5(2200) = 16000$

$a = 16000 - 11000 = 5000$

(ii) $a_n = 29200$

$\Rightarrow \quad 29200 = a + (n-1)d$

$\Rightarrow \quad 29200 = 5000 + (n-1)2200$

$\Rightarrow \quad \dfrac{24200}{2200} = n - 1$

$\Rightarrow \quad n = 12$

In 12th year the production of the company will be 29200.

(iii) $a_7 = a + 6d$

$a_4 = a + 3d$

$a_7 - a_4 = a + 6d - a - 3d$

$= 3d = 3 \times 2200 = 6600$

OR

$a_{12} - a = a + 11d - a = 11d$

$= 11 \times 2200$

$= 24200$

A-38. (i) Distance AP = Speed × time

$= \dfrac{720 \times 1000}{3600} \times 15$

$= 3000$ m

(ii) Distance AP = Speed × time

$= \dfrac{360 \times 1000}{3600} \times 15\text{m}$

$= 1500$ m

(iii) Let H be the constant height at which the jet is flying

In ΔABQ

$\tan 60^\circ = \dfrac{AQ}{BQ}$

$\Rightarrow \quad BQ = \dfrac{H}{\sqrt{3}}$

In ΔPBD

$\tan 30^\circ = \dfrac{PD}{BD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{H}{BQ + QD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{H}{\frac{H}{\sqrt{3}} + 3000} \quad$ (QD = AP)

$\Rightarrow \quad \left(\dfrac{H}{\sqrt{3}} + 3000\right)\dfrac{1}{\sqrt{3}} = H$

$\dfrac{H}{3} + \dfrac{3000}{\sqrt{3}} = H$

$H = \dfrac{3 \times 3000}{2\sqrt{3}} = 1500\sqrt{3}$ m

OR

In $\Delta ABQ = \tan 60° = \dfrac{AQ}{BQ}$

$\Rightarrow \quad BQ = \dfrac{AQ}{\sqrt{3}}$

Now in ΔPBD, $\tan 30° = \dfrac{PD}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AQ}{BQ+QD} \text{ (AQ = PD)}$$

$$AQ = \frac{BQ+QD}{\sqrt{3}}$$

$$= \frac{AQ}{\sqrt{3}\times\sqrt{3}} + \frac{QD}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{2AQ}{3} = \frac{QD}{\sqrt{3}}$$

$$\Rightarrow \quad AQ = \frac{3\times 1500}{2\times\sqrt{3}} = 750\sqrt{3}\text{m}$$

X - MATHEMATICS

SAMPLE PAPER - 3

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

Q2. If (1 – p) is a root of the equation $x^2 + px + 1 - p = 0$, then its roots are

(a) 0, 1 (b) –1, 1 (c) 0, –1 (d) –1, 2

Q3. If $p(x) = ax^2 + bx + c$ and a + c = b, then one of the zero is

(a) $\frac{b}{a}$ (b) $\frac{c}{a}$ (c) $\frac{-c}{a}$ (d) $\frac{-b}{a}$

Q4. The HCF of 2472, 1284 and a third number N is 12. If their LCM is $2^3 \times 3^2 \times 5 \times 103 \times 107$, then the number N is :

(a) $2^2 \times 3^2 \times 7$ (b) $2^2 \times 3^3 \times 103$ (c) $2^2 \times 3^2 \times 5$ (d) $2^4 \times 3^2 \times 11$

Q5. If three points (0, 0), $(0, \sqrt{3})$ and (3, k) form an equilateral triangle, then k =

(a) 2 (b) –3 (c) $-\sqrt{3}$ (d) $-\sqrt{2}$

Q6. Number of tangens to a circle which are parallel to a secant is

(a) 1 (b) 2 (c) 3 (d) Infinite

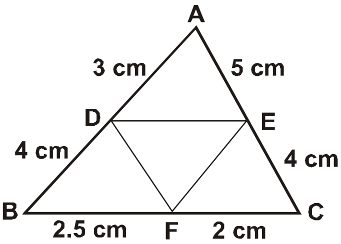
Q7. (cosec A – sin A) (sec A – cos A) – cos A (tan A + cot A) =

(a) 2 (b) –2 (c) 1 (d) –1

Q8. If $\sqrt{3}\sin\theta - \cos\theta = 0,\ 0 < \theta < 90°,$ then $\theta =$

(a) 30° (b) 45° (c) 90° (d) 60°

Q9. In given figure, AD = 3 cm, AE = 5 cm, BD = 4 cm, CE = 2 cm, BF = 2.5 cm, then



(a) DE || BC (b) DF || AC (c) EF || AB (d) none of these

Q10. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) BC.EF = AC.FD (b) AB.EF = AC.DE

(c) BC.DE = AB.EF (d) BC.DE = AB.FD

Q11. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is

(a) 5 units (b) 3 units (c) $\sqrt{34}$ units (d) 4 units

Q12. Three numbers are in an AP, having sum 24. Its middle term is

(a) 6 (b) 8 (c) 3 (d) 2

Q13. Which term of the AP : 22, 19, 16, ... is its first negative term ?

(a) 9 (b) 8 (c) 10 (d) 11

Q14. If $\Sigma f_1 = 11,\ \Sigma f_1 x_1 = 2p + 52$ and the mean of any distribution is 6, find the value of p.

(a) s4 (b) 5 (c) 6 (d) 7

Q15. A solid cube is cut into 27 small cubes of equal volume, then the ratio of the surface area of the given cube and that of one small cube is

(a) 9 : 1 (b) 1 : 9 (c) 1 : 1 (d) 3 : 3

Q16. If the mode of a data is 18 and the mean is 24, then median is

(a) 10 (b) 15 (c) 22 (d) 24

Q17. For an even E, P(E) + $P(\overline{E}) = q,$ then

(a) $a \le q < 1$ (b) $0 < q \le 1$ (c) $0 < q < 1$ (d) none of these

Q18. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b, c satisfy the relation

(a) $b^2 - a^2 = 2ac$ (b) $a^2 - b^2 = 2ac$ (c) $a^2 + b^2 = c^2$ (d) $a^2 + b^2 - 2ac$

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** Discriminant of the quadratic equation $3x^2 + 4x - 5 = 0$ is 76.

Statement R (Reason) : $D = b^2 + 4ac$

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Statement R (Reason) : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

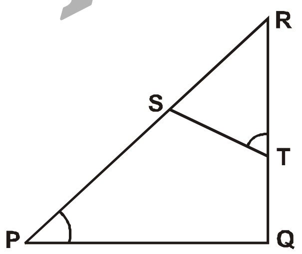
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. Which term of an Arthmetic Progression : 2, 7, 12, 17, ..., is 137 ?

Q22. In the given figure, $\angle P = \angle RTS$. Show that : $\Delta RPQ \sim \Delta RTS$ and $\frac{RQ}{RP} = \frac{RS}{RT}$.



Q23. Find the distance between two parallel tangents of a circle of radius 6 cm.

Q24. The circumference of the edge of a hemispherical bowl is 132 cm. Find the capacity of the bowl. (Use $\pi = 22/7$).

OR

Find the area of a sector of an angle A (in degree) of a circle with radius R.

Q25. If $\cos\theta = \frac{1}{2}$, find $\frac{\sec^2\theta + \tan^2\theta}{7 - 2\sec\theta \cdot \operatorname{cosec}\theta}$.

OR

Prove that $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $15+17\sqrt{3}$ be an irrational number.

Q27. Solve for x and y :

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4};\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8};\ 3x+y\neq 0,\ 3x-y\neq 0$$

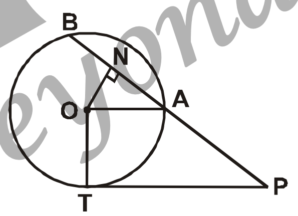
Q28. Find the zeros of the polynomial $4\sqrt{3}x^2+4\sqrt{3}x-3\sqrt{3}$. Also, verify the relationship between the zeroes and the coefficients.

OR

Solve for x : $\frac{4}{x}-3=\frac{5}{2x+3};\ x\neq 0, \frac{-3}{2}$

Q29. If sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos(A + 4B) = 0, A > B and $A+4B\leq 90°$, then find A and B.

Q30. In the given figure, PT is a tangent and PAB is a secant to a circle with centre O. ON is perpendicular to the chord AB. Prove that :

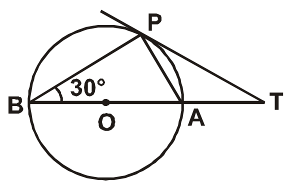


(i) PA.PB = $PN^2 - AN^2$

(ii) $PN^2 - AN^2 = OP^2 - OT^2$

OR

In the given figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA : AT = 2 : 1.



Q31. A boy standing on a horizontal plane find a kite flying at a distance of 150m from him at an angle of elevation of 30°. A girl standing on the roof of 30m high building finds the angle of elevation of the same kite to be 45°. Both boy and girl are on the opposite side of the kite. Find the distance of the kite from the girl.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

Q33. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarlity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Q34. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upight in a right circular cylinder full of water such that it touches the bottom. How many litres of water is left in the cylinder if the radius of the cylinder is 60 cm and its height is 180 cm.

OR

The cost of fencing a circular field at the rate of Rs. 24 per m is Rs. 5280. The field is to be ploughed at the rate of Rs.0.50 per m^2. Find the cost of ploughing the field.

Q35. The students of Class X of a school decided to donate their pocket money to purchase mineral water bottles for the people using contaminated water in a nearby village. They packed the mineral water bottles in different boxes. These boxes contained varying number of mineral water bottles. The following table shows the distribution of mineral water bottles according to the number of boxes :

No. of mineral water bottles	Number of boxes
50 – 52	20
53 – 55	120
56 – 58	105
59 – 61	125
62 – 64	30

Find the mean number of mineral water bottles kept in a packing box.

SECTION - E

Case study based questions are compoulsory.

Q36. Our country can be a manufacturing hub due to cheap labour cost and very high number of skilled technical man powers, which can contribute to cheaper and higher production. The manufacturing of mobile phone sets production unit increase by a fixed number every year. If manufactures 4,20,000 sets in the 5th year and 6,00,000 sets in 8th year.

(i) Find the production in the first year.

(ii) Find the common difference

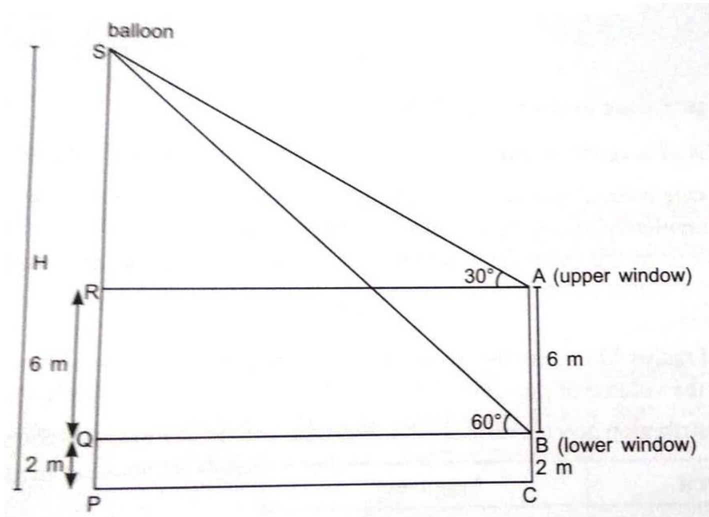
(iii) What will be the total production in the first 4 years ?

OR

What will be the total production in first 5 years ?

Q37. A building is made by keeping the lower window of a building at a particular height above the ground and upper window is constructed at some height vertically above the lower window. Position of both windows are shown in diagram.

Both windows are designed and constructed in order to have proper Sunlight.



At certain instant, the angle of elevation of balloon from these windows are shown. Balloon is flying at constant height H above the ground.

(i) Find the length AR (in terms of H)

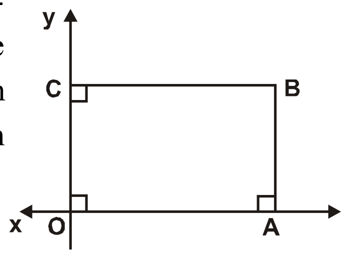
(ii) Find the height H.

(iii) Find the distance of balloon from the lower window.

OR

Find the distance of balloon from the upper window.

Q38. Rajiv decided to put a frame on a scenery which is quadrilateral in shape as shwon. He placed this scenery on coordinate axes such that one vertex coincides with origin O and one arm OA coincides with x-axis and another arm OC coincides with y-axis. Here OA = 5 units and OC = 3 units.



(i) Find the length of diagonal OB.

(ii) Find the value of $\angle ABC$.

(iii) Find the perimeter of OABC.

(iv) Find the coordinates of mid-point of OB and OC.

