KENDRIYA VIDYALAYA SANGATHAN , CHENNAI REGION

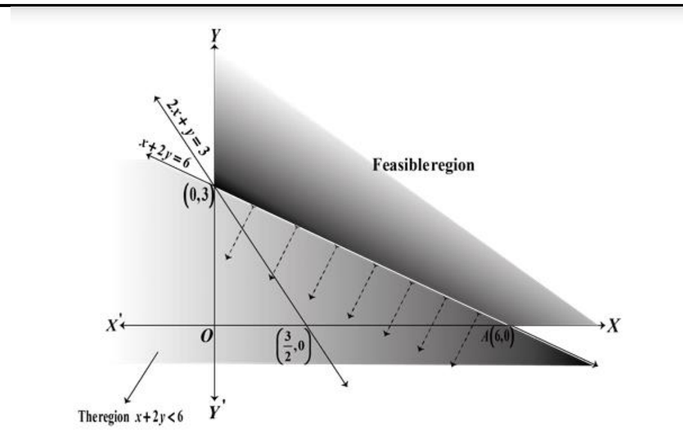
PRE BOARD-I EXAM 2024-25

Class: XII Session: 2024-25
Subject: Mathematics (041)
Marking Scheme (Theory)

S. No.	Objective Type Question Sec A	Marks
1	(b) no such x and y possible	1
2	(a) $k = 3, p = n$	1
3	(d) (0, 2)	1
4	(b) B	1
5	(d) $y = -e^x + c$	1
6	(d) – 25	1
7	(a) Skew symmetric matrix	1
8	(b) P(A′B′) = [1 – P(A)] [1 – P(B)]	1
9	(d) $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}$	1
10	(c) 5	1
11	(b) half plane that neither contains the origin nor the points on the line 2x + 3y=6	1
12	(b) $e^x \sec x + c$	1
13	(a) 0	1
14	(d) 0	1
15	(d) $\frac{1}{2}$	1
16	(c) z is maximum at (40,15), minimum at (15,20)	1
17	(b) $\frac{3}{4t}$	1
18	(d) $\frac{5}{2}$ sq units	1
19	(d) Assertion is incorrect, reason is correct.	1
20	(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.	1
	SECTION B **VERY SHORT ANSWER TYPE QUESTIONS(VSA)** **(Each question carries 2 marks)**	
21	Required value $= \frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$ $= \frac{5\pi}{4}$	1 1

22	As f is continuous at $x=2 \Rightarrow \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$ $$\lim_{x\to 2^+} 3x = \lim_{x\to 2^-} (2x+2) = k$$ $$\Rightarrow k = 6$$ OR Here, $\frac{ax}{d\theta} = a(1-\cos\theta)$ and $\frac{dy}{d\theta} = a(-\sin\theta)$ $\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{(1-\cos\theta)}$ $\therefore \left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{3}} = \frac{-\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{-\sqrt{3}/2}{1-(1/2)} = -\sqrt{3}$	1 1 1 1
23	$y^2 = 8x$ then $2y\frac{dy}{dt} = 8\frac{dx}{dt}$ $\frac{dy}{dt} = \frac{dx}{dt}$(given) Solving above 2y=8 Y=4 Solving we get point (2,4)	1 1
24	The projection of vector $\vec{a}$ on the vector $\vec{b}$ is given by $\frac{1}{\|\vec{b}\|}(\vec{a}\cdot\vec{b}) = \frac{(2\times1+3\times2+2\times1)}{\sqrt{(1)^2+(2)^2+(1)^2}} = \frac{10}{\sqrt{6}} = \frac{5}{3}\sqrt{6}$ ***OR*** $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & -1 & 1\\ 2 & -2 & 1\end{vmatrix} = \hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ and $\|\vec{a}\times\vec{b}\| = \sqrt{1^2+(-2)^2+(-6)^2} = \sqrt{41}$ Unit vector along $\vec{a}\times\vec{b} = \frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|} = \frac{\hat{\imath}-2\hat{\jmath}-6\hat{k}}{\sqrt{41}} = \frac{1}{\sqrt{41}}\hat{\imath} - \frac{2}{\sqrt{41}}\hat{\jmath} - \frac{6}{\sqrt{41}}\hat{k}$	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$
25	$\|\vec{a}+\vec{b}+\vec{c}\|^2 = 0 \Rightarrow \|\vec{a}\|^2 + \|\vec{b}\|^2 + \|\vec{c}\|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$ $\Rightarrow 3 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0 \Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -\frac{3}{2}$	1 1
	SECTION C **SHORT ANSWER TYPE QUESTIONS(SA)** **(Each question carries 3 marks**	
26	f'(x)= $\frac{3}{2}\times 4x^3 - 12x^2 - 90x$ $\Rightarrow f'(x) = 6x^3 - 12x^2 - 90x = 6x(x^2-2x-15) = 6x(x-5)(x+3)$	1

