केन्द्रीय विद्यालय संगठन, बेंगलूरु संभाग

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प्रथम प्री-बोर्ड परीक्षा (2024-25)

FIRST PRE-BOARD EXAMINATION (2024-25)

CLASS: XII — MAX MARKS:80

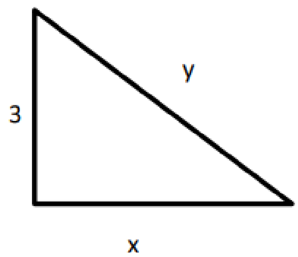
SUBJECT: MATHEMATICS — TIME: 3 HRS

SECTION-A

Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**

1. c)± 12	6. c)5	11. d) I quadrant	16. d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
2. a)4	7. a) $C(A+B')$	12. d)$-cotx - tanx + c$	17. c) 1.5
3. a)$(-\infty,-4)\cup(0,\infty)$	8. b)0.25	13. b)0	18. c) 2
4. d)8	9. c)$-\frac{1}{\sqrt{2}}\hat{\imath}-\frac{1}{\sqrt{2}}\hat{\jmath}$	14. b) not defined , 2	19. C
5. b)$\frac{1}{x}$	10. b)$\frac{7}{3}$	15. b) $\frac{\pi}{2}<y<\pi$	20 . A

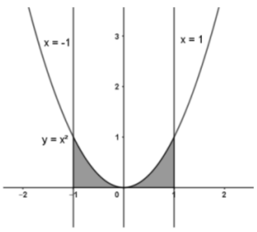
MARKING SCHEME Q No	Expected Answers/Value Points	Marks
21	$-1 \le 3x-2 \le 1$ $\Rightarrow \frac{1}{3} \le x \le 1$ or $\left[\frac{1}{3},1\right]$	1 1
22	$f'(x)=\frac{1-logx}{x^2}$ $,\therefore f'(x)=0 \Rightarrow logx=1 \Rightarrow x=e$ $f''(x)=\frac{2xlogx-3x}{x^4}$ $,f''(e)=-\frac{1}{e^3}<0$, $x=e$ is a point of local maximum	1 ½ ½
23	a) Let $x=\sin A$ and $y=\sin B$ $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1 \Rightarrow \sin A\cos B+\cos A\sin B=1$ $\Rightarrow \sin(A+B)=1$ $\Rightarrow A+B=\frac{\pi}{2}$ $\Rightarrow \sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$ Differentiating w.r.t x , we obtain $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$ b) Let $y=\tan^{-1}x$ and $z=logx$ $\frac{dy}{dx}=\frac{1}{1+x^2}$ and $\frac{dz}{dx}=\frac{1}{x}$ $\frac{dy}{dz}=\frac{\frac{1}{1+x^2}}{\frac{1}{x}}=\frac{x}{1+x^2}$	½ ½ 1 ½ + ½ ½ + ½
24	a) $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12 \Rightarrow |\vec{x}|^2-|\vec{a}|^2=12$ $\Rightarrow |\vec{x}|^2-1=12$ $\Rightarrow |\vec{x}|^2=13$	½ ½ ½