SOLUTIONS : SAMPLE PAPER - 3

A-1. (b) $a = x \times x \times x \times y \times y$

and $b = xy \times y \times y$

$\therefore$ HCF (a, b) $= x \times y \times y = x \times y^2 = xy^2$

A-2. (c) Let α and β be the roots of the equation

$$x^2 + px + 1 - p = 0$$

Let $\alpha = 1 - p$ (given)

$$\therefore \quad \alpha + \beta = \frac{-p}{1}$$

$$1 - p + \beta = -p$$

$\Rightarrow \beta = -1$

Putting $\beta = -1$ in $x^2 + px + 1 - p =$ we get

$\therefore \quad \alpha = 1 - 1 = 0$

Roots of equation are 0 and –1

A-3. (c) We have $p(x) = ax^2 + bx + c$

Putting $x = -1$, we get

$p(-1)\ a - b + c = (a + c) - b$

$= b - b = 0$ (As $a + c = b$)

$\therefore$ –1 is zero of p(x)

Let other zero be β.

$$\therefore \quad (-1)\beta = \frac{c}{a} \text{ (Product of rots)}$$

$$\beta = -\frac{c}{a}$$

A-4. (c) $2472 = 2^3 \times 3 \times 103$

$1284 = 2^2 \times 3 \times 107$

$\because$ LCM $= 2^3 \times 3^2 \times 5 \times 103 \times 107$

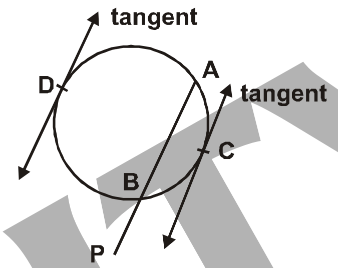
$\therefore$ N $= 2^2 \times 3^2 \times 5 = 180$

A-5. (c) $(3-0)^2 + (\sqrt{3}-0)^2 = (3-0)^2 + (k-0)^2$

$\Rightarrow \quad 3 - k^2 \Rightarrow k = \pm\sqrt{3}$

$\Rightarrow \quad k = -\sqrt{3}$

A-6. (b) Here PBA is a secand. We can draw only two tangents which are parallel to secant PBA.



A-7. (c) (cosecA – sinA)(secA – cosA)(tanA + cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \cdot \frac{1}{\sin A.\cos A}$$

$$= \frac{\cos^2 A \sin^2 A}{\sin^2 A \cos^2 A} = 1$$

A-8. (d) $\sqrt{3}\sin\theta = \cos\theta$

$\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60° \Rightarrow \theta = 60°$

A-9. (c) $\because \quad \frac{BF}{FC} = \frac{AE}{EC}$

$\therefore \quad EF \parallel AB$

A-10. (c) $\because \ \Delta ABC \sim \Delta EDF$

Then, $\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$

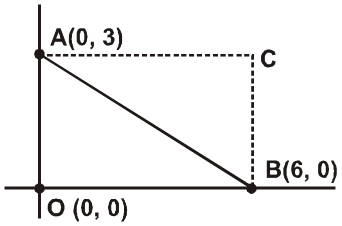
$\Rightarrow$ AB.DF = ED.BC

or AB.EF = AC.BC

or BC.EF = DF.AC

$\therefore$ BC.DE $\neq$ AB.EF

A-11. (c)



$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

A-12. (b) Let three terms in AP are

$a + d, a, a - d$

$\therefore$ $a + d + a + a - d = 24$

$\Rightarrow$ $a = 8$

$\therefore$ Middel term = a = 8

A-13. (a) Here a= 22, d = 19 – 22 = – 3

Let a_n be its first negative term

$\Rightarrow$ $a_n < 0$

$\Rightarrow$ $a + (n - 1)d < 0$

$\Rightarrow$ $22 + (n - 1)(-3) < 0$

$\Rightarrow$ $-3n < -25$

$\Rightarrow$ $n > \frac{25}{3}$

$\therefore$ 9th term is the first negative of term of the given AP.

A-14. (d)

A-15. (a) Let the sideof a solid cube be x units

Volume of a solid cube = x^3 cubic units.

This solid is cut into 27 small cubes of equal volume.

Volume of one small cube = $\frac{1}{27}x^3$ cubic units

$\Rightarrow$ Side of one small cube = $\frac{1}{3}x$ units

Now, surface area of a solid cube

$= 6 \times x^2$ sq units

Surface area of one small cube

$= 6 \times \frac{1}{9}x^2$ sq units

$$\therefore \frac{\text{Surface area of solid cube}}{\text{Surface area of one small cube}}$$

$$= \frac{6x^2}{6\times\frac{1}{9}x^2} = \frac{9}{1}$$

$\Rightarrow$ Required ratio = 9 : 1

A-16. (c)

A-17. (d) $\because$ $P(E) + P(\bar{E}) = 1$

$\therefore$ $q = 1$

A-18. (a) $\because$ $\sin\theta$ and $\cos\theta$ are the roots.

$$\sin\theta + \cos\theta = -\left(\frac{-b}{a}\right)$$

and $\sin\theta \cdot \cos\theta = \frac{c}{a}$

$$\Rightarrow (\sin\theta + \cos\theta)^2 = \left(\frac{b}{a}\right)^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = \frac{b^2}{a^2}$$

$$\Rightarrow 1 + 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2}$$

$$\Rightarrow a^2 + 2ac = b^2 \Rightarrow b^2 - a^2 = 2ac$$

A-19. (c)

A-20. (b) is correct option.

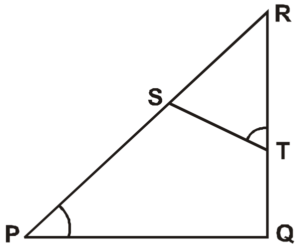
A-21. Here, a = 2, d = 7 – 2 = 5

Let $a_n = 137$

$\Rightarrow$ $a + (n + 1)d = 137 \Rightarrow n = 28$

Hence, 28th term is 137.

A-22. **Given :** $\angle P = \angle RTS$, where T and S are points on the side RQ and RP respectively.



To Prove : $\Delta RPQ \sim \Delta RTS$

and $\dfrac{RQ}{RP} = \dfrac{RS}{RT}$

Proof : In $\Delta RPQ \sim \Delta RTS$

$\angle P = \angle RTS$ (Given)

and $\angle R = \angle R$ (Common)

$\therefore$ $\Delta RPQ \sim \Delta RTS$ (AA similarity)

$\therefore$ $\dfrac{RQ}{RS} = \dfrac{RP}{RT}$

(Sides of similar triangles are in the same ratio)

$\Rightarrow$ $\dfrac{RQ}{RP} = \dfrac{RS}{RT}$ Hence Proved.

A-23. Distance between two parallel tangents of a circle is equal to the diameter of the circle.

$\therefore$ Distance between two parallel tangents of the given circle = 2 × radius

= 2 × 6 = 12 cm.

A-24. Let the radius of hemispherical bowl

= r cm

Circumference of edge of hemispherical bowl = 132 cm

$\Rightarrow$ $2\pi r = 132$

$\Rightarrow$ $2 \times \dfrac{22}{7} \times r = 132$

$\Rightarrow$ $r = \dfrac{132 \times 7}{2 \times 22} = 21$ cm

$\therefore$ Capacity of hemispherical bowl

$= \dfrac{2}{3}\pi r^3 = \dfrac{2}{3} \times \dfrac{22}{7} \times 21 \times 21 \times 21$

$= 19404 \text{ cm}^3$

OR

We have angle of sector = A

and radius of a sector = R

$\therefore$ Area of a sector $= \dfrac{\pi R^2 A}{360}$

A-25. We have $\cos\theta = \dfrac{1}{2}$

As we know that

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Now,

$$\frac{\sec^2\theta + \tan^2\theta}{7 - 2\sec\theta\,\mathrm{cosec}\,\theta} = \frac{\dfrac{1}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}}{7 - \dfrac{2}{\cos\theta}.\dfrac{1}{\sin\theta}}$$

$$= \frac{(2)^2 + (\sqrt{3})^2}{7 - 2 \times 2\dfrac{2}{\sqrt{3}}} = \frac{7\sqrt{3}}{7\sqrt{3} - 8}$$

OR

LHS = (cosec A – sin A)(sec A – cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A}.\frac{\sin^2 A}{\cos A} = \sin A \cos A$$

Taking RHS

$$\text{RHS} = \frac{1}{\tan A + \cot A} = \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$= \frac{\cos A \sin A}{\cos^2 A + \sin^2 A} = \cos A \sin A$$

LHS = RHS

A-26. Let $\sqrt{3} = \frac{a}{b}$, where a and b are coprime integers and $b \neq 0$.

Squaring both sides, we get $3 = \frac{a^2}{b^2}$.

Multiplying with b on both sides, we get

$$3b = \frac{a^2}{b}$$

LHS = $3 \times b$ = Integer

$$\text{RHS} = \frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}}$$

= Rational number

$\therefore$ LHS $\neq$ RHS

$\therefore$ Our supposition is wrong.

$\Rightarrow$ $\sqrt{3}$ is irrational

Let $15+17\sqrt{3}$ be a rational number.

$\therefore$ $15+17\sqrt{3} = \frac{a}{b}$,

where a and b are coprime integers and $b \neq 0$

$\Rightarrow$ $17\sqrt{3} = \frac{a}{b} - 15$

$$\sqrt{3} = \frac{a-15b}{17b}$$

$\frac{a-15b}{17b}$ is rational number.

But $\sqrt{3}$ irrational.

$\therefore$ $\sqrt{3} \neq \frac{a-15b}{17b}$

$\therefore$ Our supposition is wrong.

$\Rightarrow$ $15+17\sqrt{3}$ is an irrational number.

Hence Proved.

A-27. Let $\frac{1}{3x+y} = A$ and $\frac{1}{3x-y} = B$

$\therefore$ Given equations becomes

$$A + B = \frac{3}{4}$$

$\Rightarrow$ $4A + 4B = 3$ ...(i)

and $\frac{1}{2}A - \frac{1}{2}B = -\frac{1}{8}$

$\Rightarrow$ $A - B = -\frac{1}{4}$

$\Rightarrow$ $4A - 4B = -1$ ...(ii)

Adding (i) and (ii), we get

$$A = \frac{1}{4}$$

Putting $A = \frac{1}{4}$ in (i), we get

$$4\left(\frac{1}{4}\right) + 4B = 3$$

$\Rightarrow$ $4B = 2$

$\Rightarrow$ $B = \frac{1}{2}$

When $A = \frac{1}{4}$

$\Rightarrow$ $\frac{1}{3x+y} = \frac{1}{4}$

$\Rightarrow$ $3x + y = 4$ ...(iii)

When $B = \frac{1}{2}$

$\Rightarrow$ $\frac{1}{3x-y} = \frac{1}{2}$

$\Rightarrow$ $3x - y = 2$ ...(iv)

Adding (iii) and (iv), we get

x = 1 and y = 1

Hence x = 1, y = 1

A-28. Let $p(x) = 4\sqrt{3}x^2 + 4\sqrt{3}x - 3\sqrt{3}$

For zeroes of p(x), put p(x) = 0

$\Rightarrow$ $4\sqrt{3}x^2 + 4\sqrt{3}x - 3\sqrt{3} = 0$

$\Rightarrow$ $\sqrt{3}(4x^2 + 4x - 3) = 0$

$\Rightarrow$ $4x^2 + 4x - 3 = 0$

$\Rightarrow$ $[4x^2 + 6x - 2x - 3] = 0$

$\Rightarrow$ $[2x(2x + 3) - 1(2x + 3)] = 0$

$\Rightarrow$ $(2x + 3)(2x - 1) = 0$

$\Rightarrow$ $x = \frac{-3}{2}, \frac{1}{2}$

Thus, the zeroes of p(x) are $\frac{-3}{2}$ and $\frac{1}{2}$

Here, $a = 4\sqrt{3},\ b = 4\sqrt{3},\ c = -3\sqrt{3}$

Also, sum of zeroes = $\frac{-3}{2}+\frac{1}{2} = -1$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-3}{2}\times\frac{1}{2}=\frac{-3}{4}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence verified.

OR

$\frac{4}{x}-3=\frac{5}{2x+3};\ x \neq 0, \frac{-3}{2}$

$\Rightarrow \frac{4-3x}{x}=\frac{5}{2x+3}$

$\Rightarrow (4-3x)(2x+3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x + 2)(x - 1) = 0$

$\Rightarrow x + 2 = 0$

or $x - 1 = 0$

$\Rightarrow x = -2$

or $x = 1$

So, the solutions of the given equation are x = – 2 and 1.

A-29. $\sin(A + 2B) = \frac{\sqrt{3}}{2}$

So, $\sin(A + 2B) = \sin 60°$

Hence

$A + 2B = 60°$ ...(i)

Also, we have

$\cos(A + 4B) = 0$

$= \cos 90°$

$\Rightarrow A + 4B = 90°$ ...(ii)

Subtrancting (ii) from (i), we have

$B = 15°$

Put B = 15° in eq. (i), we have A = 30°.

A-30. Given : PT is a tangent and PAB is a secant to the circle with centre O. ON is perpendicular to the chord AB.

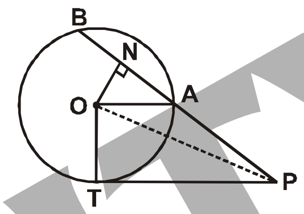
To prove :

(i) $PA.PB = PN^2 - AN^2$

(ii) $PN^2 - AN^2 = OP^2 - OT^2$

Construction : Join OP

Proof :



(i) Consider

$PA.PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$

($\because$ AN = BN, as perpendicular from of a circle to a chord bisects the chord)

$\Rightarrow PA.PB = PN^2 - AN^2$

(ii) In right-angled ΔONA,

$OA^2 = PN^2 + AN^2$

(using Pythagoras theorem) ...(i)

In right-angled ΔONA,

$OA^2 = ON^2 + AN^2$

(using Pythagoras theorem) ...(ii)

Subtracting (ii) from (i), we get

$OP^2 - OA^2 = PN^2 - AN^2$

$\Rightarrow OP^2 - OT^2 = PN^2 - AN^2$

($\because$ OA = OT, radii of the same circle)

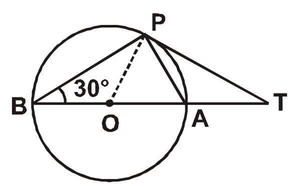
Hence Proved.

