AN EDUCATIONAL INSTITUTE

Class XII Session 2024-25

MATHEMATICS (SET -02)

(Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 markseach.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of section B, 2 Questions of section C and 2 Questions of section D has been provided. And internal choice hasbeen provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take π =22/7 wherever required if not stated.
11. Use of calculators is not allowed

SECTION-- A

(Question numbers 01 to 20 carry 1 mark each.)

Followings are multiple choice questions. Select the correct option in each one of them

1. If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is

(a) 6 (b) 7 (c) 8 (d) -8

2. The value of μ for which the matrix $\begin{vmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ \mu & 2 & -3 \end{vmatrix}$ is singular is

(a) 2 (b) 0 (c) 1 (d) -1

3. A is a square matrix of order 3 and |A| = -2, then |adj(2A) | is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

4. If f(x) = $\begin{cases} \dfrac{\tan\left(\dfrac{\pi}{4} - x\right)}{\cot 2x}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$ is continuous at x = $\dfrac{\pi}{4}$, then the value of k is

(a) 1 (b) 2 (c) ½ (d) none of these

5. The angle between a line with direction ratios (2,2,1) and a line joining the points (3,1,4) and (7,2,12) is

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{3}{2}\right)$

(c) $\tan^{-1}\left(-\frac{2}{3}\right)$ (d) none of these

6. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}}=4\frac{d^3y}{dx^3}$ respectively are

(a) 3, $\frac{2}{3}$ (b) 3, 1 (c) 3, 2 (d) 3, 3

7. A linear programming problem is as follows:
Minimize Z = 2x + y subject to constraints x ≥3, x ≤ 9, y ≥0, x -y ≥ 0, x + y ≤ 14. The feasible region has
(a) 5 corner points including (0,0) & (9,5)
(b) 5 corner points including (7,7) & (3, 3)
(c) 5 corner points including (14,0) & (9,0)
(d) 5 corner points including (3, 6) & (9, 5)

8. The vectors $3\hat{i}-\hat{j}+2\hat{k}$, $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}+\lambda\hat{j}-\hat{k}$ are coplanar if the value of λ is

(a) -2 (b) 0 (c) 2 (d) Any real number

9. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{1+\cos 2x}$ is equal to

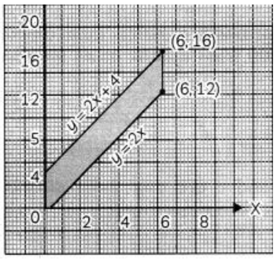
(a) 1 (b) 2 (c) 3 (d) 4

10. If A = $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$ then A^{-1} is

(a) $\begin{bmatrix} 1+bc & 1 \\ 1 & a \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

(c) $\begin{bmatrix} a & 1+bc \\ b & c \end{bmatrix}$ (d) $\begin{bmatrix} 1+bc & b \\ a & c \end{bmatrix}$

11. The feasible region of an LPP is shown in the figure. If Z = 3x – 4y, then, the minimum value of Z is



(a) 0 (b) -16 (c) – 30 (d) – 46

12. A vector of magnitude 5 and perpendicular to vectors $\hat{i}-2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}-3\hat{k}$ is

(a) $\dfrac{5\sqrt{3}\left(\hat{i}+\hat{j}+\hat{k}\right)}{3}$ (b) $\dfrac{5\sqrt{3}\left(\hat{i}-\hat{j}+\hat{k}\right)}{3}$

(c) $\dfrac{5\sqrt{3}\left(\hat{i}-\hat{j}-\hat{k}\right)}{3}$ (d) $\dfrac{5\sqrt{3}\left(\hat{i}+\hat{j}-\hat{k}\right)}{3}$

13. The sum of a matrix and its transpose is $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$, then the matrix is

(a) $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

14. If A and B are 2 events such that $P\overline{\left(A\cup B\right)}=\frac{1}{6}$, $P\left(A\cap B\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{4}$, then the events A and B are