	$\Rightarrow \lvert x \rvert = \sqrt{13}$ OR b) $\vec{a}+\vec{b} = 6\hat{\imath} - 2\hat{\jmath} + (7+\lambda)\hat{k}$ and $\vec{a}-\vec{b} = -4\hat{\imath} + (7-\lambda)\hat{k}$ $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$ $\Rightarrow -24 + (7+\lambda)(7-\lambda) = 0$ $\lambda^2 = 25 \Rightarrow \lambda = \pm 5$	½ ½ 1 ½
25	$\overrightarrow{d_1} = \vec{a}+\vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a}-\vec{b} = -6\hat{\jmath} - 8\hat{k}$ Area of the parallelogram $= \frac{1}{2}\lvert\overrightarrow{d_1} \times \overrightarrow{d_2}\rvert = \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} =$ $\frac{1}{2}\lvert 4\hat{\imath} + 32\hat{\jmath} - 24\hat{k}\rvert = \lvert 2\hat{\imath} + 16\hat{\jmath} - 12\hat{k}\rvert = \sqrt{404} = 2\sqrt{101}$	½ ½ ½ + ½
26	$f(x) = sin3x \Rightarrow f'(x) = 3cos3x = 0 \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$ $f'(x) \geq 0$ for all $x \in \left[0, \frac{\pi}{6}\right] \Rightarrow f(x)$ is increasing on $\left[0, \frac{\pi}{6}\right]$ $f'(x) \leq 0$ for all $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right] \Rightarrow f(x)$ is decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$	1 1 1
27	 $x^2 + 3^2 = y^2$ $When\ y = 5\ then\ x = 4, now\ 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ $4\,(200) = 5\,\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = 160$ cm/s	½ ½ 1 1
28	a) $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{b} \times \vec{d} - \vec{c} \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ OR b) Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$ is of the form $(2\lambda + 1, 3\lambda + 2, 4\lambda + a)$ and any point on the line $\frac{x-4}{5} = \frac{y-1}{2} = z$ is of the form $(5\mu + 4, 2\mu + 1, \mu)$. Solving $2\lambda + 1 = 5\mu + 4$ and $3\lambda + 2 = 2\mu + 1$ to get $\lambda = -1$ and $\mu = -1$. The lines will be skew if , $4\lambda + a \neq \mu$ $4(-1) + a \neq (-1)$ $a \neq 3$, $\therefore a \in R - \{3\}$	½ ½ ½ ½ ½ ½ 1 ½ + ½ ½ ½

<table>
<tr><td>29</td><td>a) Let $x^{\frac{3}{2}} = t \Rightarrow \frac{3}{2}x^{\frac{1}{2}}dx = dt$
$\int \sqrt{\frac{x}{1-x^3}}\,dx = \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}\,dx$
$= \frac{2}{3}\sin^{-1} t + c = \frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right) + c$
OR
b) Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
$\int_0^1 x(1-x)^n\,dx = \int_0^1 (1-x)x^n\,dx = \int_0^1 (x^n - x^{n+1})dx$
$= \frac{x^{n+1}}{n+1}\Big]_0^1 - \frac{x^{n+2}}{n+2}\Big]_0^1$
$= \frac{1}{n+1} + \frac{1}{n+2}$</td><td>1
1
½ + ½

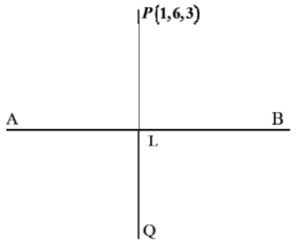
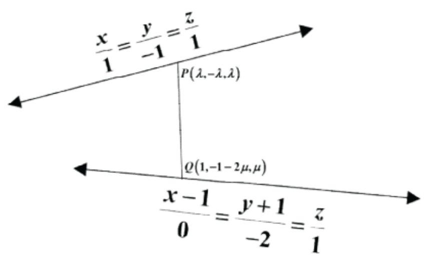
1 + ½
½
1</td></tr>
<tr><td>30</td><td>Value of z at (15, 15) = 15p + q
Value of z at (0, 20) = q
15p + q = q or p = 0</td><td>1
1
1</td></tr>
<tr><td>31</td><td>a) P(H) = $\frac{3}{4}$ P(T) = $\frac{1}{4}$
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{9}{16}$</td><td>$\frac{6}{16}$</td><td>$\frac{1}{16}$</td></tr><tr><td>X P(X)</td><td>0</td><td>$\frac{6}{16}$</td><td>$\frac{2}{16}$</td></tr></table>Mean = $\frac{8}{16}$ or $\frac{1}{2}$

b) $P(A' \cup B') = \frac{1}{4} \Rightarrow 1 - P(A \cap B) = \frac{1}{4}$
$\Rightarrow P(A \cap B) = \frac{3}{4}$
Since $P(A \cap B) \neq 0$, A and B are not mutually exclusive.
$P(A).P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq P(A \cap B)$
A and B are not independent</td><td>½
½
1
½
½