OR

Given : TP is tangent to the circle having centre O, ∠PBT = 30°

To Prove : BA : AT = 2 : 1

Proof :

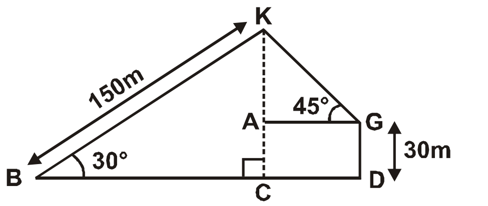


We have ∠BPA = 90° (Angle in a semicircle)

In ΔBPA,

$\angle ABP + \angle BPA + \angle PAB = 180°$
(Angle sum property of triangle)
$\Rightarrow 30° + 90° + \angle PAB = 180°$
$\Rightarrow \angle PAB = 60°$
Also $\angle POA = 2\angle PBA$
$\Rightarrow \angle POA = 2 \times 30° = 60°$
$= \angle PAB$
$\Rightarrow OP = AP$ ...(i)
(Sides opposite to equal angles are equal)
In ΔOPT, $\angle OPT = 90°$
$\Rightarrow \angle POT = 60°$ and $\angle PTO = 30°$
(Angle sum property of a triangle)
Also, $\angle APT + \angle ATP = \angle PAO$
(Exterior angle property)
$\therefore \angle APT + 30° = 60°$
$\Rightarrow \angle APT = 30° = \angle ATP$
$\therefore AP = AT$ ...(ii)
(Sides opposite to equal angles are equal)
From (i) and (ii), we get
$AT = OP$
= radius of the circle
and $AB = 2r$
$\Rightarrow AB = 2AT$
$\Rightarrow \frac{AB}{AT} = 2$
$\Rightarrow AB : AT = 2 : 1$ Hence Proved

A-31. Let boy be at B, girl be at G and kite be at K.
$\therefore \angle KBC = 30°$ and $\angle AGK = 45°$



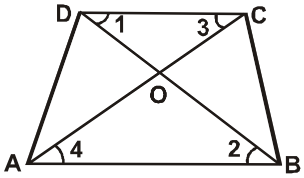
In right-angled ΔBCK,
$$\frac{KC}{KB} = \sin 30°$$
$$\Rightarrow \frac{KC}{KB} = \frac{1}{2}$$
$\Rightarrow KC = 75$ m
Now, $AC = GD = 30$ m
$\therefore AK = KC - AC$
$= 75 \text{ m} - 30 \text{ m} = 45$ m
In right-angled ΔKAG,
$$\frac{AK}{GK} = \sin 45°$$
$$\Rightarrow \frac{45}{GK} = \frac{1}{\sqrt{2}}$$
$\Rightarrow GK = 45\sqrt{2}$ m
$= 63.63$ m

A-32. Let the speed of the train be x km/h
Distance travelled = 360 km
$\therefore$ Time taken $= \frac{360}{x+5}$ hours
According to equation,
$$\frac{360}{x} = \frac{360}{x+5} + 1$$
$$\Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1$$
$$\Rightarrow \frac{360(x+5) - 360x}{x(x+5)} = 1$$
$\Rightarrow 1800 = x^2 + 5x$
$\Rightarrow x^2 + 5x - 1800 = 0$
$\Rightarrow x^2 + 45x - 40x - 1800 = 0$
$\Rightarrow (x + 45)(x - 40) = 0$
$\Rightarrow x = -45$ or $x = 40$
Rejecting $x = -45$
$\therefore$ Speed of the train = 40 km/h

OR

Let the two numbers be x and x – 5
According to question,
$$\frac{1}{x-5} - \frac{1}{x} = \frac{1}{10} \quad \left(\text{since } \frac{1}{x-5} > \frac{1}{x}\right)$$
$$\Rightarrow \frac{x - x + 5}{(x-5)x} = \frac{1}{10}$$
$\Rightarrow (x - 5)x = 50$
$\Rightarrow x^2 - 5x - 50 = 0$
$\Rightarrow (x - 10)(x + 5) = 0$
$\Rightarrow x = 10$ or $x = -5$
When $x = 10$, then $x - 5 = 10 - 5 = 5$
When $x = -5$, then $x - 5 = -5 - 5 = -10$
Thus, the required numbers are either 10 and 5 or –5 and – 10.

A-33. **Given :** Diagonals AC and BD intersect at O.



AB || DC

To prove : $\frac{OA}{OC} = \frac{OB}{OD}$

Proof : In ΔAOB and ΔCOD

$\angle 1 = \angle 2$

$\angle 3 = \angle 4$ [Alternate angle]

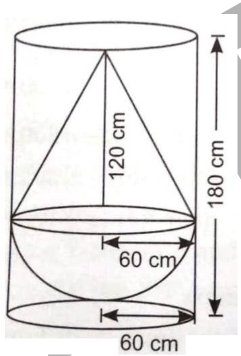
$\therefore$ $\Delta AOB \sim \angle COD$ [Alternate angle]

$\Rightarrow$ $\frac{OA}{OC} = \frac{OB}{OD}$

[Corresponding sides of similar triangles]

A-34. Radius of cone = 60 cm

Height of cone = 120 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (60)^2 \times 120$

$= 14400\pi$ cm^3.

Radius of hemipshere = 60 cm

$\therefore$ Volume of hemisphere

$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (60)^3$

$= 144000\pi$ cm^3

$\therefore$ Volume of solid

=Vol. of cone + Vol. of hemisphere

$= 144000\pi$ cm$^3 + 144000\pi$ cm^3

$= 288000\pi$ cm^3

Now, Vol. of cylinder = $\pi r^2 h$

$= \pi \times (60)^2 \times 180 = 648000\pi$ cm^3

Vol. of water left in the cylinder

= Vol. of cylinder – Vol. of solid

$= 648000\pi$ cm$^3 - 288000\pi$ cm^3

$= 360000\pi$ cm^3

$= 360000 \times \frac{22}{7}$ cm^3

= 1131428.57 cm^3

$= \frac{1131428.57}{1000} l = 1131.42\ l$

OR

Rs. 24, is the cost for fencing 1m of circular field.

Rs. 5280, is the cost for fencing

$= \frac{1}{24} \times 5280 = 220$ m of circular field

Circumference of the field = 220 m

$\Rightarrow$ $2\pi r = 220$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 220$

$\Rightarrow$ $r = \frac{220 \times 7}{44} = 35$m

$\therefore$ Area of the field = $\pi r^2 = \pi(35)^2$

$= 1225\pi$ cm^2

Cost of ploughing = Rs. 0.50 per m^2

Total cost of ploughing the field

= Rs. $1225\pi \times 0.50$

= Rs. $\frac{1225 \times 22 \times 1}{7 \times 2}$ = Rs. 175×11

= Rs. 1925

A-35. Let A = 57, h = 3

No. of mineral water bottles	No. of boxes (f_i)	Class marks (x_i)	$u_i = \frac{x_i = A}{h}$	$f_i u_i$
49.5 – 52.5	20	51	–2	–40
52.5 – 55.5	120	54	–1	–120
55.5 – 58.5	105	57 = A	0	0
58.5 – 61.5	125	60	1	125
61.5 – 64.5	30	63	2	60
Total	n = 400			$\Sigma f_i u_i = 25$

Here A = 57, h = 3, n = 400 and

$\Sigma f_i u_i = 25$

By step-deviation method

Mean, $\bar{x} = A + h \times \frac{1}{n} \times \Sigma f_i u_i$

$= 57 + 3 \times \frac{1}{400} \times 25$

$= 57 + \frac{75}{400}$

$= 57 + 0.1875$

$= 57.1875 \approx 57.19$ (app.)

A-36. (i) $a_5 = a + (n - 1)d$

$\Rightarrow$ $420000 = a + 4d$ ...(i)

$a_8 = a + (n - 1)d$

$\Rightarrow$ $600000 = a + 7d$ ...(ii)

Subtracting (i) from (ii), we get

$\Rightarrow$ $d = 60000$

Now putting the value of d in eqn. (i), we get

$\Rightarrow$ $a = 180000$

Production in first year = 180000

(ii) Common difference = 60000

(iii) $a_1 = 180000$

$a_2 = 180000 + 60000$

$= 240000$

$a_3 = 180000 + 120000$

$= 300000$

$a_4 = 180000 + 180000$

$= 360000$

Total production of first four years

$= a_1 + a_2 + a_3 + a_4$

$= 180000 + 240000 + 300000 + 360000$

$= 1080000$ sets

OR

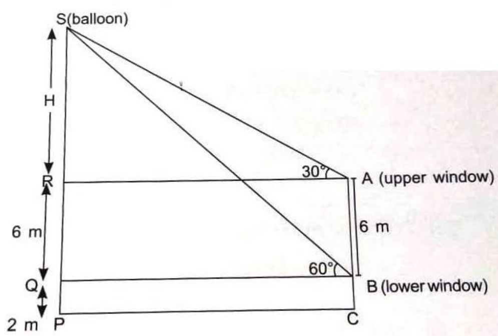
The production in first 5 years

$= \frac{5}{2}[2 \times 180000 + 4 \times 60000]$

= 15,00,000 sets

A-37. (i) Legnth AR = length BQ

$\Rightarrow$ length AR = $\left(\frac{H - 2}{\sqrt{3}}\right)$m ...(i)



Also in ΔARS,

$\tan 30° = \frac{SR}{AR}$

$$\frac{1}{\sqrt{3}} = \frac{H-8}{AR}$$

$$AR = \sqrt{3}(H-8)\text{m} \quad ...(ii)$$

(ii) equating (i) and ii), we get

$$\frac{H-2}{\sqrt{3}} = \sqrt{3}(H-8)$$

$\Rightarrow$ H = 11 m

(iii) In ΔSQB, $\sin 60° = \frac{SQ}{SB}$

$$\Rightarrow SB = (11-2)\times\frac{2}{\sqrt{3}}$$

$$= \frac{18\times\sqrt{3}}{3} = 6\sqrt{3}\text{m}$$

Hence distance of the balloon from lower window is $6\sqrt{3}$ m.

OR

In ΔSRA,

$$\sin 30° = \frac{SR}{AS}$$

$\Rightarrow$ AS = (11 – 8) × 2 = 6 m

Hence, distance of the balloon from upper window is 6 m.

A-38. (i) Coordinates of O are (0, 0) and co-ordinates of B are (5, 3).

Length of diagonal OB

$$= \sqrt{(5-0)^2 + (3-0)^2}$$

$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

(ii) In quadrilateral OABE

$\angle O + \angle A + \angle B + \angle C = 360°$

$\Rightarrow 90° + 90° + \angle B + 90° = 360°$

$\angle B = 360° - 270°$

$= 90°$

(iii) Perimeter of OABC

OA = 5 units

$$AB = \sqrt{(5-5)^2 + (3-0)^2}$$

$$= \sqrt{0 + (3)^2} = 3 \text{ units}$$

$= OC$

$$BC = \sqrt{(5-0)^2 + (3-3)^2}$$

$$= \sqrt{25} = 5$$

Perimeter OABC

= OA + AB + BC + OC

= 5 + 3 + 5 + 3 = 16 units

OR

Coordinates of O and B are (0, 0) and (5, 3).

Mid-point of OB = $\frac{5+0}{2} = \frac{5}{2}$ and $\frac{0+3}{2} = \frac{3}{2}$

Coordinates A and C are (5, 0) and (0, 3)

Mid-point of AC = $\frac{5+0}{2}$ and $\frac{0+3}{2}$

$$= \frac{5}{2} \text{ and } \frac{3}{2} = \left(\frac{5}{2}, \frac{3}{2}\right)$$

Coordinate of mid-point of AC and OB are $\left(\frac{5}{2}, \frac{3}{2}\right)$.

