STUDENT ID

No. of pages - 24

(M)

PRE-BOARD EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours | **Maximum Marks : 80**

समय : 3 घंटे | **अधिकतम अंक - 80**

सामान्य निर्देश:

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न 19 एवं 20 अभिकथन एवं तर्क आधारित एक-एक अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के दो-दो अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक लघुउत्तरीय (SA) प्रकार के तीन-तीन अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के पाँच- पाँच अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित चार-चार अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में, खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, questions No. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, questions No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, questions No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, questions No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, questions No. 36 to 38 are case study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION-A

This section comprises multiple choice questions (MCQ's) of 1 mark each.

1. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 1 & -2 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is : 1

(a) -1 (b) 1

(c) 0 (d) -2

2. If A, B and AB are matrices of order 3×2 , $a \times b$ and 3×4 respectively, then number of elements in matrix B is : 1

(a) 6 (b) 8

(c) 4 (d) 12

3. Let $A = \begin{bmatrix} 2 & 4 \\ 0 & 6 \end{bmatrix}$ and $(3A + 2B)$ is a null matrix, then B is equal to : 1

(a) $\begin{bmatrix} 3 & 9 \\ 0 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -9 \\ 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -6 \\ 0 & -9 \end{bmatrix}$

4. For what value of K may the function $f(x) = \begin{cases} K(x^2+1) & , \quad x \leq 0 \\ x+1 & , \quad x > 0 \end{cases}$ become continuous? 1

(a) 1 (b) 0

(c) 2 (d) No value

5. Two events A and B will be independent, if: 1

(a) A and B are mutually exclusive

(b) $P(A) = P(B)$

(c) $P(\bar{A}\,\bar{B}) = [1 - P(A)][1 - P(B)]$

(d) $P(A) + P(B) = 1$ 1

6. If $y = \sec^2(\tan^{-1} x)$, then $\frac{d^2y}{dx^2}$ at $x = 1$ is equal to :

(a) 0

(b) 1

(c) 2

(d) 0.5

7. The derivative of x^{2x} with respect to x^x is : 1

(a) $2x$

(b) $2x^x(1 + \log x)$

(c) $2x^x$

(d) $2x^{2x}(1 + \log x)$

8. If $3^x + 3^y = 3^{x+y}$, then $\frac{dy}{dx}$ is **NOT** equal to : 1

(a) $\frac{3^{x+y} - 3^x}{3^y - 3^{x+y}}$

(b) $1-3^y$

(c) $\frac{1}{1-3^x}$

(d) 3^{y-x}

9. The function $f(x) = \frac{x}{5} + \frac{5}{x}$ has a local minima at x equal to : 1

(a) 0

(b) -5

(c) 5

(d) 2

10. The Value of $\int_{-2}^{2} x|x|\,dx$ is : 1

(a) 0 (b) 4

(c) –4 (d) 8

11. The rate of change of surface area of a sphere with respect to its radius 'r', when $r = 2$ cm is : 1

(a) $60\,\pi\ \text{cm}^2/\text{cm}$ (b) $32\,\pi\ \text{cm}^2/\text{cm}$

(c) $16\,\pi\ \text{cm}^2/\text{cm}$ (d) $8\,\pi\ \text{cm}^2/\text{cm}$

12. $\int_{0}^{2a} f(x)\,dx = 0$, if: 1

(a) $f(-x) = -f(x)$ (b) $f(2a-x) = f(x)$

(c) $f(2a-x) = -f(x)$ (d) $f(a-x) = f(x)$

13. The degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 = x$ is : 1

(a) 1 (b) 2

(c) 3 (d) 4

14. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is : 1

(a) $\frac{\sin x + \sin y}{x}$ (b) $\frac{x-y}{x+y}$

(c) $\frac{x^2+y^2}{x^2-y^2}$ (d) $\frac{x^2+xy}{y^2}$

15. The unit vector perpendicular to both vectors $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$ is : 1

(a) $\hat{i}$ (b) $\hat{j}$

(c) $\hat{k}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

16. Let $\vec{b}$ be any vector such that $|\vec{b}|=b$. The value of $|\vec{b}\times\hat{i}|^2+|\vec{b}\times\hat{j}|^2+|\vec{b}\times\hat{k}|^2$ is : 1

(a) 0 (b) b^2

(c) $2b^2$ (d) $3b^2$

17. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is **NOT** true? 1

(a) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ (b) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

(c) $\cos^2\alpha+\cos^2\beta=\sin^2\gamma$ (d) $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=1$

18. $\int e^x(x^3+4x^2+2x)\,dx$ is equal to : 1

(a) $e^x(x^3+x^2)+c$ (b) $e^x(x^3)+c$

(c) $e^x\left(\frac{x^3}{3}+\frac{x^2}{2}\right)+c$ (d) $e^x(x^2+x)+c$

ASSERTION-REASON BASED QUESTIONS

Question number 19 and 20 each carry one mark.

