AN EDUCATIONAL INSTITUTE

Class XII Session 2024-25

MATHEMATICS (SET -01)

(Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 markseach.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of section B, 2 Questions of section C and 2 Questions of section D has been provided. And internal choice hasbeen provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take π =22/7 wherever required if not stated.
11. Use of calculators is not allowed

SECTION-- A

(Question numbers 01 to 20 carry 1 mark each.)

Followings are multiple choice questions. Select the correct option in each one of them

01. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value (s) of x is/are

(a) ± 25 (b) 0 (c) ± 5 (d) 25

02. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then $A^{2024} =$

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2024 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2024 & 0 \\ 0 & 2024 \end{bmatrix}$

03. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

04. If the vector $\hat{i} - b\hat{j} + k$ is equally inclined to the coordinate axes, then the value of b is

(a) -1 (b) 1 (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

05. If $\frac{d}{dx}(f(x)) = \log x$, then $f(x)$ equals

(a) $x(\log x - x) + c$ (b) $x(\log x - 1) + c$

(c) $x(\log x + x) + c$ (d) $\frac{1}{x} + c$

06. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is

(a) $y = 2x + \frac{x^3}{3} + c$ (b) $y = 2\log x + \frac{x^3}{3} + c$

(c) $y = \frac{x^2}{2} + c$ (d) $y = 2\log x + \frac{x^2}{2} + c$

07. The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is

(a) 2 (b) 3 (c) 4 (d) None of these

08. In ΔABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2k$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4k$. Let D is the mid-point of BC, then vector $\overrightarrow{AD}$ is equal to

(a) $4\hat{i} + 6k$ (b) $2\hat{i} - 2\hat{j} + 2k$ (c) $\hat{i} - \hat{j} + k$ (d) $2\hat{i} + 3k$

09. $\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$

10. If $|A| = |kA|$, where A is a non-singular square matrix of order 2×2, then sum of all possible values of k is

(a) 1 (b) -1 (c) 2 (d) 0

11. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to

(a) 40 (b) 96 (c) 144 (d) 136

12. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

(a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ

13. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $(x + y + z)$ is

(a) 10 (b) 6 (c) 8 (d) 0

14. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{2}$

15. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}$?

(a) 3 (b) 2 (c) 6 (d) not defined

16.if $|\vec{a}|=\frac{\sqrt{3}}{2}, |\vec{b}|=4$ and angle between them is 60^0 then the value of $\vec{a}.\vec{b}$ is equal to

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) none of these

17. The value of λ for which the angle between the lines $\vec{r}=\hat{i}+\hat{j}+k+p(2\hat{i}+\hat{j}+2k)$ and $\vec{r}=(1+q)\hat{i}+(1+q\lambda)\hat{j}+(1+q)k$ is $\frac{\pi}{2}$, is

(a) -4 (b) 4 (c) 2 (d) -2

18. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

19.Assertion (A): The range of the function $f(x)=2\sin^{-1}x+\frac{3\pi}{2}$, where $x\in[-1,1]$, is $\left[\frac{\pi}{2},\frac{5\pi}{2}\right]$.

Reason (R): The range of the principal value branch of $\sin^{-1}(x)$ is $[0,\pi]$.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

20.Assertion (A): A line through the points $(4,7,8)$ and $(2,3,4)$ is parallel to a line through the point $(-1,-2,1)$ and $(1,2,5)$.

Reason (R): Lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are parallel, if $\vec{b}_1\cdot\vec{b}_2=0$.

SECTION - B

(Question numbers 21 to 25 carry 2 marks each.)

21. Draw the graph of $f(x)=\sin^{-1}x$, $x\in\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$.

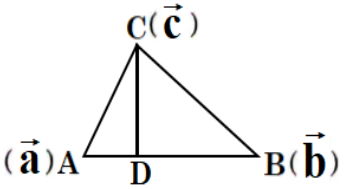
OR

A function $f:A\to B$ defined as $f(x)=2x$ is both one-one and onto. If $A=\{1,2,3,4\}$, then find the set B.

22. Show that the function $f(x)=\frac{16\sin x}{4+\cos x}-x$, is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$.

23. If $\vec{r}=3\hat{i}-2\hat{j}+6k$, then find the value of $(\vec{r}\times\hat{j})\cdot(\vec{r}\times k)-12$.

24. Let A, B and C are non-collinear points with position vectors $\vec{a}$, $\vec{b}$ and, $\vec{c}$ respectively.



Show that the length of perpendicular (CD) drawn from C on AB is $\frac{|(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})|}{|\vec{b}-\vec{a}|}$

OR

If the angle between the lines $\frac{x-5}{\alpha}=\frac{y+2}{-5}=\frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1}=\frac{y}{0}=\frac{z}{1}$ is $\frac{\pi}{4}$, then find the relation between α and β.