– Notes –

X - MATHEMATICS
SAMPLE PAPER - 4

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. The ratio between LCM and HCF of 5, 15, 20 is

(a) 9 : 1 (b) 4 : 3 (c) 11 : 1 (d) 12 : 1

Q2. If $(1 - p)$ is a root of the equation $x^2 + px + 1 - p = 0$, then roots are

(a) 0, 1 (b) –1, 1 (c) 0, –1 (d) –1, 2

Q3. The HCF of 2472, 1284 and a third number N is 12. If their LCM is $2^3 \times 3^2 \times 5 \times 103 \times 107$, then the number N is :

(a) $2^2 \times 3^2 \times 7$ (b) $2^2 \times 3^3 \times 10^3$ (c) $2^2 \times 3^2 \times 5$ (d) $2^4 \times 3^2 \times 11$

Q4. The roots of the equation $x + \frac{1}{x} = 5\frac{1}{5}$ are

(a) $5, \frac{1}{5}$ (b) 5, – 5 (c) –5, – 5 (d) 2, –2

Q5. The distance of the point (α, β) from y-axis is

(a) α units (b) $|\alpha|$ units (c) β units (d) $|\beta|$ units

Q6. The height of mountains is found out using the idea of indirect measurements which is based on the

(a) principle of congruent figures (b) principle of similarity of figures

(c) principle of equality of figures (d) none of these

Q7. If $\tan\theta + \cot\theta = 4$, $\tan^4\theta + \cot^4\theta =$

(a) 196 (b) 194 (c) 192 (d) 190

Q8. $\dfrac{1+\cot^2 A}{1+\tan^2 A} =$

(a) $\tan^2 A$ (b) $\cot^2 A$ (c) $\operatorname{cosec}^2 A - 1$ (d) $1 - \sin^2 A$

Q9. In ΔPQR and ΔMNS, $\dfrac{PQ}{NS} = \dfrac{QR}{MS} = \dfrac{PR}{MN}$, then symbolically we write it as

(a) $\Delta PQR \sim \Delta MNS$ (b) $\Delta PQR \sim \Delta SMP$ (c) $\Delta QRP \sim \Delta NSM$ (d) $\Delta QRP \sim \Delta SMN$

Q10. If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of the other triangle, then the two triangles are similar. This may be referred to as the

(a) AAA similarity criterion (b) ASSS similarity criterion

(c) SAS similarity criterion (d) RHS similarity criterion

Q11. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle POA$ is equal to

(a) 50° (b) 60° (c) 70° (d) 80°

Q12. The area of the square inscribed in circle of diameter p is

(a) $p^2\ cm^2$ (b) $\dfrac{p^2}{2} cm^2$ (c) $\dfrac{p}{2} cm^2$ (d) $\dfrac{p^2}{\sqrt{2}} cm^2$

Q13. Two cylindrical cans have equal base areas. If one of the can is 15 cm high and other is 20 cm high, find the ratio of their volumes.

(a) 2 : 3 (b) 3 : 4 (c) 4 : 3 (d) 3 : 2

Q14. The median from the table is

Value	7	8	9	10	11	12	13
Frequency	2	1	4	5	6	1	3

(a) 11 (b) 10 (c) 12 (d) 11.5

Q15. If the circumference of a circle is 352 metres, then its area in square metres is

(a) 5986 (b) 6589 (c) 7952 (d) 9856

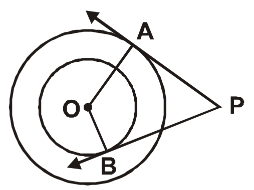
Q16. If the difference of mode and median of data is 26, then the difference of median and mean is

(a) 13 (b) 26 (c) 8 (d) 32

Q17. If two towers of heights h_1 and h_2 subtend angles of 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2 =$

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

Q18. In the figure, there are two concentric circles with centre O and radii 5 cm and 3 cm. PA and PB are tangents to these circles from an external point P. If PA = 12 cm, then length of PB (in cm) is



(a) 10 (b) $4\sqrt{10}$ (c) 12 (d) $\sqrt{178}$

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** If one zero of the polynomial $p(x) = (k^2 + 4)x^2 + 9x + 4k$ is the reciprocal of the other zero then $k = 2$.

Statement R (Reason) : If $(x – a)$ is a factor of the polynomial $p(x)$, then a is zero of $p(x)$.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** The point (0, 6) lies on y-axis.

Statement R (Reason) : The x co-ordinate on the point on y-axis.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

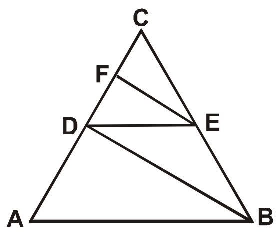
SECTION - B

Section B consists of 5 questions of 2 marks each.

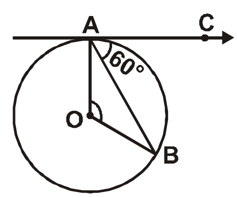
Q21. Find the number of solutions of the following pair of linear equations :

$3x – 3y = 5, \; 7x – 2y = 2$

Q22. In figure, AB || DE and BD || EF. Prove that $DC^2 = CF \times AC$.



Q23. In the given figure, O is the centre of circle. AB is a chord and the tangent AC at A makes an angle iof 60° with AB. Find $\angle AOB$.



Q24. A chord of circle of a radius 28 cm subtends a right angle at the centre. What is the area of the minor sector ?

OR

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Q25. For θ = 30°, verify that : $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.

OR

If $x = p\cos^3\theta$ and $y = q\sin^3\theta$, prove that $\left(\frac{x}{p}\right)^{2/3} + \left(\frac{y}{q}\right)^{2/3} = 1$.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Prove that $\sqrt{5}$ is an irrational.

Q27. The difference of an integer and its reciprocal is $\frac{143}{12}$. Find the integer.

OR

Find the positive value of k, for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots.

Q28. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β.

Q29. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0° < A + B \le 90°$; A > B, find A and B.

Q30. The lengths of tangents drawn from an external point (point outside the circle) to a circle are equal. Prove it.

OR

ABC is an isosceles triangle, in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

Q31. Radhika, a good student has ability to save her pocket money into her own piggy bank. Saving money is a skill that will be useful at all stages in person's life.

Radhika's piggy bank contains hundred 50p coins, fifty Rs. 1 coins, twenty Rs. 2 coins and ten Rs. 5 coins. One day she decided to take out money from her piggy bank. If it is equally likely that one of the coins will fall out when the piggy bank is turned upside down, find the probability that the coin (i) will be a 50p coin (ii) will not be a Rs. 5 coin (iii) will be Rs. 2 coin.

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

OR

If the zeroes of $x^2 - px + 6$ are in the ratio 2 : 3, find p.

Q33. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Q34. A cylindrical vessel with internal radius 5 cm and height of 10.5 cm is full of water. A solid cone of base radius 3.5 cm and height 6 cm is completely in water. Find the volume of

(i) water displaced out of the cylindrical vessel

(ii) water left in the cylindrical vessel.

OR

A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of the cone.

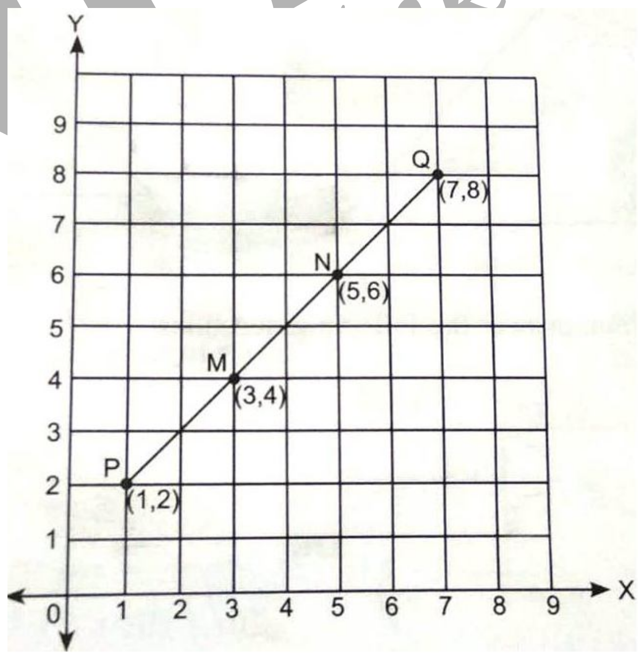
Q35. The median of the distribution given below is 14.4. Find the values of x and y, if the sum of frequency is 20.

Class interval	Frequency
0 – 6	4
6 – 12	x
12 – 18	5
18 – 24	y
24 – 30	1

SECTION - E

Case study based questions are compoulsory.

Q36. SOM, a firm organised an athletic meet. They made a rectangular grid on their ground.



Points P(1, 2) and Q(7, 8) were marked for disc throw competition. Disc were made to throw from point P towards point Q.

(i) Find the PM.

(ii) Find PN.

(iii) Find the ratio in which M divides PQ.

OR

Find PM : QN.

Q37. The farmers in the field make a heap of wheat in the field in the form of a cone. The base diameter of heap formed in the field is 24 m and height of heap formed is 3.5 m.

Answer the questions based on above.

(i) What will be the slant height of heap formed in the field ?

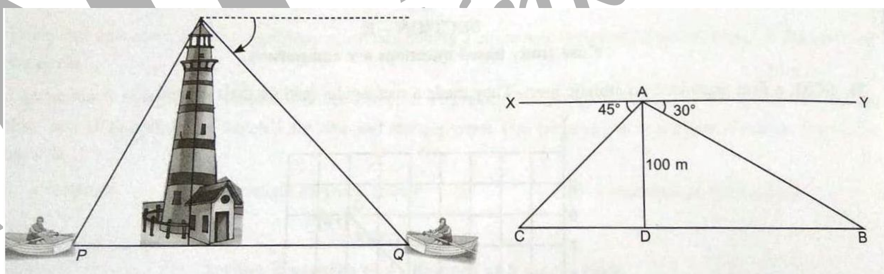
(ii) How much canvas cloth is required to just cover the heap ?

(iii) Find the volume of heap of wheat ?

OR

Farmer packed the wheat into bags. If volume of each bag of wheat is 0.48 m^3, then two many bags of wheat can be made ?

Q39. A boy is standing on the top of light house. He observed boat P and boat Q are approaching to light house from opposite directions. He finds that angle depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.



Based on the above information, answer the following questions :

(i) Find $\angle ACD$.

(ii) Find the length of CD.

(iii) Find the length of BD.

OR

Find the length of AC.

SOLUTIONS : SAMPLE PAPER - 4

A-1. (d) 5, 15 = 5 × 3, 20 = 2 × 2 × 5

LCM (5, 15, 20) = 5 × 3 × 2 × 2 = 60

HCF (5, 15, 20) = 5

Ratio = $\frac{LCM}{HCF} = \frac{60}{5} = \frac{12}{1}$ = 12 : 1

A-2. (c) (1 – p) is a root

$\therefore$ $(1-p)^2 + p(1-p) + 1 - p = 0$

$\Rightarrow$ $(1-p)[1-p+p+1] = 0$

$\Rightarrow$ $(1-p)(2) = 0$

$\Rightarrow$ p = 1

$x^2 + x = 0$

One root = 0 and another root = –1

$\therefore$ roots are 0 and –1

A-3. (c) $2472 = 2^3 \times 3 \times 103$

$1284 = 2^2 \times 3 \times 107$

$\because$ $LCM = 3^3 \times 3^2 \times 5 \times 103 \times 107$

$\therefore$ $N = 2^2 \times 3^2 \times 5 = 180$

A-4. (a) We have

$x + \frac{1}{x} = 5\frac{1}{5}$

$\frac{x^2+1}{x} = \frac{26}{5}$

$5x^2 + 5 = 26x$

$5x^2 - 26x + 5 = 0$

$5x^2 - 25x - +5 = 0$

5x(x – 5) – 1(x – 5) = 0

(5x – 1)(x – 5) = 0

$\Rightarrow$ $x = \frac{1}{5}$ and x = 5

A-5. (b) Distance = | α | units

A-6. (b)

A-7. (b) We have $\tan\theta + \cot\theta = 4$

Squaring both sides,

$\tan^2\theta + \cot^2\theta = 14$

Squaring both sbides

$\tan^4\theta + \cot^4\theta + 2 = 196$

$\tan^4\theta + \cot^4\theta = 194$

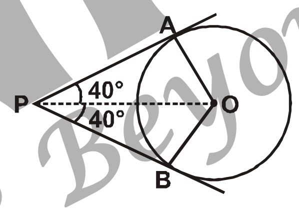
A-8. (b) $\frac{1+\cot^2 A}{1+\tan^2 A} = \frac{\operatorname{cosec}^2 A}{\sec^2 A}$

$= \frac{\cos^2 A}{\sin^2 A} = \cot^2 A$

A-9. (d) ΔQRP ~ ΔSMN

A-10. (d) RHS similarity criterion

A-11. (a) In ΔOAP and ΔOBP



OA = OB [Radii]

PA = PB [Length of tangent from an external point are equal]

OP = OP [Common]

$\therefore$ ΔOAP ≅ ΔOBP

[SSS congruence rule]

$\Rightarrow$ ∠POA = ∠POB = ½ ∠AOB

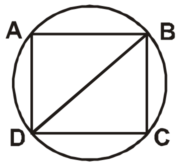
$\Rightarrow$ ∠AOB + ∠APB = 180°

$\Rightarrow$ ∠AOB + 80° = 180°

$\Rightarrow$ ∠AOB = 180° – 80° = 100°

$\Rightarrow$ $\angle POA = \frac{1}{2} \times 100° = 50°$

A-12. (b)



Suare is inscribed in a circle of diameter p cm are so dash angle of square = 90°

∠BAD = ∠BCD = 90°

There are angle in semicircle.

$\therefore$ diameter of circle is equal to diagonal of square.

In ΔBCD

$$DB^2 = DC^2 + BC^2$$

$$DB^2 = 2BC^2 \ (BC = DC)$$

$$BC^2 = \frac{DR^2}{2} = \frac{p^2}{2} cm^2$$

A-13. (b) Let the base area of first cylinder is πr^2.

$\therefore$ Base area of second cylinder is also πr^2

$$h_1 = 15cm,\ h_2 = 20cm$$

$$\text{Ratio of volumes} = \frac{\pi r^2 h_1}{\pi r^2 h_2} = \frac{15}{20}$$

$$= \frac{3}{4}$$

Volume of first cylinder : Volume of second cylinder = 3 : 4

A-14. (b)

Value	7	8	9	10	11	12	13
f	2	1	4	5	6	1	3
c.f.	2	3	7	12	18	19	22

$n = 22, \Rightarrow \frac{n}{2} = 11,$ so, median is 10.