	For strictly increasing, $f'(x) > 0$ $\Rightarrow \quad 6x(x-5)(x+3) > 0$ $\Rightarrow \quad x \in (-3,0) \cup (5,\infty)$ For strictly decreasing, $f'(x) < 0$ $\Rightarrow \quad 6x(x-5)(x+3) < 0$ $\Rightarrow \quad x \in (-\infty,-3) \cup (0,5)$	1 1
27	Let V and S be the volume and surface area of a cube of side x cm respectively. Given $\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$ We require $\left.\frac{dS}{dt}\right]_{x=10 \text{ cm}}$ Now $V = x^3$ $\Rightarrow \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \Rightarrow 9 = 3x^2 \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{9}{3x^2} = \frac{3}{x^2}$ Again, $\because S = 6x^2$ [By formula for surface area of a cube] $\Rightarrow \frac{dS}{dt} = 12 \cdot x \cdot \frac{dx}{dt} = 12x \cdot \frac{3}{x^2} = \frac{36}{x}$ $\Rightarrow \left.\frac{dS}{dt}\right]_{x=10 \text{ cm}} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{sec}.$	1 1 1
28	W $x^3 - x \geq 0$ on $[-1,0]$ and $x^3 - x \leq 0$ on $[0,1]$ and that $x^3 - x \geq 0$ on [1, 2]. $\int_{-1}^{2} \lvert x^3 - x\rvert dx = \int_{-1}^{0} (x^3 - x)dx + \int_{0}^{1} -(x^3 - x)dx + \int_{1}^{2} (x^3 - x)dx$ $= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$ $= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{4} + \frac{1}{2} = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4}$ **OR** Let $x + 3 = A\frac{d}{dx}(5 - 4x - 2x^2) + B = A(-4 - 4x) + B$ On comparing the coefficients of like term, We have $A = -\frac{1}{4}$ and $B = 2$ $\Rightarrow I = -\frac{1}{2}\sqrt{5 - 4x - 2x^2} + \sqrt{2}\sin^{-1}\left[\sqrt{\frac{2}{7}}(x+1)\right] + C$	1 1 1 1 1 1

29	Let $\vec{d} = x\hat{\imath} + y\hat{\jmath} + z$ $\vec{d}$ is perpendicular to $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$ $\therefore \vec{d} \cdot \vec{b} = 0$ and $\vec{d} \cdot \vec{c} = 0$ hence $x - 4y + 5z = 0$ (1) and $3x + y - z = 0$ …….(2) Also, $\vec{d} \cdot \vec{a} = 21$, where $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$ $\Rightarrow 4x + 5y - z$=21 …….(3) Eliminating z from (i) and (ii), we get $16x + y = 0$ Eliminating z from (ii) and (iii), we get $x + 4y = 21$ Solving we get $x = \frac{-1}{3}, y = \frac{16}{3}$ and $z = \frac{13}{3}$ $\therefore \bar{d} = \frac{-1}{3}\hat{\imath} + \frac{16}{3}\hat{\jmath} + \frac{13}{3}\hat{k}$ is the required vector. OR Any point on the line (1) is $(3r - 1, 5r - 3, 7r - 5)$ Any point on the line(2) is$(k + 2, 3k + 4, 5k + 6)$ Let these points be common point of intersection So $3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$ On solving these, we get $r = \frac{1}{2}, k = -\frac{3}{2}$ $\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$	 ½ ½ 1/2 ½ ½ ½ 1 1
30	 The corner points of the **unbounded** feasible region are $A(6,0)$ and $B(0,3)$. The values of Z at these corner points are as follows:	1

Corner point	Value of the objective function $Z = x + 2y$
$A(6,0)$	6
$B(0,3)$	6

1

We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z = 6$. 1

It can be seen that the value of Z at points **A** and **B** is same. If we take any other point on the line $x + 2y = 6$ such as (2,2) on line $x + 2y = 6$, then $Z = 6$.

Thus, the minimum value of Z occurs for more than 2 points, and is equal to 6 .

