

## FIRST PRE BOARD EXAMINATION 2024-25

## XII MATHEMATICS TIME:3 HOURS

GENERAL INSTRUCTIONS MAX MARKS : 80

Read the following instructions very carefully and strictly follow them:

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**
  - (ii) This Question paper is divided into **five** Sections-**A, B, C, D** and **E**.
  - (iii) In **Section A**, Questions no. **1 to 18** are **multiple choice questions (MCQs)** and Questions no. **19 and 20** are **Assertion-Reason based** questions of **1 mark each**.
  - (iv) In **Section B**, Questions no. **21 to 25** are **Very Short Answer (VSA)-type** questions, carrying **2 mark each**.
  - (v) In **Section C**, Questions no. **26 to 31** are **Short Answer (SA)-type** questions, carrying **3 mark each**.
  - (vi) In **Section D**, Questions no. **32 to 35** are **Long Answer (LA)-type** questions, carrying **5 mark each**.
  - (vii) In **Section E**, Questions no. **36 to 38** are **Case study-based** questions, carrying **4 mark each**.
  - (viii) There is **no overall choice**. However, an **internal choice** has been provided in **2** questions in **Section B**, **3** questions in **Section C**, **2** questions in **Section D** and **one** subpart each in **2** questions of **Section E**.
  - (ix) Use of calculator is **not allowed**
- SECTION A [1×20=20]**

1. If  $A$  is a square matrix of order 3, and  $|adj A| = 729$ , then  $|A|$  is equal to

- (a) 3                      (b) 9                      (c)  $\pm 81$                       (d) None of these

2. Value of  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  is

- (a)  $\tan^{-1}x$                       (b)  $2 \tan^{-1}x$                       (c)  $\frac{1}{2} \tan^{-1}x$                       (d) None of these

3. If  $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$ , then find the value of  $x$  and  $y$  are

- (a)  $x=3, y=1$                       (b)  $x=2, y=3$                       (c)  $x=2, y=4$                       (d)  $x=3, y=3$

4. If  $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$  then the value of  $x$  is

- (a) 3                      (b)  $\pm 3$                       (c)  $\pm 6$                       (d) 6

5. If there are two values of  $p$  for which makes determinant,  $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & p & -1 \\ 0 & 4 & 2p \end{vmatrix} = 86$ , then the sum of these number is

- (a) 4                      (b) 5                      (c) -4                      (d) 9

6. If  $A$  is a square matrix and  $A^2 = A$ , then  $(I + A)^2 - 3A$  is equal to

- (a)  $I$                       (b)  $A$                       (c)  $2A$                       (d)  $3I$

7. The function  $f(x) = \frac{x}{2} + \frac{2}{x}$  has a local minima at  $x$  is equal to

- (a)  $2$                       (b)  $1$                       (c)  $0$                       (d)  $-2$

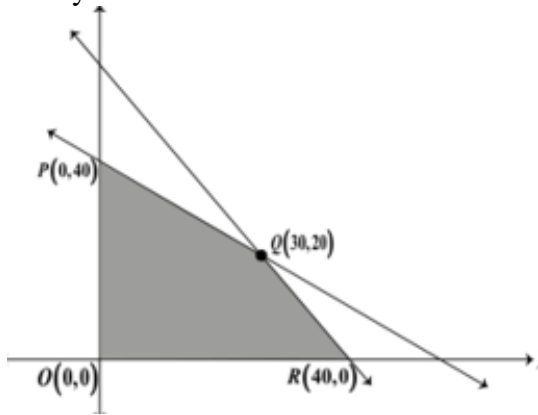
8.  $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$  is equal to

- (a)  $2\sqrt{\cot x}$               (b)  $\frac{\sqrt{\tan x}}{2}$               (c)  $\frac{2}{\sqrt{\tan x}}$               (d)  $2\sqrt{\tan x}$

9.  $\int_0^{\sqrt{5}} [x] dx$  is

- (a)  $2\sqrt{5}$                       (b)  $2\sqrt{5} - 1$               (c)  $2\sqrt{5} - 2$                       (d)  $2\sqrt{5} - 3$

10. For the linear programming problem(LPP), the objective function is  $Z = x + y$ . The feasible region determined by a set of constraints is shown in the graph



Which of the following statements is true?