Median = mean of 11th and 12th observations = 10

A-15. (d) $\because$ $2\pi r = 352 \Rightarrow r = \frac{176}{\pi}$

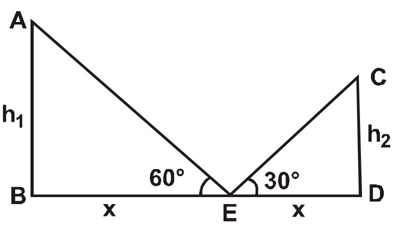
$\therefore$ Area = πr^2.

$$= \frac{\pi \times 176 \times 176}{\pi \times \pi} = \frac{176 \times 176 \times 7}{22}$$

$$= 9856 \text{ m}^2$$

A-16. (a)

A-17. (d)



$$\frac{h_1}{h} = \tan 60° = \sqrt{3}$$

$$\Rightarrow h_1 = \sqrt{3}x \quad ...(i)$$

$$\frac{h_2}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h_2 = \frac{1}{\sqrt{3}} x$$

$$\frac{h_1}{h_2} = \frac{\sqrt{3}x}{\frac{1}{\sqrt{3}}x} = \frac{3}{1}$$

$$\Rightarrow h_1 : h_2 = 3 : 1$$

A-19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-20. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-21. Given equations are

$$3x - 3y = 5;\ 7x - 2y = 2$$

Here, $a_1 = 3,\ b_1 = -3,\ c_1 = -5$

$$a_2 = 7,\ b_2 = -2,\ c_2 = -2$$

$$\frac{a_1}{a_2} = \frac{3}{7},\ \frac{b_1}{b_2} = \frac{-3}{-2}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore$ Given pair of linear equations has a unique solution.

A-22. **Given :** ΔABC in which DE || AB and BD || EF.

To Prove : $DC^2 = CF \times AC$

Proof : In ΔABC,

DE || AB

$$\Rightarrow \frac{CD}{AC} = \frac{CE}{BC} \quad ...(i)$$

(Baisc Proportionallity Theorem)

Again in ΔCDB, EF || BD

$$\Rightarrow \frac{CF}{CD} = \frac{CE}{CB} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{CD}{AC} = \frac{CF}{CD}$$

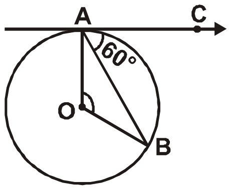
$\Rightarrow$ $CD^2 = CF \times AC$

Hence Proved.

A-23. AC is tangent and OA is radius

$OA \perp AC$

[$\because$ Tangent to a circle is perpendicular to radius through point of contact]



$\Rightarrow$ $\angle OAC = 90^\circ$

$\Rightarrow$ $\angle OAB + \angle BAC = 90^\circ$

$\Rightarrow$ $\angle OAB + 60^\circ = 90^\circ$

($\because$ Given $\angle BAC = 60^\circ$)

$\Rightarrow$ $\angle OAB = 30^\circ$

In ΔAOB,

$OA = OB$ (Radii of the same circle)

$\therefore$ $\angle OAB = \angle OBA$ (Angle opposite to equal sides of triangle are equal)

$\Rightarrow$ $\angle OBA = 30^\circ$

In ΔAOB,

$\Rightarrow$ $\angle AOB + \angle OBA + \angle OAB = 180^\circ$

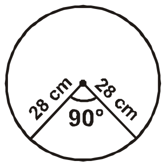
(Angle sum property of triangle)

$\Rightarrow$ $\angle AOB + 30^\circ + 30^\circ + 30^\circ = 180^\circ$

$\Rightarrow$ $\angle AOB = 180^\circ - 60^\circ = 120^\circ$

A-24. Area of the sector of angle θ

$$= \frac{\theta}{360} \times \pi r^2$$



$$= \frac{90}{360} \times \frac{22}{7} \times (28)^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$$

$= 616\ cm^2$

OR

Length of minute hand of clock = 14 cm

Angle swept in 5 minutes $= \frac{360^\circ}{60^\circ} \times 5$

$= 30^\circ$

Area swept in 5 minutes

$$= \frac{22}{7} \times \frac{14 \times 14 \times 30^\circ}{360^\circ}$$

$$= \frac{11 \times 2 \times 14}{6} = \frac{11 \times 14}{3}$$

$$= \frac{154}{3} cm^2$$

A-25. LHS $= \sin 2\theta = \sin(2 \times 30^\circ) = \sin 60^\circ$

$$= \frac{\sqrt{3}}{2} \quad ...(i)$$

$$RHS = \frac{2\tan\theta}{1+\tan\theta} = \frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} \quad ...(ii)$$

From (i) and (ii), LHS = RHS.

Hence Proved.

OR

We have $x = p\cos^3\theta$

and $y = q\sin^3\theta$

as $x = p\cos^3\theta$

$$\Rightarrow \quad \frac{x}{p} = \cos^3\theta$$

$$\Rightarrow \quad \cos\theta = \left(\frac{x}{p}\right)^{1/3}$$

Similarly

$$\sin\theta = \left(\frac{y}{q}\right)^{1/3}$$

as we know $\sin^2\theta + \cos^2\theta = 1$

So, $\left(\frac{x}{p}\right)^{2/3} + \left(\frac{y}{q}\right)^{2/3} = 1$

Hence proved.

A-26. Let $\sqrt{5}$ is a rational number and $\sqrt{5} = \frac{a}{b}$, where a and b are coprime and $b \neq 0$.

Now, $(\sqrt{5})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow \quad 5b^2 = a^2 \qquad$ ...(i)

$\Rightarrow$ 5 is a factor of a^2

$\therefore$ a is also divisible by 5.

Let a =5c, where c is some integer

Substituting a = 5c in (i), we get

$5b = (5c)^2 \Rightarrow 5b^2 = 25c^2$

$\Rightarrow \quad b^2 = 5c^2$

$\Rightarrow$ 5 is a factor of b^2

$\therefore$ 5 is a factor of b.

$\therefore$ 5 is a common factor of a and b

This contradicts the fact that a and b are coprime so, our assumption is wrong.

Hence, $\sqrt{5}$ is irrational.

A-27. Let the integer be x and its reciprocal be $\frac{1}{x}$.

According to question,

$$x - \frac{1}{x} = \frac{143}{12}$$

$\Rightarrow \quad \frac{x^2 - 1}{x} = \frac{143}{12}$

$\Rightarrow \quad 12x^2 - 12 = 143x$

$\Rightarrow \quad 12x^2 - 143x - 12 = 0$

$\Rightarrow \quad 12x^2 - 144x + x - 12 = 0$

$\Rightarrow \quad 12x(x - 12) + 1(x - 12) = 0$

$\Rightarrow \quad (x - 12)(12x + 1) = 0$

$\Rightarrow \quad x = 12$ or $x = -\frac{1}{12}$

Rejecting $x = -\frac{1}{12}$, because x is an integer.

$\therefore \quad x = 12$

$\therefore$ The required integer is 12.

OR

If the equation $x^2 + kx + 64 = 0$ has real roots, then $D \geq 0$.

$\Rightarrow \quad k^2 - 4 \times 1 \times 64 \geq 0$

$\Rightarrow \quad k^2 \geq 256$

$\Rightarrow \quad k^2 \geq (16)^2$

$\Rightarrow \quad k \leq 16 \qquad [\because k > 0] \qquad$ ...(i)

If the equation $x^2 - 8k + k = 0$ has real roots, then $D \geq 0$

$\Rightarrow \quad 64 - 4k \geq 0 \Rightarrow 4k \leq 64$

$\Rightarrow \quad k \leq 16 \qquad$ ...(ii)

From (i) and (ii), we get

$k = 16$

A-28. Let $\quad p(x) = 4x^2 + 4x + 1$

$\because \quad \alpha, \beta$ are zeroes of p(x)

$\therefore \quad \alpha + \beta = \text{sum of zeroes} = \frac{-b}{a}$

$\Rightarrow \quad \alpha + \beta = \frac{-4}{4} = -1 \qquad$ ...(i)

Also $\quad \alpha \cdot \beta = \text{Product of Zeroes} = \frac{c}{a}$

$\Rightarrow \quad \alpha \cdot \beta = \frac{1}{4} \qquad$ ...(ii)

Now a quadratic polynomial whose zeroes are 2α and 2β

x^2–(Sum of zeroes)x + Product of zeroes

$= x^2 + (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 + 2(\alpha + \beta)x + 4\alpha\beta$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$

[Using eqn. (i) and (ii)]

$= x^2 + 2x + 1$

A-29. $\tan(A + B) = \sqrt{3}$

$\Rightarrow$ $\tan(A + B) = \tan 60°$

$\Rightarrow$ $A + B = 60°$ ...(i)

$\tan(A + B) = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan(A - B) = \tan 30°$

$\Rightarrow$ $A - B = 30°$ ...(ii)

Adding (i) and (ii), we get

$2A = 90°$

$\Rightarrow$ $A = \frac{90°}{2} = 45°$

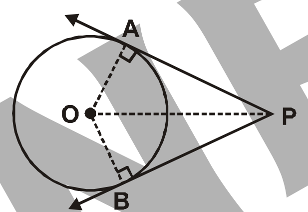
From (i),

$45° + B = 60°$

$\Rightarrow$ $B = 60° - 45° = 15°$

Hence, $\angle A = 45°, \angle B = 15°$

A-30. Given : A circle C(O, r) is a point outside the circle and PA and PB are tangents to a circle.



To Prove : PA = PB

Construction : Join OA, OB and OP

Proof : In ΔOAP and ΔOBP,

$\angle OAP = \angle OBP = 90°$

(Radius is perpendicular to the tangent at the point of contact)

OA = OB (Radii of the same circle)

OP = OP (Common)

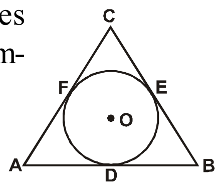
$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS congruence rule)

$\Rightarrow$ PA = PB [CPCT]

Hence proved.

OR

Given : In an isosceles ΔANC, AB = AC, circumscribed a circle.



To prove :

BD = DC

Proof : Here,

AB = AC (Given) ...(i)

AF = AE

(Tangent from an external point A to a circle are equal) ...(ii)

Subtracting (ii) from (i), we get

AB – AF = AC – AE

$\Rightarrow$ BF = CE ...(iii)

Now, BF = BD

(Tangents from an external point B to a circle are equal)

Also, CE = CD

(Tangents from an external point C to a circle are equal)

$\Rightarrow$ BD = CD

$\therefore$ BC is bisected at the point of contact. Hence Proved.

A-31. Total number of coins

= 100 + 50 + 20 + 10 = 180

(i) Number of 50p coins = 100

$\therefore$ Probability of getting a 50p coins

$= \frac{100}{180} = \frac{5}{9}$

(ii) Number of Rs. 5 coins = 10

Number of coins other than a Rs. 5 coin = 180 – 10 = 170

$\therefore$ Probability of not getting a Rs. 5 coin $= \frac{170}{180} = \frac{17}{18}$

(iii) Number of Rs. 2 coin = 20

$\therefore$ Probability of getting Rs. 2 coin

$= \frac{20}{180} = \frac{1}{9}$

A-32. Here $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of p(y), p(y) = 0

$\Rightarrow$ $7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow$ $21y^2 - 11y - 2 = 0$

$\Rightarrow$ $21y^2 - 14y + 3y - 2 = 0$

$\Rightarrow$ $7y(3y - 2) + 1(3y - 2) = 0$

$\Rightarrow \quad (7y+1)(3y-2) = 0$

$\Rightarrow \quad y = \dfrac{-1}{7}, \dfrac{2}{3}$

$\therefore$ zeroes are $\dfrac{-1}{7}$ and $\dfrac{2}{3}$

Also $a = 7$, $b = \dfrac{-11}{3}$, $c = \dfrac{-2}{3}$

Sum of zeroes $= \dfrac{-1}{7} + \dfrac{2}{3} = \dfrac{-3+14}{21} = \dfrac{11}{21}$

Also, $\dfrac{-b}{a} = \dfrac{-(-11/3)}{7} = \dfrac{11}{21}$

$\Rightarrow$ Sum of zeroes $= \dfrac{-b}{a}$

and product of zeroes $= \dfrac{-1}{7} \times \dfrac{2}{3} = \dfrac{-2}{21}$

Also $\dfrac{c}{a} = \dfrac{\frac{-2}{3}}{7} = \dfrac{-2}{21}$

$\Rightarrow$ Produt of zeroes $= \dfrac{c}{a}$

OR

$p(x) = x^2 - px + 6$

Let zeroes aer 2m and 3m

Sub of zeroes $= -\dfrac{b}{a}$

$\Rightarrow \quad 2m + 3m = \dfrac{-(-p)}{1}$

$\Rightarrow \quad 5m = p \qquad ...(i)$

Product of zeroes $= \dfrac{c}{a}$

$\Rightarrow \quad 2m \times 3m = \dfrac{6}{1}$

$\Rightarrow \quad 6m^2 = 6$

$\Rightarrow \quad m^2 = 1$

$\Rightarrow \quad m = \pm 1$

When m = 1, eq. (i) becomes

$5 \times 1 = p$

$\Rightarrow \quad p = 5$

when m = –1, eq. (i) becomes

$5 \times -1 = p$

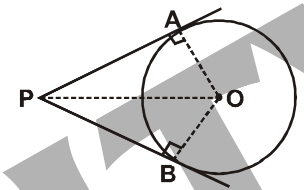
$\Rightarrow \quad p = -5$

$\therefore \quad p = \pm 5$

A-33. **Given :** In circle, O is the centre. P is an external point and PA and PB are the tangents drawn.