(a) Independent but not equally likely
(b) Mutually exclusive & Independent
(c) Equally likely & mutually exclusive
(d) Equally likely but not independent

15. Integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{x^3+y}{x}$ is

(a) x e^x (b) $\frac{x}{e^x}$ (c) $\frac{e^x}{x}$ (d) e^x

16. If x = t^2 and y = t^3 , then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{3t}{2}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4t}$

17. If y = $\log\left(\frac{1-x^2}{1+x^2}\right)$, |x| < 1 then $\frac{dy}{dx}$ equal to

(a) $\frac{4x^3}{1-x^4}$ (b) $\frac{-4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $\frac{-4x^3}{1-x^4}$

18. The vector equation of the line passing through the points A(3, 4, -7) and B(1, -1, 6) is

(a) $\overline{r}=3\hat{i}+4\hat{j}-7\hat{k}+\lambda\left(\hat{i}-\hat{j}+6\hat{k}\right)$ (b) $\overline{r}=3\hat{i}+4\hat{j}-7\hat{k}+\lambda\left(-2\hat{i}-5\hat{j}+13\hat{k}\right)$

(c) $\overline{r}=\hat{i}-\hat{j}+6\hat{k}+\lambda\left(3\hat{i}+4\hat{j}-7\hat{k}\right)$ (d) $\overline{r}=\hat{i}-\hat{j}+6\hat{k}+\lambda\left(4\hat{i}+3\hat{j}-\hat{k}\right)$

19. **Assertion(A):** Let a and b $\in$ R be such that the function f given by f(x) = log |x| + bx^2 + ax, x ≠0 has local maximum at x = -1 and x = 2. Then the values of a =$\frac{1}{2}$ and b = $\frac{1}{4}$ $A = \pi r^2$.

Reason(R) : x = k, is a point of local maxima if f'(k) = 0, and f''(k) > 0.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

20. Let S be a relation on set R of real numbers defined by S = {(a,b): 1+ab > 0, a,b $\in$ R}

Assertion (A): S on R is a transitive relation.

Reason (R): Symmetric relation is a binary relation R defined on a set A for elements a, b ∈ A, such that aRb, that is, (a, b) ∈ R, implies bRa, that is, (b, a) ∈ R.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

SECTION - B

(Question numbers 21 to 25 carry 2 marks each.)

21. Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

OR

Find the domain of the function y = $\cos^{-1}|x-1|$.

22. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the edge of the cube.

OR

A man of height 2m walks at a uniform speed of 5km/h away from a lamp post which is 6m high. Find the rate at which the length of the shadow increases.

23. Find the intervals in which the function f(x) defined by f(x) = sin x + cos x, 0 ≤ x ≤ 2π is strictly increasing or strictly decreasing.

24. Evaluate $\int \frac{x\tan x}{\sec x.\operatorname{cosec} x}$ dx

25. Find the points of local maxima and local minima of the function f(x) = $3x^4 - 4x^3 + 5$ in [-1, 2].

SECTION-C

(Question numbers 26 to 31 carry 3 marks each.)

26. Evaluate $\int \frac{dx}{\sin x + \sin 2x}$.

27. Out of a group of 60 architects, 40 are qualified and co-operative, while the remaining are qualified and remains reserved. Two architects are selected from the group at random. Find the Probability distribution and the expected number of architects who are qualified and co-operative.

28. Evaluate $\int_0^1 \frac{2x+3}{5x^2+1}dx$

OR

Evaluate $\int_{-1}^{2} |x^3 - x| dx$

29. Given $x + (y+1)\frac{dy}{dx} = 2$

(a) Solve the differential equation and show that the solution represents a family of circles.
(b) Find the radius of a circle belonging to the above family that passes through the origin.

OR

Find the particular solution of the differential equation $(x^2 + y^2)dy = xy\,dx$, given $y(1) = 1$

30. If y = $e^{x\sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

31. Minimize and maximize Z = 5x + 2y subject to constraints
$x - 2y \le 2$, $3x + 2y \le 12$, $-3x + 2y \le 3$, $x \ge 0$, $y \ge 0$.

