SET – 1(B)

केंद्रीय विद्यालय संगठन, अहमदाबाद संभाग

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प्री – बोर्ड परीक्षा: **2024 – 25**

PRE – BOARD EXAMINATION: 2024 – 25

SUBJECT : MATHEMATICS (041) **TIME: 3 HOURS**

CLASS : XII **MAX. MARKS: 80**

General Instructions:

1. This Question paper contains – **five sections A, B, C, D and E.** Each section is compulsory. However, there are internal choices in some questions.
2. **Section – A** has 20 MCQ's, 18 **Very Short Answer (Type – 1)** and **02 Assertion – Reason based** questions of **1 mark each.**
3. **Section – B** has 5 **Very Short Answer (VSA)** – type questions of **2 marks each.**
4. **Section – C** has 6 **Short Answer (SA)** – type questions of **3 marks each.**
5. **Section – D** has 4 **Long Answer (LA)** – type questions of **5 marks each.**
6. **Section – E** has 3 **source based/ case based / passage based/integrated** units of assessment **(4 marks each)** with sub parts.

SECTION – A

1. The number of all possible matrices of order 2×2 with each entry 0 or 1 or – 1 is:

(a) 27 **(b)** 3^4

(c) 3^9 **(d)** 81

2. The order and degree of the differential equation $\left\{5+\left(\frac{dy}{dx}\right)^2\right\}^{5/3} = x^5\left(\frac{d^2y}{dx^2}\right)$ is m and n, then the value of $(2m+3n)$ is:

(a) 4 **(b)** 3

(c) 10 **(d)** Not defined

3. If $A = \begin{bmatrix} x & 1 \\ -1 & -x \end{bmatrix}$ such that $A^2 = O$, then $x =$

(a) 1 **(b)** ± 1

(c) 0 **(d)** 1

4. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents:

(a) circle **(b)** straight lines

(c) ellipse **(d)** parabolas

5. The principal value of $cos^{-1}\left(-sin\frac{7\pi}{6}\right)$ is:

(a) $\frac{5\pi}{3}$ **(b)** $\frac{7\pi}{6}$

(c) $\frac{\pi}{3}$ **(d)** $-\frac{\pi}{3}$

6.	The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is: **(a)** $e^x + e^{-y} = C$ **(b)** $e^x + e^y = C$ **(c)** $e^{-x} + e^{-y} = C$ **(d)** $e^{-x} + e^y = C$
7.	If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to: **(a)** $\frac{2}{\pi}\ units$ **(b)** $\frac{1}{\pi}\ unit$ **(c)** $\frac{\pi}{2}\ units$ **(d)** $\pi\ units$
8.	The vector equation of the line passing through the point $(2, -3, 4)$ and parallel to the line $\frac{x-3}{2} = \frac{2y-1}{4} = \frac{2-z}{-5}$ is: **(a)** $\vec{r} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k} + \omega(2\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ **(b)** $\vec{r} = 2\hat{\imath} + 2\hat{\jmath} + 5\hat{k} + \omega(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$ **(c)** $\vec{r} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} + \omega(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$ **(d)** $\vec{r} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k} + \omega(2\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$
9.	Let $f(x) = \|cos\ x\|$. Then, **(a)** $f(x)$ is everywhere diffrentiable **(b)** $f(x)$ is everywhere continuous but not differentiable at $x = n\pi,\ n \in Z$ **(c)** $f(x)$ is everywhere continuous but not differentiable at $x = (2n+1)\frac{\pi}{2},\ n \in Z$ **(d)** None of these
10.	If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is a non – singular matrix and $a \in A$, then the set A is: **(a)** $\boldsymbol{R}$ **(b)** $\{0\}$ **(c)** $\{4\}$ **(d)** $\boldsymbol{R} - \{4\}$
11.	The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio $1 : 3$ externally, is: **(a)** $\frac{\vec{a} + 2\vec{b}}{3}$ **(b)** $3(\vec{a} - \vec{b})$ **(c)** $\frac{\vec{a} + 4\vec{b}}{3}$ **(d)** $\frac{4\vec{a} + \vec{b}}{3}$
12.	The value of $\int_0^1 x\sqrt{1-x}\ dx$ is: **(a)** $\frac{2}{15}$ **(b)** $\frac{4}{15}$ **(c)** $-\frac{2}{15}$ **(d)** $-\frac{4}{15}$
13.	The value of $\int \frac{cos\sqrt{x}}{\sqrt{x}}\ dx$ is: **(a)** $2\ cos\sqrt{x} + C$ **(b)** $\sqrt{\frac{cos\ x}{x}} + C$ **(c)** $sin\sqrt{x} + C$ **(d)** $2\ sin\sqrt{x} + C$