To prove : PA = PB

Construction : Join OA, OB and OP



Proof : Since PA and PB are the tangents and OA and OB are the radii of a circle.

$\therefore \quad OA \perp PA$ and $OB \perp PB$

[Tangent to a circle makes angle 90° with the radius at the point of contact]

$\Rightarrow \quad \angle OAP = \angle OBP = 90°$

Now, in ΔOAP and ΔOBP,

$OA = OB$ (Radii)

$OP = OP$ (Common)

$\angle OAP = \angle OBP$ (Each 90°)

$\therefore \quad \Delta OAP \cong \Delta OBP$

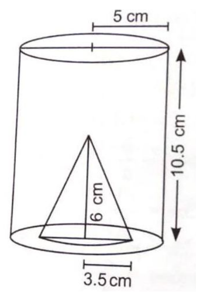
[By RHS congruence rule]

$PA = PB$ [By CPCT]

Hence Proved.

A-34. Height of cylinder = h = 10.5 cm

Radius of cylinder = r = 5 cm



$\therefore$ Volume of cylinder $= \pi r^2 h$

$= \dfrac{22}{7} \times 5 \times 5 \times \dfrac{105}{10} \text{ cm}^3$

$= 825 \text{ cm}^3$

Now, radius of base of cone = R = 3.5cm

Height of cone = H = 6cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi R^2 H$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times 6$

$= 77 \text{ cm}^3$

(i) Volume of water displaced out of the cylinderical vessel = Volume of cone $= 77 \text{ cm}^3$.

(ii) Volume of water left in cylindrical vessle

= Vol. of cylinder – Vol. of cone

$= 825 - 77 = 748 \text{ cm}^3$

OR

Length of the arc $= \frac{\theta\pi \times r}{180°}$

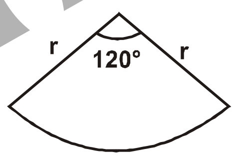
$= \frac{120°}{180°} \times \pi \times 12$

= Circumference of the base of the cone

Let the radius of cone be R cm

$\Rightarrow \quad 2 \times \pi \times R = \frac{120°}{180°} \times \pi \times 12$

$\Rightarrow \quad R = \frac{2}{3} \times \frac{12}{2} = 4 \text{ cm}$



Now, $R = 4 \text{ cm}, l = 12 \text{ cm}$

$\Rightarrow \quad h^2 = l^2 - R^2 = 12^2 - 4^2$

$= 144 - 16$

$\Rightarrow \quad h^2 = 128$

$\Rightarrow \quad h = \sqrt{128} = 8\sqrt{2} \text{ cm}$

$\therefore$ Volume of the cone

$= \frac{1}{3} \times \pi \times R^2 \times h$

$= \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 8 \times \sqrt{2}$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 8 \times 1.414 \text{ cm}^3$

$= 189.61 \text{ cm}^3$.

A-35.

Class Interval	f	cf
0 – 6	4	4
6 – 12	x	4 + x
12 – 18	5	9 + x
18 – 24	y	9 + x + y
24 – 30	1	10 + x + y
Total	10 + x + y	

Here $n = 20$

$\Rightarrow \quad \frac{n}{2} = 10$, Median = 14.4

$\therefore$ Median class = 12 – 18

Here $l = 12$, cf = 4 + x

f = 5, h = 6

$\therefore$ Median $= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$\Rightarrow \quad 14.4 = 12 + \left(\frac{10 - (4 + x)}{5}\right) \times 6$

$\Rightarrow \quad x = 4, y = 6$ (As x + y = 10)

A-36. (i) $\sqrt{8}$ units

(ii) $4\sqrt{2}$ units

(iii) 1 : 2 OR 1 : 1

A-37. (i) Diameter of base of heap = 24 m

Radius of base of heap $= \frac{24}{2}$ m

$= 12$ m

Height of heap = 3.5 m

Let l be the slant height of heap

$\therefore \quad l = \sqrt{r^2 + h^2}$

$= \sqrt{(12)^2 + (3.5)^2}$

$= \sqrt{144 + 12.25}$

$= \sqrt{156.25}$

$l = \sqrt{156.25} = 12.5$m

(ii) Canvas cloth required to cover the heap $= \pi r l$

$= \frac{22}{7} \times 12 \times 12.5 = 471.42 \text{ m}^2.$

(iii) Volume of heap of wheat

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$

$= 22 \times 4 \times 12 \times 0.5 = 528 \text{ m}^3$

OR

Volume of one bag = 0.48 m^3

Number of bags required

$= = \frac{528}{0.48} = 1100$

A-38. (i) $\angle ACD = \angle CAX$ (Alternate angles)

$\therefore \angle ACD = 45°$

(ii) In right-angled ΔADC,

$\tan 45° = \frac{AD}{CD}$

$\Rightarrow CD = AD = 100$ m

(ii) In right-angled ΔADB,

$\tan 30° = \frac{AD}{DB}$

$[\because \angle ABD = \angle BAY]$

$\Rightarrow BD = AD \cot 30°$

$= 100 \times \sqrt{3}$ m

OR

In ΔADC,

$\sin 45° = \frac{AD}{AC}$

$\Rightarrow AC = AD \times \sqrt{2} = 100\sqrt{2}$ m

$\left\{\sin 45° = \frac{1}{\sqrt{2}}\right\}$

X - MATHEMATICS

SAMPLE PAPER - 5

Time Allowed : 3 Hours] **[Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 MCQs carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each)with sub-parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However an internal choice in 2 Qs. of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

SECTION - A

Section A consists of 20 questions of 1 mark each.

Q1. 4 bells toll together at 9.00 am. They toll after 7, 9, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours ?

(a) 3 (b) 4 (c) 5 (d) 6

Q2. A quadratic polynomial whose zeroes are $\frac{3}{5}$ and $-\frac{1}{2}$ are ________.

(a) $10x^2 - x - 3$ (b) $10x^2 + x - 3$ (c) $10^2 - x + 3$ (d) none of these

Q3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = 1$ (c) $a = 2, b = -6$ (d) $a = 0, b = -6$

Q4. Three runners running around a circular track, can complete one revolution in 2, 3 and 4 hrs respectively. They will meet again at the starting point after

(a) 8 hrs (b) 6 hrs (c) 12 hrs (d) 18 hrs

Q5. If A and B are the points (–3, 4) and (2, 1) respectively, then the coordinates of the point on AB produced such that AC = 2BC are

(a) (2, 4) (b) (3, 7) (c) (7, – 2) (d) none of these

Q6. What is the largest number that divides each one of 1152 and 1664 exactly ?

(a) 32 (b) 64 (c) 128 (d) 256

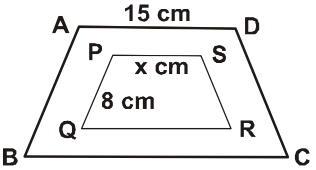
Q7. In right triangle, $\angle B = 90°$, AB = 24 cm, BC = 7 cm, then cos C =

(a) $\frac{7}{24}$ (b) $\frac{24}{25}$ (c) $\frac{25}{24}$ (d) $\frac{7}{25}$

Q8. In ΔABC, $\angle C = 90°$, then tan A + tan B =

(a) $\frac{b^2}{ac}$ (b) a + b (c) $\frac{a^2}{bc}$ (d) $\frac{c^2}{ab}$

Q9. If quadrilateral ABCD and PQRS are similar, then x =



(a) 4 cm (b) 5 cm (c) 6 cm (d) 7 cm

Q10. Distance between two parallel tangents is 14cm, then the radius of circle is

(a) 6 cm (b) 7 cm (c) 12 cm (d) 14 cm

Q11. sin 45° – cos 45° is equal to

(a) 2 cos θ (b) 0 (c) 2 sin θ (d) 1

Q12. The length of the minute hand a wall clock is 7 cm, then how much area does it sweep in 20 minutes ?

(a) 51 cm^2 (b) 49.33 cm^2 (c) 51.33 cm^2 (d) 52 cm^2

Q13. The curved surface area of a cylinder of height 14 cm is 88 cm^2, then diameter of the cylinder is

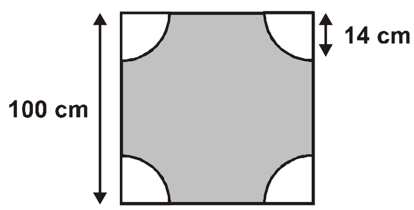
(a) 8.5 cm (b) 1 cm (c) 1.5 cm (d) 2 cm

Q14. The relationship between mean, median and mode for a moderately skewed distribution is

(a) mean = median – 2 mean (b) mode = 3 median – 2 mean

(c) mode = 2 median – 3 mean (d) mode = median – mean

Q15. In figure, at each corner of square side 100 cm, a quadrant of radius 14 cm is formed, then area of shaded region is



(a) 9834 cm^2 (b) 9348 cm^2 (c) 9384 cm^2 (d) 9884 cm^2

Q16. The mean age of combined group of men and women is 30 years. If the mean of the age of men and women are respectively 32 and 27, then the percentage of women in the group is

(a) 30 (b) 20 (c) 50 (d) 40

Q17. Radius of circumcircle of a triangle ABC is $5\sqrt{10}$ units. If point P is equidistant from A(1, 3), B(–3, 5) and C(5, –1), then AP =

(a) 5 units (b) $5\sqrt{5}$ units (c) 25 units (d) $5\sqrt{10}$ units

Q18. If $\sin\theta - \cos\theta = 0,$ then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

Direction : *In the question number 19 and 20, a statement of* ***Assertion (A)*** *is followed by a statement of* ***Reason (R)****. Choose the correct option.*

Q19. **Statement A (Assertion) :** Pair of linear equations : $9x + 3y + 12 = 0$, $8x + 6y + 24 = 0$ have infinitely many solutions.

Statement R (Reason) : Pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q20. **Statement A (Assertion) :** PA and PB are two tangents to a circle with centre O, such that $\angle AOB = 110°$, then $\angle APB = 90°$.

Statement R (Reason) : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

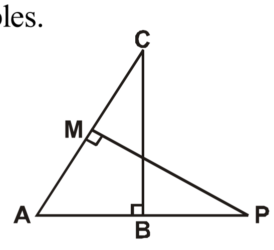
(d) Assertion (A) is false but reason (R) is true.

SECTION - B

Section B consists of 5 questions of 2 marks each.

Q21. 5 books and 7 pens together cost Rs. 79, whereas 7 books and 5 pens together cost Rs. 77. Represent this situation in the form of linear equation in two variables.

Q22. Amandeep draws two right-angled triangle ABC and AMP right-angled at B and M respectively, as shown in figure.

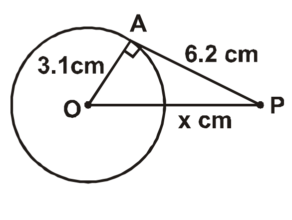


Prove that :

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Q23. In the given figure, O is the centre of the circle. The radius of the circle is 3.1 cm and PA is a tangent drawn to the circle from point P. If OP = x cm and AP = 6.2 cm, then find the value of x.



OR

AB is a tangent drawn from a point A to a circle with centre O and BOC is a diameter of the circle such that $\angle AOC = 110°$. Find $\angle OAB$.

Q24. Find the area of the quadrant of a circle whose circumference is 44 cm.

Q25. If $\tan\theta = \frac{1}{\sqrt{3}}$, then evaluate $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$.

OR

If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, find A and B.

SECTION - C

Section C consists of 6 questions of 3 marks each.

Q26. Manju and Manish participate in a cycle race, organised for National integration. Manju takes 18 minutes to complete one round, while Manish takes 12 minutes for the same. Suppose they both start at the same time and go in the same direction. After how many minutes, will they meet again at the starting point ?

Q27. Solve for x : $\frac{x-2}{x-4} + \frac{x-6}{x-8} = 6\frac{2}{3}, \ (x \neq 4, 8)$

Q28. Abhishek is planning a journey by ship to Andaman. Andaman trip in itself is an advaenture. There are three port in India from where you can sail to Andaman : Kolkata, Chennai and Vishakhapatnam. Abhishek did not know the length of journey so he took the help of an expert who helped him by solving a simple mathematical situation related to ships.

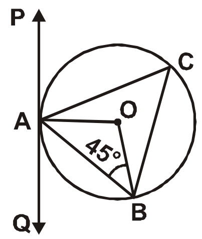
The ship covered a certain distance at a uniform speed. If the speed would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And if the speed of ship were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

OR

Two pipes running together can fill a cistern in 6 minutes. If one pipe takes 5 minutes more than the other to fil the cistern, find the time in which eace pipe would fill the cistern.

Q29. From a window (120 metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60° and 45° respectively. Show that the height of the opposite house is $120(1+\sqrt{3})$ metres.

Q30. In the given figure, PAQ is a tangent to the circle with centre O at a point A. If $\angle OBA = 45°$, find the value of $\angle BAQ$ and $\angle ACB$.



OR

The incircle of ΔABC touches the sides BC, CA and AB at D, E and F respectively. Show that AF + BD + CE + AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC).

Q31. Cards marked with numbers 4 to 99 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is :

(i) a perfect square

(ii) a multiple of 7

(iii) a prime mumber less than 30

SECTION - D

Section D consists of 4 questions 5 marks each.