25. If $f(x)=\begin{cases} ax+b;\ 0<x\le 1 \\ 2x^2-x;\ 1<x<2 \end{cases}$ is a differentiable function in $(0,\ 2)$, then find the values of a and b.

SECTION-C

(Question numbers 26 to 31 carry 3 marks each.)

26. Evaluate $\int_0^{\frac{\pi}{2}}[\log(\sin x)-\log(2\cos x)]dx$.

27. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$.

OR

Find $\int\frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$.

28. Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$.

29. Chandrayaan, India's lunar exploration program designed by ISRO, has two types of missions: those focused on orbiter missions and those focused on lander and rover missions. Historically, 70% of Chandrayaan missions have been orbiters, and 30% have been lander and rover missions.

Due to various technical challenges, orbiter missions have a success rate of 80%, while lander and rover missions have a success rate of 60%.

If a Chandrayaan mission is randomly selected and it is known to be successful, then what is the probability that it was an orbiter mission?

30. Solve the following differential equation $(x^2-y^2)dx+2xydy=0$

OR

Solve the differential equation: $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y\,dx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$

31. Solve the following linear programming problem graphically.

Maximize z = 5x + 3y

Subject to the constraints $3x+5y\le 15,\ 5x+2y\le 10,\ x\ge 0,\ y\ge 0.$

SECTION-D

(Question numbers 32 to 35 carry 5 marks each.)

32. Determine the area of the region bounded by the curves $x^2 = y,\ y = x + 2$ and x-axis, using the concept of integration.

OR

Find the area of the region bounded by the line $y = \sqrt{3}x$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant.

33.A relation R is defined on a set of real number as

$R = \{(x, y) : x.y$ is an irrational number$\}$.

Check whether R is reflexive, symmetric and transitive or not.

OR

A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an on-to function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

34. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

35. If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the product AB.

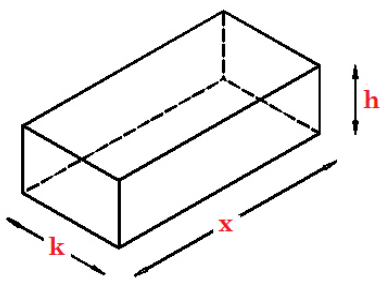
Hence, use the product AB to solve the following system of equations.

x – 2y = 3 , 2x – y –z = 2 , -2y + z = 3

SECTION-E

(Question numbers 36 to 38 carry 4 marks each.)

36.CASE-STUDY I: Read the following passage and then answer the questions given below.



A foreign client approaches ISHA BRICKS COMPANY for a special type of bricks.

The client requests for few samples of bricks as per their requirement.

The solid rectangular brick is to be made from 1 cubic feet of clay of special type.

The brick must be 3 times as long as it is wide.

(i)According to the figure shown, the length of brick is 'x', width is 'k' and height is 'h'. Obtain an expression in terms of 'h' and 'k'.

(ii) Express the surface area (S) of the brick, as a function of 'k'.

(iii) Find $\frac{dS}{dK}$ At what value of k, $\frac{dS}{dK} = 0$?

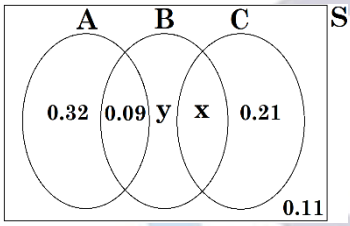
(iv) Show that $\frac{d^2S}{dk^2}$ is positive, at this obtained value of k. What does it signify?

OR

Find the minimum value of S, using second derivative test.

37.CASE STUDY II: Read the following passage and then answer the questions given below.
There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayama as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C yoga is 0.44.



(i) Find the value of x.

(ii) Find the value of y.

(iii) Find $P(A|B)$ and $P(C|B)$.

OR

(iii) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

38.CASE STUDY III: Read the following passage and then answer the questions given below.

The Indian Cost Guard (ICG) while patrolling, saw a suspicious boat with some men. They were not looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. One of the officer observed that the boat is moving along a plane surface.

At an instant, the co-ordinates of the position of coast guard helicopter and boat are at the points A(2, 3, 5) and B(1, 4, 2) respectively.

(i) Write the direction cosines of line AB

(ii) When the position of coast guard helicopter is at the point C(1, o, -3), then the position of the boat is at the point D(3, -2, 3). Check if the line CD is parallel to line AB. Justify.