14.	The function $f(x) = x - [x]$, where $[x]$ denotes the greatest integer function, is: **(a)** continuous everywhere **(b)** continuous at integer points only **(c)** continuous at non – integer points only **(d)** differentiable everywhere
15.	The value of $\int \frac{x+3}{(x+4)^2} e^x \, dx$ is: **(a)** $\frac{e^x}{x+4} + C$ **(b)** $\frac{e^x}{x+3} + C$ **(c)** $\frac{1}{(x+4)^2} + C$ **(d)** $\frac{e^x}{(x+4)^2} + C$
16.	Which of the following point satisfy both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$? **(a)** $(-2, 4)$ **(b)** $(3, 2)$ **(c)** $(-5, 6)$ **(d)** $(4, 2)$
17.	The value of $\int \frac{2}{(e^x + e^{-x})^2} \, dx$ is: **(a)** $\frac{-e^{-x}}{e^x + e^{-x}} + C$ **(b)** $-\frac{1}{e^x + e^{-x}} + C$ **(c)** $-\frac{1}{(e^x + 1)^2} + C$ **(d)** $\frac{1}{e^x - e^{-x}} + C$
18.	The objective function $Z = ax + by$ of an LPP has maximum value 42 at $(4, 6)$ and minimum value 19 at $(3, 2)$. Which of the following is true? **(a)** $a = 9, b = 1$ **(b)** $a = 5, b = 2$ **(c)** $a = 3, b = 5$ **(d)** $a = 5, b = 3$
	ASSERTION – REASON QUESTIONS Each of these questions contains two statements, **Assertion and Reason**. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below. **(a)** Assertion is correct, reason is correct; reason is a correct explanation for assertion. **(b)** Assertion is correct, reason is correct; reason is not a correct explanation for assertion. **(c)** Assertion is correct, reason is incorrect. **(d)** Assertion is incorrect, reason is correct.
19.	**Assertion (A):** For two independent events A and B, if $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{5}$, then $P(\bar{A} \cup \bar{B}) = \frac{13}{15}$. **Reason (R):** For two independent events A and B, $\bar{A}$ and $\bar{B}$ are also independent.

20.	**Assertion (A):** The value of λ for which the two vectors $2\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is 5. **Reason (R):** Two vectors $\vec{a}$ *and* $\vec{b}$ are collinear, then $\vec{a} \times \vec{b} = \vec{0}$.
	SECTION – B
21.	Find the interval for which $f(x) = x^4 - 2x^2$ is strictly increasing.
22.	Find the principal value of $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + sin^{-1}\left\{sin\left(\frac{17\pi}{6}\right)\right\} + tan^{-1}(-1)$ **OR** Find the domain of $cos^{-1}[x^2 - 9]$.
23.	If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that $2x\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}$.
24.	If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, prove that $A^2 - A + 2I = 0$. **OR** Write $A = \begin{bmatrix} 5 & 1 \\ -6 & 8 \end{bmatrix}$ as sum of a symmetric and a skew – symmetric matrix.
25.	If $\vec{a} = 5\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.
	SECTION – C
26.	Integrate: $\int \frac{x + 3}{\sqrt{5 - x - 2x^2}} dx$ **OR** Integrate: $\int \frac{1 - x^2}{x(1 - 2x)} dx$
27.	A random variable X has the following probability distribution: X: 0, 1, 2, 3, 4, 5, 6, 7 $P(X)$: 0, k, $2k$, $2k$, $3k$, k^2, $2k^2$, $7k^2 + k$ Find $P(0 < X < 5)$.
28.	Integrate: $\int_0^{\pi/2} \frac{cos\,x}{1 + cos\,x + sin\,x} dx$
29.	Solve the following Linear Programming Problem graphically: Minimize $Z = 20x + 10y$ subject to the constraints: $x + 2y \leq 40$, $3x + y \geq 30$, $4x + 3y \geq 60$, $x \geq 0, y \geq 0$.
30.	If $(cos\,x)^y = (sin\,y)^x$, then find $\frac{dy}{dx}$. **OR** Differentiate $tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ with respect to $sin^{-1}\left(\frac{2x}{1 + x^2}\right)$, if $-1 < x < 1$, $x \neq 0$.
31.	Solve the differential equation: $(x^2 - y^2)\,dx + 2xy\,dy = 0$