Q32. Determine graphically the coordinates o the vertices of triangle formed by the equation 2x – 3y + 6 =0 and 2x + 3y – 18 = 0; and the y-axis.Also, find the area of this triangle.

OR

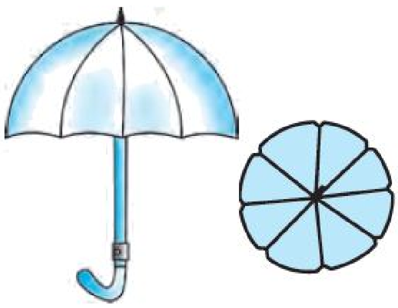
Eight times a two-digit number is equal to three times the number obtained by reversing the order of the digits. If the difference between the digits of the number is 5, find the number.

Q33. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Q34. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

OR

An umbrella has 8 ribs which are equally speed (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Q35. Find mean, median and mode of the following data :

Classes	Frequency
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

SECTION - E

Case study based questions are compoulsory.

Q36. The houses of four friends are located by point A, B, P and Q shown in figure.

If coordinates of A and B with respect to coordinate axes are known and P and Q trisect the AB. Then answer the following questions based on it

(i) Find the coordinates of P.

(ii) Find the coordinates of Q.

(iii) Find the distance PQ.

OR

Find the distance AB.

Q37. Deepak and Sanju works together in a bank in Delhi. Hometown of both of them is Rampur in Uttar Pradesh which is at a distance of 300 km from Delhi. To reach Rampur from Delhi they travel partly by train and partly by bus. This Diwali they travelled separately to Rampur. Deepak travels 60 km by train and remaining by bus and taken 4 hrs. Sanju travels 100 km by train and remaining by bus and takes 4 hrs. 10 minuts.

(i) If speed of train is x km/h and speed of bus is y km/h then write algebraic representation of the situation.

(ii) Find the speed of the bus.

(iii) If speed of the train 90 km/h and speed of the bus is 60 km/h then find time taken by Deepak to travel 60 km by train and 240 km by bus.

OR

If speed of the train is 120 km/h and speed of bus is 60 km/h then find time taken by Sanju to travel 120 km by train and 180 km by bus.

Q38. Akshat appears for a multiple choice questions test with four choices one of which is right. He either guesses or copies or known the answer to a question. Total number of questions in the test is 50.

He knows the answer to 50% of the questions, he guesses the answer of 15 questions and copies the answer of remaining questions.

(i) What is the probability that he knows the answer of a question ?

(ii) What is probability that Akshat guesses the answer of a question ?

(iii) What is the probability that Akshat copies the answer of a question ?

OR

What is the probability that Akshat does not copy the answer of a question ?

SOLUTIONS : SAMPLE PAPER - 5

A-1. (c) LCM of 7, 8, 11, 12 = 1848

$\therefore$ Bells will toll together after every 1848 sec.

$\therefore$ In next 3 hrs, number of times the bells will toll number

$$= \frac{3 \times 3600}{1848} = 5.84$$

$\Rightarrow$ 5 times

A-2. (a) Zeroes of quadraticpolynomial are $\frac{3}{5}$ and $-\frac{1}{2}$

$\therefore$ quadratic polynomial

= k[x^2 – (Sum of zeroes) product of zeroes]

$$= k\left[x^2 - \left[\frac{3}{5} + \left(\frac{-1}{2}\right)\right]x + \frac{3}{5} \times \left(-\frac{1}{2}\right)\right]$$

$$= k\left(x^2 - \frac{x}{10} - \frac{3}{10}\right)$$

$$= \frac{k}{10}[10x^2 - x - 3]$$

where k is any constant.

A-3. (d) $x^2 + (a + 1)x + b$

$\because$ x = 2 is a zero

and x = – 3 is another zero

$\therefore$ $(2)^2 + (a + 1)2 + b = 0$

and $(-3)^2 + (a + 1)2 + b = 0$

$\Rightarrow$ 4 + 2a + 2 + b = 0

and 9 – 3a – 3 + b = 0

$\Rightarrow$ 2a + b = – 6 ...(i)

and –3a + b = – 6 ...(ii)

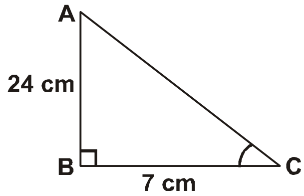
Solving (i) and (ii), we get a = 0 and b = – 6

A-4. (c)

A-5. (c)

A-6. (c

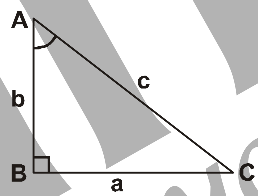
A-7. (d) In ΔABC, ∠B = 90°



$$\cos C = \frac{BC}{AC} = \frac{7}{\sqrt{(24)^2 + (7)^2}}$$

$$= \frac{7}{25}$$

A-8. (d) In ΔABC, ∠C = 90°



$$\tan A = \frac{BC}{AC}$$

$$\tan B = \frac{AC}{BC}$$

$$\therefore \quad \tan A + \tan B = \frac{BC}{AC} + \frac{AC}{BC}$$

$$= \frac{BC^2 + AC^2}{AC \cdot BC}$$

$$\tan A + \tan B = \frac{AB^2}{AC \cdot BC}$$

$$= \frac{c^2}{b \cdot a} = \frac{c^2}{ab}$$

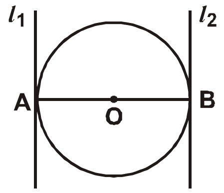
A-9. (c) Quadrilateral ABCD and quadrilateral PQRS are similar

$$\therefore \quad \frac{PS}{AD} = \frac{PQ}{AB} \text{ (Similarly criteria)}$$

$$\frac{x}{15} = \frac{8}{20}$$

$$x = \frac{8}{20} \times 15 = 6\text{cm}$$

A-10. (b) Distance between two parallel tangents is always equal to the diameter of circle.



Here tangents l_1 and l_2 are parallel and AB is diameter of circle.

Hence, AB = 14 cm

$\therefore$ Radius = $\frac{1}{2}$AB = 7 cm

A-11. (b)

A-12. (c) Angle subtended by minute hand in 20 minutes

$$= \frac{360°}{60} \times 20 = 120°$$

$\therefore$ Area swept in 20 minutes

$$= \frac{22}{7} \times \frac{7 \times 7 \times 120°}{360°}$$

$= 51.33\text{ cm}^2$

A-13. (d) Height of cylinder = 14 cm

Radius of cylinder - r

$\therefore$ Curved surface area = $2\pi rh$

$$88 = 2 \times \frac{22}{7} \times r \times 14$$

Diameter = 2r

$$= \frac{88 \times 7}{22 \times 14} = 2\text{ cm}$$

A-14. (b) Mode = 3 median – 2 mean

A-15. (c) Radius of quadrant = 14 cm

$$\text{Area of quadrant} = \frac{\pi(14)^2 \times 90°}{360°}$$

$$= \frac{22}{7} \times \frac{14 \times 14 \times 90°}{360°}$$

$= 154\text{ cm}^2$

Area of four quadrants = 4(154)

$= 616\text{ cm}^2$

Angle of shaded region

= area of square – (ara of four quadrants)

$= 10000\text{ cm}^2 - 616\text{ cm}^2$

$= 9384\text{ cm}^2$

A-16. (d) Let no. of men be x, and women be y.

Total age of the group = 30(x + y)

Total age of men = 32x years

Total age of women = 27y years

$\Rightarrow$ 30(x + y) = 32x + 27y

$\Rightarrow$ 30x + 30y = 32x + 27y

$\Rightarrow$ $x = \frac{3}{2}y$

$\therefore$ % of women = $\frac{y}{x+y} \times 100$

$\Rightarrow$ $\frac{y}{\frac{3}{2}y + y} \times 100 = 40\%$

A-17. (d)

A-18. (c) $\sin\theta - \cos\theta = 0$

$\Rightarrow$ $(\sin\theta - \cos\theta)^2 = 0$

$\Rightarrow$ $(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta) = 0$

$\Rightarrow$ $-2\sin\theta\cos\theta = -1$

$\Rightarrow$ $\sin^2\theta\cos^2\theta = \frac{1}{4}$

$$\sin^4\theta + \cos^4\theta = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta$$

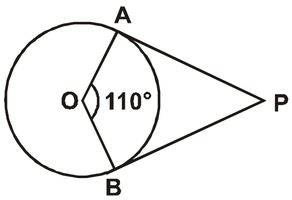
$$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$$

$$= (1)^2 - 2 \times \frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}$$

A-19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

A-20. (d) Assertion (A) is false but reason (R) is true.

As per information given in question we have figure given below :



Radius is perpendicular to the tangent at point of conctant.

Thus, OA ⊥ AL and OB ⊥ PB

In quadrilateral, OAPB, we have

$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$

$90° + \angle APB + 90° + 110° = 360°$

$\angle APB = 70°$

Assertion (A) is false but reason (R) is true.

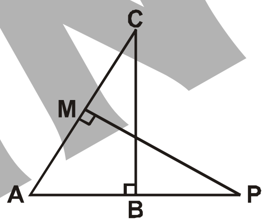
A-21. Let the cost of 1 book be Rs. x and the cost of 1 pen be Rs. y.

According to question,

$5x + 7y = 79$ ...(i)

and $7x + 5y = 77$ ...(ii)

A-22. **Given :** In ΔABC, ∠B = 90° and in ΔAMP, ∠M = 90°



To Prove : (i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Proof :

(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ (Each 90°)

$\angle BAC = \angle MAP$ (Common)

∴ ΔABC ~ ΔAMP (AA similarlity)

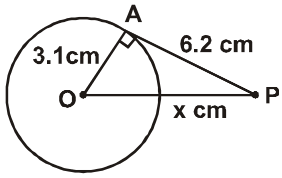
(ii) As ΔABC ~ ΔAMP,

$\therefore \frac{AC}{AP} = \frac{BC}{MP}$

(Ratios of the corresponding sides of similar triangles

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ Hence Proved.

A-23. In right-angeld ΔOAP,



$OP^2 = OA^2 + AP^2$

(Using pythagoras theorem)

$x^2 = (3.1)^2 + (6.2)^2$

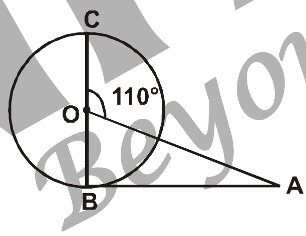
$x^2 = 9.61 + 38.44$

$x^2 = 48.05$

$x = 6.93$ cm

OR

$\angle AOB + \angle AOC = 180°$ (linear pair)



$\therefore \angle AOB = 180° - \angle AOC$

$= 180° - 110° = 70°$

In ΔAOB,

$\angle OBA + \angle OAB + \angle AOB = 180°$

$\therefore 90° + \angle OAB + 70° = 180°$

$\angle OAB = 180° - 160° = 20°$

A-24. Circumference of the circle = 44 cm

$\Rightarrow 2\pi r = 44$ cm

$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7$ cm

∴ Area of the quadrant of a circle

$= \frac{1}{4}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} cm^2$

A-25. Given $\tan\theta = \frac{1}{\sqrt{3}}$

As we know that $\tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

Putting θ = 30° in $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$, we get

$$\frac{\operatorname{cosec}^2 30° - \sec^2 30°}{\operatorname{cosec}^2 30° + \sec^2 30°} = \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{8}{16} = \frac{1}{2}$$

OR

$$\sin(A - B) = \frac{1}{2} = \sin 30°$$

$\Rightarrow$ A – B = 30° ...(i)

$$\cos(A + B) = \frac{1}{2} = \cos 60°$$

$\Rightarrow$ A + B = 60° ...(ii)

Adding (i) and (ii), we get

2A = 90° $\Rightarrow$ A = 45°

Putting the value in (i), we get

45° – B = 30°

$\Rightarrow$ B = 15°

A-26. Factors of 12 = 2 × 2 × 3 = 2^2 × 3

Factor of 18 = 2 × 3 × 3 = 2 × 3^2

LCM (12, 18) = 2^2 × 3^2 = 36

$\therefore$ After 36 mintues, they will meet again at the staring point.

A-27. $$\frac{x-2}{x-4} + \frac{x-6}{x-8} = 6\frac{2}{3}$$

$$\Rightarrow \frac{(x-2)(x-8) + (x-6)(x-4)}{(x-4)(x-8)} = \frac{20}{3}$$

$\Rightarrow$ $14x^2 - 180x + 520 = 0$

$\Rightarrow$ $7x^2 - 90x + 260 = 0$

Here, a = 7, b = – 90, c = 260

$\therefore$ Discriminate,

D = $b^2 - 4ac$

= $(-90)^2 - 4 \times 7 \times 260$

= 820

Using quadratic formula, we have

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{90 \pm \sqrt{820}}{2 \times 7}$$

$$= \frac{90 \pm \sqrt{820}}{14} = \frac{45 \pm \sqrt{205}}{7}$$

Hence, $x = \frac{45 + \sqrt{205}}{7}, \frac{45 - \sqrt{205}}{7}$

A-28. Let the usual speed of ship be x km/h and the usual time be y hours.