OR

Minimize Z = 3x + 5y subject to the constraints
$X + 2y \ge 10$, $x + y \ge 6$, $3x + y \ge 8$, $x \ge 0$, $y \ge 0$.

SECTION-D

(Question numbers 32 to 35 carry 5 marks each.)

32. Using integration, find the area of the region bounded by the line 2y = 5x + 7, x – axis and the lines x = 2 and x = 8.

33. A relation R on Z (set of all integers) is defined by R = { (a,b) : |a – b| ≤ 3}, Check whether R is an equivalence relation or not.

OR

Show that the function $f: R \to \{x : x \in R, -1 < x < 1\}$ given by $f(x) = \frac{x}{1+|x|}$ is one-one and onto.

34. The Resident Welfare Association of a colony has 3 different subcommittees with total of 12 members. First subcommittee is adult education committee, which looks after the literacy needs of the adults, the second subcommittee is health and cleanliness committee, which looks after health and cleanliness needs of the colony and third subcommittee is safety committee, which looks after the safety needs of the colony. The number of members of the first subcommittee is half of the sum of the members of the other two subcommittees and the number of members of the second subcommittee is the sum of the members of the other two subcommittees. Reduce the information in the form of algebraic statements and find the number of members in each committee using matrices.

35. Find the image of the point P(2, -1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also, find the length of the perpendicular from the point P(2, -1, 5) to the line.

OR

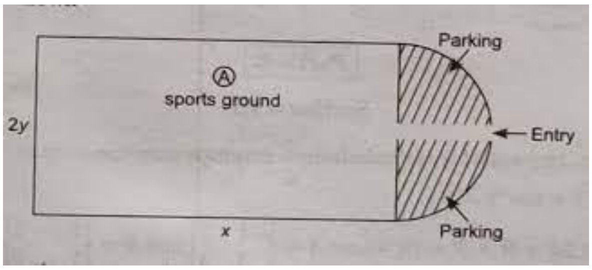
Check whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-5}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew lines or not. Also find the distance between them.

SECTION-E

(Question numbers 36 to 38 carry 4 marks each.)

36. Case study – 1

The government of a state, which has mostly hilly area decided to have adventurous playground on the top of the hill having plane area and space for 10000 persons to sit at a time. After survey it was decided to have rectangular playground with a semicircular parking at one end of playground. The total perimeter of the field is measured as 1000m.



Based on the above passage answer the following questions:
(a) Find the relation between x and y.
(b) Find the area of the sports ground in terms of x.
(c) Find the value of x for which sports ground has the maximum area. Also find the maximum area.

OR

If government wants to maximize the area including the parking space, find the value of y to maximize the total area.

37. <u>CASE STUDY –</u> Read the following passage and answer the questions given below:
During the time of need and otherwise also people help the needy. In a survey it was found that out of 200 people surveyed in a city 50 help the needy on a regular basis, 120 contribute to Prime Minister Relief Fund and the rest help through NGO;s. A person is selected who needs a help, the probabilities of help through persons on regular basis, from Prime Minister relief fund and through NGO's are 0.15, 0.06 and 0.10 respectively.

(a) Find the probability that the needy person received the help..
(b) Find the probability of helping the needy through Prime Minister relief fund.

OR

Find the probability that needy person was helped through person on regular basis.
(c) Find the probability of a help through NGO's.

38.<u>CASE STUDY – 3</u>
The adjoining figure shows an air plant holder which is in the shape of a tetrahedron. Let A (1, 1, 1), B(2, 1, 3), C(3, 2, 2) and D(3, 3, 4) be the vertices of the air plant holder.

Based on the above passage answer the following questions:
(a) Find unit vector along $\overrightarrow{AB}$.
(b) Find vectors $\overrightarrow{AB} \& \overrightarrow{AD}$.
(c) Find the area of ΔABC

OR

Find the area of ΔABD