31

Let $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$ 1

Required probability $= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$ $\frac{1}{2} + \frac{1}{2}$

$$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$$ 1

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{5}{6} = 1 - \frac{5}{12} = \frac{7}{12}$$

OR

Solution Let E be the event that the man reports that six occurs in the throwing of the die and let S_1 be the event that six occurs and S_2 be the event that six does not occur.

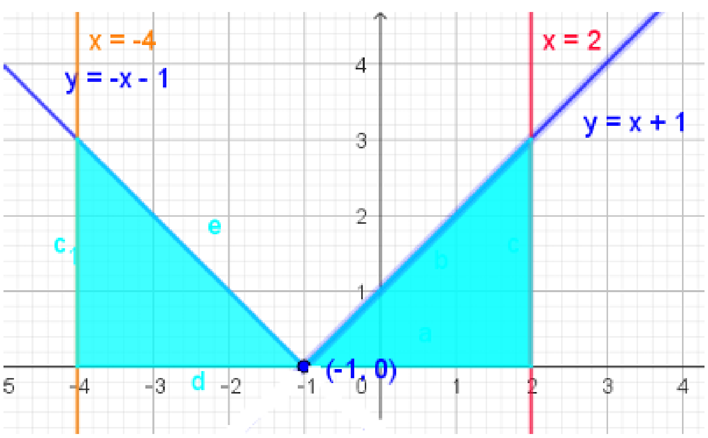
Then $P(S_1) = \frac{1}{6}$, $P(S_2) = \frac{5}{6}$ 1

$P(E|S_1)$= Probability that the man speaks the truth $= \frac{3}{4}$ 1

$P(E|S_2)$= Probability that the man does not speak the truth $= 1 - \frac{3}{4} = \frac{1}{4}$

Now $P(S_1 | E) = \frac{P(S_1)P(E|S_1)}{P(S_1)P(E|S_1) + P(S_2)P(E|S_2)}$

1

	$=\frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}=\frac{1}{8}\times\frac{24}{8}=\frac{3}{8}$	
	<u>SECTION D</u> <u>LONG ANSWER TYPE QUESTIONS(LA)</u> (Each question carries 5 marks)	
32	$\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}=\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}$ Gives identity matrix of order 3. Hence $\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}^{-1}=\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$ Now the system of equation can be written in the form AX = B gives x =0, y = 5 and z = 3	2 1 1 1
33	$A=\left\|\int_{-4}^{-1}(x+1)\,dx\right\|+\int_{-1}^{2}(x+1)\,dx = 9$ sq.un 	3 2
34	$\frac{dy}{dt}=\frac{(\sqrt{\cos 2t})\cdot\frac{d}{dt}(\cos^3 t)-(\cos^3 t)\cdot\frac{d}{dt}(\sqrt{\cos 2t})}{(\sqrt{\cos 2t})^2}$ Simplifying:	$1\frac{1}{2}$

$\frac{dy}{dt}=\frac{-3\cos^2 t\sin t\sqrt{\cos 2t}+\cos^3 t\cdot\frac{\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$

And $\frac{dx}{dt}=\frac{(\sqrt{\cos\ 2t})\cdot\frac{d}{dt}(\sin^3\ t)-(\sin^3\ t)\cdot\frac{d}{dt}(\sqrt{\cos\ 2t})}{(\sqrt{\cos\ 2t})^2}$

Simplifying:

$\frac{dx}{dt}=\frac{3\sin^2\ t\cos\ t\sqrt{\cos\ 2t}+\sin^3\ t\cdot\frac{\sin\ 2t}{\sqrt{\cos\ 2t}}}{\cos\ 2t}$ $1\frac{1}{2}$

Now substituting $\frac{dy}{dt}$ and $\frac{dx}{dt}$ into the formula for $\frac{dy}{dx}$:

This simplifies to:

$$\frac{dy}{dx}=\frac{-3\cos^2\ t\sin\ t\sqrt{\cos\ 2t}+\cos^3\ t\cdot\frac{\sin\ 2t}{\sqrt{\cos\ 2t}}}{3\sin^2\ t\cos\ t\sqrt{\cos\ 2t}+\sin^3\ t\cdot\frac{\sin\ 2t}{\sqrt{\cos\ 2t}}}$$ 2

$\frac{dy}{dx}=\frac{-(4cos^3x-3\cos x)}{3sinx-4sin^3x}$ =-cotx

OR

Left-Hand Limit = $\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\frac{\sin\ (a+1)x+\sin\ x}{x}$