$\therefore$ Distance covered = xy km

Case I :

When speed = (x + 6) km

then time taken = (y – 4) hour

Now, distance covere = xy

$\Rightarrow$ (x + 6)(y – 4) = xy

$\Rightarrow$ 2x – 3y = – 12 ...(i)

Case II :

When speed = (x – 6) km/h

then time taken = (y + 6) hour

Now distance covered = xy

$\Rightarrow$ (x – 6)(y + 6) = xy

x – y = 6 ...(ii)

Solving (i) and (ii), we get

x = 30, y = 24

$\therefore$ Distance covered = xy = 30 × 24

= 720 km

$\therefore$ The length of the journey = 720 km

OR

Let the time taken by first pipe to fill the cistern be x minutes

$\therefore$ In 1 minute, it can fill $\frac{1}{x}$ of cistern.

Time taken by second pipe to fill the cister

= (x + 5) minutes

$\therefore$ In 1 minute, it fill $\frac{1}{x+5}$ of cistern.

According to question

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$\Rightarrow$ $x^2 - 7x - 30 = 0$

$\Rightarrow$ (x – 10)(x + 3) = 0

$\Rightarrow$ x = 10, –3

$\Rightarrow$ x = 10 [x = –3 is rejected]

$\therefore$ Time taken by first pipe = 10 minute

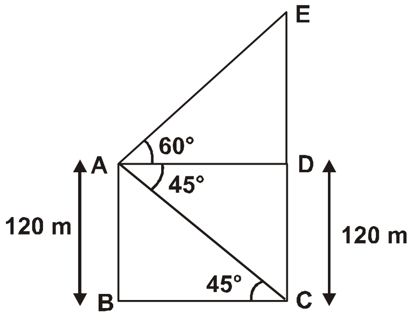
Time taken by second pipe = 15 minutes

A-29. Let bet he window and CE be the opposite house.

Now, CD = AB = 120 m ...(i)

(Opposite sides of a rectangle)

In right-angled ΔABC, $\tan 45° = \frac{AB}{BC}$



$\Rightarrow \quad 1 = \frac{120}{BC}$

$\Rightarrow$ BC = 120 m ...(ii)

Now, AD = BC

(Opposite sides of a rectangle)

ADS = 120 m [From (ii)]...(iii)

In right-angled ΔADE,

$\tan 60° = \frac{DE}{AD}$

$\Rightarrow \quad \sqrt{3} = \frac{DE}{120}$ [From (iii)]

$\Rightarrow$ DE = $120\sqrt{3}$ m

$\therefore$ Height of the opposite house

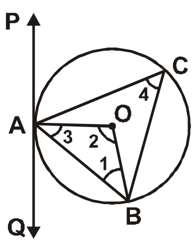
CE = CD + DE

= 120 m + $120\sqrt{3}$ m

= $120(1+\sqrt{3})$m

A-30. **Given :** PAQ is a tangent to th circle with centre O at a piont A and $\angle$OBA = 45°.

To find : $\angle$BAQ and $\angle$ACB



We have OA = OB

(Radii of the same circle)

$\Rightarrow \quad \angle 3 = \angle 1$

(Angle opposite to equal sides of a triangle are equal)

$\Rightarrow \quad \angle 3 = 45°$ ($\because \angle 1 = 45°$, given)

Also, in ΔOAB

$\angle 1 + \angle 2 + \angle 3 = 180°$

(Angle sum property of a triangle)

$\Rightarrow \quad 45° + \angle 2 + 45° = 180°$

$\Rightarrow \quad \angle 2 = 180° - 90° = 90°$

Now $\angle 4 = \frac{1}{2}\angle 2 = 45°$

(Degree measure theorem)

$\Rightarrow \quad \angle ACB = 45°$

Now, $\angle BAQ = \angle OAQ - \angle 3$

$= 90° - 45° = 45°$

[OA $\perp$ AQ]

OR

Given : Sides AB, BC and CA of ΔABC, touches the incircle at D, E and F respectively.

To prove :

AF + BD + CD = AE + BF + CD

$= \frac{1}{2}$ (perimeter of ΔABC)

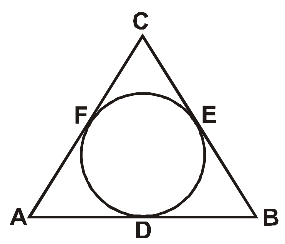
Proof : Since lengths of the tangents drawn from an external point to a circle are equal

Therefore,

AF = AE ...(i)

BD = BF ...(ii)

CE = CD ...(iii)



Adding (i), (ii) and (iii), we get

AF + BD + CE = AE + BF + CD

Now,

perimeter of $\Delta ABC = AB + BC + CA$

$\therefore$ Perimeter of ΔABC

$= (AF + FB) + (BD + CD) + (EC + AE)$

$= (AF + AE) + (BD + BF) + (EC + CD)$

$= 2(AF + BD + CE)$

$\Rightarrow$ $AF + BD + CE$

$= \frac{1}{2}$ (perimeter of ΔABC)

So, $AF + BD + CE = AE + BF + CD$

$= \frac{1}{2}$ (perimeter of ΔABC)

Hence proved.

A-31. Total number of cards = 96

Number of ways to draw one card = 96

(i) Let A be the event of number on the card is a perfect square.

Perfect squares are 4, 9, 16, 25, 36, 49, 64, 81

Outcomes favourable to A = 8

$\therefore$ $P(A) = \frac{8}{96} = \frac{1}{12}$

(ii) Let B be the event of number on the card is a multiple of 7.

Multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 79, 77, 84, 91, 98

Outcomes favourable to B = 14

$\therefore$ $P(B) = \frac{14}{96} = \frac{7}{48}$

(iii) Let C be the event of number on the card is a prime number less than 30.

Prime numbers are 5, 7, 11, 13, 17, 19, 23, 29.

Outcomes favourable to C = 8

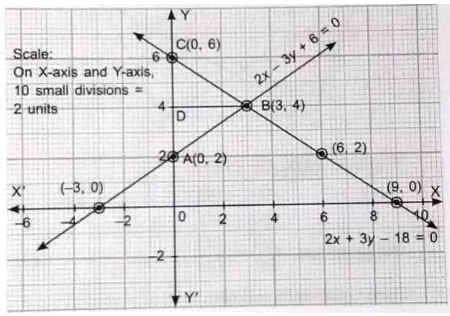
$\therefore$ $P(C) = \frac{8}{96} = \frac{1}{12}$

A-32. The solution table for $2x - 3y + 6 = 0$ is

x	0	–3	3
y	2	0	4

The solution table for $2x + 3y - 18 = 0$ is

x	0	9	6
y	6	0	2



Coordinates of the vertices of a triangle are A(0, 2), B(3, 4) and C(0, 6).

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ base × height

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4 \times 3$

= 6 units

OR

Let the digit at unit's place be x and the digt at ten's place be y.

$\therefore$ Required number = $10y + x$

When the digts are reversed, the number becomes $10x + y$

According to question,

$8(10y + x) = 3(10x + y)$

$\Rightarrow$ $80y + 8x = 30x + 3y$

$\Rightarrow$ $77y - 22x = 0 \Rightarrow 7y - 2x = 0$ ...(i)

Also $x - y = 5$ (keeing $x > y$) ...(ii)

Multiplying (ii) by 2 and adding to (i), we get

$y = 2$

Putting $y = 2$ in (ii), we get

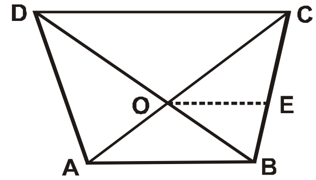
$x - 2 = 5$

$\Rightarrow$ $x = 7$

$\therefore$ Required number

$10y + x = 10 \times 2 + 7 = 27$

A-33. Given : A quadrilateral ABCD, whose diagonals intersect at O.



and $\frac{AO}{BO} = \frac{CO}{DO}$ or $\frac{AO}{OC} = \frac{BO}{DO}$

To prove : ABCD is a trapezium

Construction : Draw EO || AB

Proof : In ΔABC, OE || AB

$\therefore \quad \frac{AO}{OC} = \frac{BE}{EC}$ [By B.P.T.] ...(i)

But given that

$\frac{AO}{OC} = \frac{BO}{DO}$ ...(ii)

From equation (i) and (ii)

$\frac{BO}{DO} = \frac{BE}{EC}$

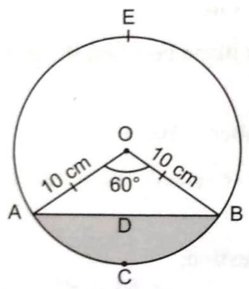
$\Rightarrow$ OE || DC

[By converse of B.P.T.]

OE || AB and OE || DC $\Rightarrow$ AB || DC

$\therefore$ ABCD is a trapezium.

A-34.



Radius of the circle 10 cm

Central angle subtended by chord AB

= 60°

Area of minor sector OACB

$= \frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \times \frac{(10)^2 \times 60°}{360°}$

$= \frac{22}{7} \times \frac{10 \times 10}{6}$

$= \frac{1100}{21} \text{cm}^2 = 52.38 \text{ cm}^2$.

Area of equilateral triangle OAB formed by radii and chord

$= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (10)^2$

$= \frac{1.732}{4} \times 100$

$= 43.3 \text{ cm}^2$

$\therefore$ Area of minor segment ACBD

= Aea of sector OACB – Area of triangle OAB

$= (52.38 - 43.30) \text{ cm}^2$

$= 9.08 \text{ cm}^2$

Areaw of circle = πr^2

$= \frac{22}{7} \times (10)^2$

$= \frac{22 \times 100}{7} \text{cm}^2$

$= 314.28 \text{ cm}^2$

$\therefore$ Area of major segment ADBE

= Area circle – Areaof minor segment

$= (314.28 - 9.08) \text{ cm}^2$

$= 305.20 \text{ cm}^2$

OR

Radius of the circle 45 cm

Number of ribs = 8

Angle between two consecutive ribs

$= \frac{\text{central angle of the circle}}{\text{number of the sectors (ribs)}}$

$= \frac{360°}{8} = 45°$

Area between two consecutive ribs

= Area of one sector of circle

$= \frac{\pi r^2 \theta}{360°}$

$= \frac{22}{7} \times \frac{45° \times 45° \times 45°}{360°}$

$= \frac{11 \times 45 \times 9 \times 5}{7 \times 4} \text{cm}^2$

$= \frac{22275}{28} \text{ cm}^2$

A-35.

Classes	Frequency	Cumulative frequency	
0 – 20	6	6	
20 – 40	8	14	
40 – 60	10	24	← Median Class
60 – 80	12	36	
80 – 100	6	42	
100 – 120	5	47	
120 – 140	3	50	
	n = 50		

$\therefore \quad \frac{n}{2} = 25$

Median class = (60 – 80)

$l = 60$, f = 12, c.f. = 24, h = 20

$$\text{Median} = l + \frac{\frac{n}{2} - \text{c.f.}}{f} \times h$$

$$= 60 + \frac{25-24}{12} \times 20$$

$$= 60 + \frac{1 \times 5}{3} = \frac{180+5}{3}$$

$$= \frac{185}{3} = 61.6$$

Modal class = (60 – 80) as its frequency is 12

h = 20, $l = 60$, $f_1 = 12$, $f_0 = 10$, $f_2 = 6$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \frac{12-10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{2}{8} \times 20 = 65$$

Now, Mode = 3 Median – 2 Mean

65 = 3(61.6) – 2Mean

2 Mean = 184.8 – 65

2 Mean = 119.8

$\Rightarrow \quad \text{Mean} = \frac{119.8}{2} = 59.9$

$\therefore$ Mean = 59.9; Median = 61.6, Mode = 65

A-36. (i) As P divides AB in the ratio 1 : 2.

$\therefore$ coordinates of P are

$$\text{x-coordinate} = \frac{1(-2) + 2(4)}{1+2}$$

$$= \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$\text{y-coordinate} = \frac{1(-3) + 2(-2)}{1+2}$$

$$= \frac{-3-2}{3} = \frac{-5}{3}$$

Coordinate of P are $\left(2, \frac{-5}{3}\right)$

(ii) Coordinate of Q are as Q divides AB in the ratio 2 : 1

x-coordinate y-coordinate

$$= \frac{2(-3) + 1(-1)}{1+2}$$

$$= \frac{-6-1}{3} = \frac{-7}{3}$$

Coordinate of Q are $\left(0, \frac{-7}{3}\right)$

(iii) P(2, –5/3) ——— Q(0, –7/3)

Distance PQ

$$= \sqrt{(0-2)^2 + \left(\frac{-7}{3} + \frac{5}{3}\right)^2}$$

$$= \sqrt{(-2)^2 + \left(\frac{-2}{3}\right)^2}$$

$$= \sqrt{4 + \frac{4}{9}}$$

$$= \sqrt{\frac{40}{9}}$$

$$= \frac{1}{3}\sqrt{40} \text{ units}$$

OR

Distance AB

$$= \sqrt{(-2-4)^2 + (-3+1)^2}$$

$$= \sqrt{(-6)^2 + (-2)^2}$$

$$= \sqrt{36+4}$$

$$= \sqrt{40} \text{ units}$$

A-37. (i) $\frac{60}{x} + \frac{240}{y} = 4$

$\frac{100}{x} + \frac{200}{y} = 4\frac{1}{6}$

(ii) 80 km/h

(iii) $\frac{14}{3}$ hours OR 4 hours

A-38. (i) $\frac{1}{2}$

(ii) $\frac{3}{10}$

(iii) $\frac{1}{5}$ OR $\frac{4}{5}$

— *Notes* —