1
1
1</td></tr>
<tr><td>32</td><td>

Required area = $2\int_0^1 x^2 dx$
$= 2\frac{x^3}{3}\Big]_0^1$
$= \frac{2}{3}$</td><td>1
1
2
1</td></tr>
<tr><td>33</td><td>Assumes the number of litres of orange juice, beetroot juice and kiwi juice as x, y and z, respectively to frame equations as follows:
$500x + 20y + 800z = 1860$
$2x + 5y + 3z = 22$
$100x + 120y + 200z = 760$
Writes the above system of equations in the matrix form using AX = B</td><td>1</td></tr>
</table>

	as $\begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$ Finds |A| = 110000 ≠ 0 and hence writes that A is non-singular and has a unique solution. Finds adj A as: $\begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$ Finds A^{-1} using |A| and adj A as: $A^{-1} = \frac{1}{110000}\begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$ Writes that X = A^{-1}B and finds X as $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ Concludes that 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.	½ 2 ½ 1
34	a) $(ax+b)e^{\frac{y}{x}} = x \Rightarrow \frac{y}{x} = \log\left(\frac{x}{a+bx}\right) = logx - \log(a+bx)$ differenetaiting w.r.t. x , $\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$ $\Rightarrow \frac{x\frac{dy}{dx} - y}{x} = \frac{a}{a+bx}$---(1) $\Rightarrow x\frac{dy}{dx} - y = \frac{ax}{a+bx}$ Again differentiating w.r.t x , $\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - ax.b}{(a+bx)^2} = \frac{a^2}{(a+bx)^2}$ $\Rightarrow x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2} = \left[\frac{a}{a+bx}\right]^2 = \left[\frac{x\frac{dy}{dx} - y}{x}\right]^2$ from (1) $\Rightarrow x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$. b) $(x-a)^2 + (y-b)^2 = c^2$ Differentiating both sides w.r.t x , $2(x-a) + 2(y-b)\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$ Again differentiating w.r.t. x $\frac{d^2y}{dx^2} = -\left[\frac{(y-b) - (x-a)\frac{dy}{dx}}{(y-b)^2}\right]$	½ 1 ½ ½ 1 1 ½ ½ ½ 1 ½+ ½ ½

	$$= -\left[\frac{(y-b)-(x-a)\left\{-\frac{x-a}{y-b}\right\}}{(y-b)^2}\right] = -\frac{c^2}{(y-b)^3}$$ $$1+\left(\frac{dy}{dx}\right)^2 = 1+\left[-\frac{x-a}{y-b}\right]^2 = \frac{c^2}{(y-b)^2}$$ $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{c^3}{(y-b)^3}$$ $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$, which is independent of a and b	½ 1
35	a)Let P(1,6,3) be the given point, and let ' ' L be the foot of the perpendicular from ' ' P to the given line AB (as shown in the figure below). The coordinates of a general point on the given line are given by  Coordinates of L is $(\lambda, 1+2\lambda, 2+3\lambda)$ So the direction ratios of PL are $\lambda-1, 2\lambda-5, 3\lambda-1$ Since PL ⊥ AB, $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow \lambda = 1$ ⇒ Coordinates of L are (1,3,5) Let Q (α,β,γ) be the image , then L is the midpoint of PQ $\left(\frac{\alpha+1}{2}, \frac{\beta+6}{2}, \frac{\gamma+3}{2}\right) = (1,3,5) \Rightarrow \alpha = 1, \beta = 0, \gamma = 7$ ∴ Image of P(1,6,3) ,is (1,0,7) The distance of (1,0,7) from y axis is $\sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$ units OR b)  Let the coordinates of P be $(\lambda,-\lambda,\lambda)$) and Q be $(1,-2\mu-1,\mu)$ PQ is perpendicular to both the lines , direction ratio of PQ are	½ ½ 1 1 1 1 ½ ½