In the following questions a statement of Assertion (A) is followed by statement of Reason (R) is given. Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but Reason R is not correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

19. Assertion (A) : $\cos^{-1}\left(\cos\frac{25\pi}{3}\right)$ is equal to $\frac{\pi}{3}$. 1

Reason (R): The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

20. Assertion (A) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ 1

Reason (R): For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

SECTION-B

This section comprises of Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Find the direction cosines of a line whose vector equation is given as $\vec{r}=(1-\mu)\hat{i}+(2\mu-1)\hat{j}+(2\mu+3)\hat{k}$. 2

OR

(b) For vectors $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{c}=2\hat{i}+2\hat{j}+\hat{k}$, determine $|\vec{a}\times(\vec{b}+\vec{c})|$. 2

22. (a) Evaluate : $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ 2

OR

(b) Function f : A $\rightarrow$ B defined as f(x) = 4x is both one-one and onto. If A = {1, 2, 3, 4}, then find the set B. 2

23. If $y=\sqrt{2024x+2025}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$ 2

24. Show that $f(x)=e^{2x}-e^{-2x}+x-\tan^{-1}x$ is strictly increasing in its domain. 2

25. Find : $\int\frac{\sin(x-a)}{\sin(x+a)}dx$ 2

SECTION-C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. (a) Find a matrix A, such that $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. Also a find $(A^T)^{-1}$. 3

OR

(b) If $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix}$, then find AB and use it to solve the following system of equations : 3

$$3x + 7y = 10$$
$$x + 2y = 3$$

27. (a) Evaluate : $\int \frac{2}{(1+x)(1+x^2)}\,dx$ 3

OR

(b) Evaluate : $\int_1^4 (|x-1| + |x-3|)\,dx$ 3

28. If $x = \sin^3 t$, $y = \cos^3 t$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$. 3

29. (a) Find the particular solution of the differential equation given by $x^2 \frac{dy}{dx} - xy = x^2 \cos^2\left(\frac{y}{2x}\right)$ given that when $x = 1$, $y = \frac{\pi}{2}$. 3

OR

(b) Solve the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$. 3

30. Find the distance between the lines: 3

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

31. Let X be a discrete random variable. The probability distribution of X is given below: 3

x	0	1	2	3
P(X=x)	q	$4P^2$	P	$0.7–4p^2$

Find the values of p and q for which the mean of X, $(E(x))$ is largest.

SECTION-D

Question number 32 to 35 are Long type answer (LA). Each carries 5 marks.

32. (a) Let A= R – {5} and B = R – {p}. Find the value of 'p' such that the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-7}{x-5}$ is onto. Also, check whether the given function is one-one or not 5

OR

(b) A Relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y) : (xy)$ is an irrational number$\}$.

Check whether R is reflexive, symmetric and transitive or not. 5

33. Sketch the graph of $y = 2x + 1$ and hence, using integration find the area bounded by this curve, x-axis and the ordinates $x = -3$ and $x = 4$. 5

34. Solve the following linear programming problem graphically: 5

Maximize $z = x + 2y$

Subject to: $x + y \leq 300,$

$2x + 3y \leq 720,$

$x \geq 0, y \geq 0$

35. (a) Find the vector and cartesian equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines : 5

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{Z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

OR

(b) Find the coordinates of the foot of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Hence, write the equation of this perpendicular line. 5

SECTION-E

This section comprises 3 case study based questions of 4 marks each.

Case Study-I

36. A professional typist having his shop in a busy market charges ₹ 310 for typing 5 English and 7 Hindi pages, while he charges ₹ 210 for typing 7 English and 5 Hindi pages.

Based on the above information, answer the following questions :

(i) If he charges ₹ x for one page of English and ₹ y for one page of Hindi, express the above as a pair of linear equations. 1

(ii) Express the information in terms of matrix equation AX = B. 1

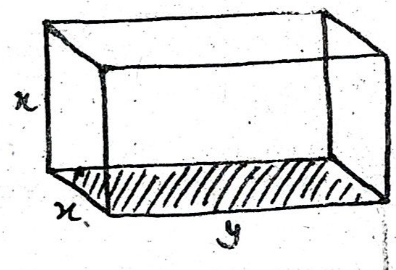
(iii) (a) Using matrix method, find the values of x and y. 2

OR

(b) Find the value of $\frac{|2A^T| + |adj.A|}{|A^{-1}|}$ 2

Case Study-II

37. Raj makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume of container is $36m^3$. Let A (x) be the outside surface area of the container.

Based on the above information, answer the following questions :

(i) Show that $A(x) = \frac{108}{x} + 2x^2$ 1

(ii) Find $A'(x) = \frac{dA(x)}{dx}$ 1

(iii) (a) Given that the outside surface area is minimum, find the height of the container. 2

OR

(b) Find the minimum outer surface area of the container. 2

Case Study-III

38. Sarvesh is a keen chess player who plays one game of chess every night before going to bed. In anyone of those games, the probabilities of Sarvesh winning, drawing or losing are 0.5, 0.2 and 0.3 respectively. Following each game, the probabilities of Sarvesh sleeping well after winning, drawing or losing are 0.7, 0.8 and 0.3 respectively.

Based on the above information, answer the following questions :

(i) Find the probability that on a randomly chosen night Sarvesh sleeps well. 2

(ii) Given that Sarvesh sleeps well, then find the probability that his chess game did not end in a draw. 2