Table for Q27:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

	OR Solve the differential equation: $y\,dx + (x - y^3)\,dy = 0$
	SECTION – D
32.	If $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 1 \\ 0 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -4 & 2 \\ 2 & 2 & -1 \\ -4 & -4 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the following system of linear equations: $x - y = 3\,,\quad 2x + 3y + 4z = 17\,,\quad y + 2z = 7$
33.	Draw the rough sketch of curves $y = 1 + \lvert x+2 \rvert$, $x = -3$, $x = 3$ and $y = 0$. Find the area bounded by the curves $y = 1 + \lvert x+2 \rvert$, $x = -3$, $x = 3$ and $y = 0$.
34.	Find the image of the point $(1\,,\ 6\,,\ 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also write the equation of the line joining the given point and its image and find the length of the line – segment joining the given point and its image. **OR** Find the shortest distance between lines whose Cartesian equations are given by $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Also, find the equation of the line determining the shortest distance.
35.	Let $f : N \cup \{0\} \rightarrow N \cup \{0\}$ be a function defined as $f(x) = \begin{cases} n+1\ , & if\ n\ is\ even \\ n-1\ , & if\ n\ is\ odd \end{cases}$. Show that f is a bijection. **OR** Show that relation R on the set $A = \{x \in Z : 0 \leq x \leq 20\}$, given by $R = \{(a\ ,\ b) : \lvert a - b \rvert\ is\ a\ multiple\ of\ 4\}$ is an equivalence relation. Find the set of all elements related to 3.
	SECTION – E
36.	**<u>CASE – STUDY 1</u>: (2 + 2)** A shopkeeper sells three types of flower seeds A_1 , A_2 , A_3 . They are sold in the form of a mixture, where the proportions of these seeds are $4 : 5 : 3$ respectively. The germination rates of the three types of seeds are 45% , $60\ and\ 35\%$ respectively. **Based on the above information, calculate the probability that:** **(i)** a randomly chosen seed will germinate. **(ii)** the seed is of the type A_2 , given that a randomly chosen seed germinates.
37.	**<u>CASE – STUDY 2</u>: (1 + 1 + 2)** Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension $25\,cm\ by\ 40\,cm$ to make container packets without top. Let $x\,cm$ be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.

Based on the above information answer the following:

(i) Express the volume (V) of each container as function of x only.

(ii) Find $\frac{dV}{dx}$.