Using the limit properties, we can rewrite this as:

$$\lim_{x\to0^-}\left(\frac{\sin\ (a+1)x}{(a+1)x}\times(a+1)+\frac{sinx}{x}\right)$$

$$\lim_{x\to0^-} f(x)=(a+1)+1=a+2$$

Right-Hand Limit $\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}}$ 2

To simplify this, we can factor out $x^{1/2}$:

= $\lim_{x\to0^+}\frac{x^{1/2}((1+bx)^{1/2}-1)}{xbx^{1/2}}$

= $\lim_{x\to0^+}\frac{(1+bx)^{1/2}-1}{(1+bx)-1}$ 2

=$\frac{1}{2}(1)^{\frac{1}{2}-1}$ = $\frac{1}{2}$

And f(0)=c

now $\lim_{x\to0^-} f(x)$= $\lim_{x\to0^+} f(x)$= f(0)

1

	⇨ $a+2=\frac{1}{2}=c$ hence $a=-\frac{3}{2}$, $c=\frac{1}{2}$ and b is any non zero real no. ⇨	
35	$$\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+3\hat{k}\ ,\ \overrightarrow{a_2}=2\hat{\imath}+4\hat{\jmath}+5\hat{k}$$ $$\overrightarrow{b_1}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}\ ,\ \overrightarrow{b_2}=3\hat{\imath}+4\hat{\jmath}+5\hat{k}$$	1
	$$\overrightarrow{a_2}-\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$$ $$\overrightarrow{b_1}\times\overrightarrow{b_2}=-\hat{\imath}+2\hat{\jmath}-\hat{k}$$	1 1
	$$\left\|\overrightarrow{b_1}\times\overrightarrow{b_2}\right\|=\sqrt{6}$$ $$(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})=1$$ S.D. = d = $\left\| \frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{\left\|\overrightarrow{b_1}\times\overrightarrow{b_2}\right\|} \right\|$ Shortest distance d = $\frac{1}{\sqrt{6}}$ The lines do not intersect	2
	OR Eq. of line $\vec{r}=\vec{a}+\lambda\vec{b}$ Line passes through (1,2,-4) and let (a,b,c) be theD,Ratio of line thenEq of line is $$\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(a\hat{\imath}+b\hat{\jmath}+c\hat{k})$$ Line is perpendicular to the lines $$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}\quad and\quad \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$	2
	$$\vec{a}\times\vec{b}\ is\ perpendicular\ to\ \vec{a}\ and\ \vec{b}\ both$$ $$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}$$ $= 24\hat{\imath}+36\hat{\jmath}+72\hat{k}$ Hence D' Ratio of line is 24,36,72 Eq. of line $\frac{x-1}{24}=\frac{y-2}{36}=\frac{z+4}{72}$	
	$$\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(24\hat{\imath}+36\hat{\jmath}+72\hat{k})$$	2
	$$\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$$	1

	SECTION E **(3 case study questions carries 4 marks each)**	
36	Ans a) R is reflexive and transitive but not symmetric. b) No. Because $n(B)$ is greater than $n(A)$ c) R is neither reflexive nor symmetric nor transitive OR No. of relations $= 2^{12}$	2 1 1 1
37	1) Area,A(x) =$(10 + x)\sqrt{100-x^2}$ 2) A'(x)= $\frac{-2x^2-10x+100}{\sqrt{100-x^2}}$ 3) Put A'(x)=0 we get x $= 5$ A''(x)= $\frac{\sqrt{100-x^2}(-4x-10)-(-2x^2-10x+100)\frac{(-2x)}{2\sqrt{100-x^2}}}{100-x^2}$ $=\frac{2x^3-300x-1000}{(100-x^2)^{\frac{3}{2}}}$ (on simplification) A''(5)= $\frac{2(5)^3-300(5)-1000}{(100-(5)^2)^{\frac{3}{2}}} = \frac{-2250}{75\sqrt{75}} = \frac{-30}{\sqrt{75}} < 0$ thus, area of trapezium is maximum at $x = 5$ OR Maximum area of trapezium ,$A(5) = (5+10)\sqrt{100-(5)^2} = 15\sqrt{75} = 75\sqrt{3}$ cm^2	1 1 2 2
38	ANS a) K=0.14 b) A: even face is turn up B: face show 2 or 4 S=$\{1, 2, 3, 4, 5, 6\}$, P(A)=P(2)+P(4)+P(6)=0.24+0.18+0.14=0.56 B={2,4} B∩A={2, 4} P(B∩A)= P(2)+P(4)= 0.24+0.18=0.42 P(B/A)=$\frac{P(B\cap A)}{P(A)}=\frac{0.42}{0.56}=\frac{3}{4}$ =0.75	2 2