	$\lambda-1,-\lambda+2\mu+1,\lambda-\mu,$ Since PQ is perpendicular to both the lines $1(\lambda-1)+(-1)(-\lambda+2\mu+1)+1(\lambda-\mu)=0 \Rightarrow 3\lambda-3\mu=2$ -- (1) $0(\lambda-1)+(-2)(-\lambda+2\mu+1)+1(\lambda-\mu)=0 \Rightarrow 3\lambda-5\mu=2$ -- (2) Solving (1) and (2) we get $\lambda=\frac{2}{3}$ and $\mu=0$ $\therefore P\left(\frac{2}{3},-\frac{2}{3},\frac{2}{3}\right)$ and $Q(1,-1,0)$ So the required shortest distance is $\sqrt{\left(1-\frac{2}{3}\right)^2+\left(-1+\frac{2}{3}\right)^2+\left(-\frac{2}{3}\right)^2}$ $=\sqrt{\frac{2}{3}}$ units	½ ½ 1 1 1
36	(i) Let $(l_1,l_2)\epsilon R \Rightarrow l_1$is parallel to $l_2 \Rightarrow l_2$is parallel to l_1 $\Rightarrow (l_2,l_1)\epsilon R$, $\therefore R$ is symmetric. (ii) Let $(l_1,l_2),(l_2,l_3)\epsilon R \Rightarrow l_1$is parallel to l_2 and $l_2$is parallel to l_3 $\Rightarrow l_1$is parallel to l_3 $\Rightarrow (l_1,l_3)\epsilon R$, $\therefore R$ is transitive. (iii) a) The set of rail lines in R related to the line $y=3x+2$ is the set of all lines parallel to it , $\{l: l$ is a line of type $y=3x+c, c\in R\}$ OR (iii) b) Let $(l_1,l_2)\epsilon S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2,l_1)\epsilon S$ $\therefore R$ is symmetric. Let $(l_1,l_2),(l_2,l_3)\epsilon R \Rightarrow l_1 \perp l_2$ and $l_2 \perp l_3$ $\Rightarrow l_1$is parallel to l_3 $\therefore R$ is not transitive.	1 1 2 1 1
37	(i) When V = 40 km/h , F $=\frac{1600}{500}-\frac{40}{4}+14=\frac{36}{5}\ l/100km$ (ii) $\frac{dF}{dV}=\frac{2V}{500}-\frac{1}{4}=\frac{V}{250}-\frac{1}{4}$ (iii) a) For minimum $\frac{dF}{dV}=0 \Rightarrow \frac{V}{250}-\frac{1}{4}=0 \Rightarrow V=62.5$ km/h Also $\frac{d^2F}{dV^2}=\frac{1}{250}>0$ Hence F is minimum when V = 62.5 km/h OR (iii) b) $\frac{dF}{dV}=-0.01 \Rightarrow \frac{V}{250}-\frac{1}{4}=-0.01 \Rightarrow V=60$ km/h F $=\frac{3600}{500}-\frac{60}{4}+14=6.2\ \ l/100km$ Quantity of fuel required for 600 km = $6.2*6=37.2l$	1 1 1 ½ ½ ½ 1 ½
38	(i) Probability of a randomly chosen seed to germinate $=\frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}=\frac{490}{1000}=0.49$ (ii) Probability that the randomly selected seed is of type of A_1 , given that it germinates $=\frac{\frac{4}{10}\times\frac{45}{100}}{\frac{490}{1000}}=\frac{18}{49}$	1 ½ + ½ 1 +1