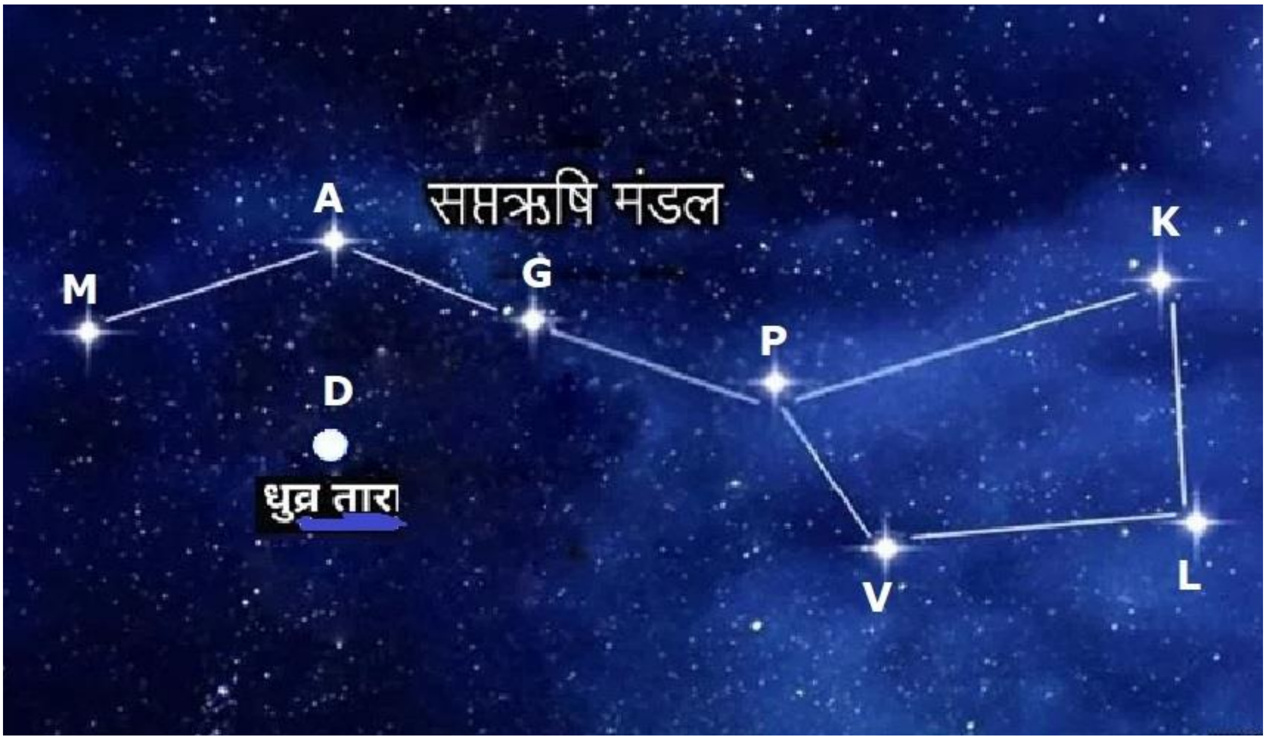
(iii) (a) For what value of x, the volume of each container is maximum?

OR

(iii) (b) Check whether V has a point of inflection at $x = \frac{65}{6}$ or not?

38. **<u>CASE – STUDY 3</u>: (1 + 1 + 2)**

According to Vishnu Purana in Hindu religion, Kritak Trailokya – Bhuh, Bhuvah and Swah – these three worlds together are called Kritak Trailokya. Saptarishi Mandal is one lakh yojana above Saturn Mandal. Saptarishi Mandal is named after seven sages (Marichi, Atri, Angira, Pulah, Kratu, Pulast, Vashishtha).



The position vectors of points M, A, G, P, V, L and K (taking D as origin) are $-3\,\hat{i} - \hat{j} - 6\,\hat{k}$, $\hat{i} - 2\,\hat{j} - 8\,\hat{k}$, $5\,\hat{i} - 2\,\hat{k}$, $11\,\hat{i} + 3\,\hat{j} + 7\,\hat{k}$, $\hat{j} + \hat{k}$, $3\,\hat{i} + \hat{j} + 5\,\hat{k}$ and $3\,\hat{j} + 3\,\hat{k}$ respectively.

Based on the above information answer the following:

(i) What are the direction cosines of $\overrightarrow{MA}$.

(ii) Are ***A, G and P*** collinear? Justify your answer.

(iii) (a) Find the area of triangle ΔVLK.

OR

(iii) (b) Find the projection of $\overrightarrow{VP}$ on $\overrightarrow{KL}$.

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