(a) Maximum value of Z is at P (40,0)                      (b) Maximum value of Z is at Q(30,20)

(c) Value of Z at Q (30,20) is less than value of Z at R (40,0)

(d) Value of Z at R(40,0) is less than value of Z at P(40,0)

11. The solution of differential equation  $\tan y \sec^2 x + \tan x \sec^2 y = 0$  is

(a)  $\tan x + \tan y = C$     (b)  $\tan x - \tan y = C$     (c)  $\tan x \cdot \tan y = C$     (d)  $\frac{\tan x}{\tan y} = C$

12. Find the integrating factor of  $(1 - x^2) \frac{dy}{dx} - xy = 1$

(a)  $-x$                       (b)  $\frac{x}{1+x^2}$                       (c)  $\sqrt{1-x^2}$                       (d)  $\frac{1}{2} \log(1-x^2)$

13. If  $|\vec{a}| = 10$ ,  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = 12$ , then the value of  $|\vec{a} \times \vec{b}|$  is

(a) 5                      (b) 10                      (c) 14                      (d) 16

14. If A and B are two events such that  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B/A) = 0.6$

then  $P(A \cup B)$  is equal to

(a) 0.24                      (b) 0.3                      (c) 0.48                      (d) 0.96

15. The lines  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and  $\frac{x-1}{-2} = \frac{y+3}{-4} = \frac{z-5}{-6}$  are

(a) parallel                      (b) Intersecting                      (c) skew                      (d) coincident

16. If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ ,  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$ ,  $|\vec{c}| = 7$  then the angle between  $\vec{a}$  and  $\vec{b}$  is

(a)  $\frac{\pi}{6}$     (b)  $\frac{2\pi}{3}$     (c)  $\frac{5\pi}{3}$     (d)  $\frac{\pi}{3}$

17. Area of the region bounded by the curve  $y = \cos x$  between  $x = 0$  and  $x = \pi$  is

(a) 1 sq units                      (b) 2 sq units                      (c) 3 sq units                      (d) 4 sq units

18. If the function  $f(x) = \begin{cases} 4 \times 5^x & x < 0 \\ 8a + x & x \geq 0 \end{cases}$  is continuous, then the value of a is

(a)  $\frac{1}{2}$                       (b) 2                      (c) 2                      (d) 4

### ASSERTION-REASON BASED QUESTIONS

**Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.**

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A)  
 (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).  
 (c) (A) is true but (R) is false  
 (d) (A) is false but (R) is true.

19. **Assertion (A):** A function  $f : A \rightarrow B$  can not be onto if  $n(A) < n(B)$

**Reason (R):** A function  $f$  is onto if every element of co-domain has at least one pre image in the domain

20. **Assertion (A):** If  $f(x) = |\cos x|$ , then  $f'(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$  and  $f'(\frac{3\pi}{4}) = \frac{1}{\sqrt{2}}$

**Reason (R) :**  $f(x) = |\cos x| = \begin{cases} \cos x & x \in (0, \frac{\pi}{2}) \\ -\cos x & x \in (\frac{\pi}{2}, \pi) \end{cases}$

**SECTION B**

**[2 X 5 = 10]**

21. Solve the equation  $\cos(\tan^{-1} x) = \sin(\cot^{-1} \frac{3}{4})$

22. Differentiate  $\frac{x}{\sin x}$  with respect to  $\sin x$

23. Find  $\frac{dy}{dx}$ , if  $\tan^{-1}(x^2 + y^2) = a$

OR

Find the derivative of  $(\sin x)^{\cos x}$

24. Find the unit vector in the direction of the sum of the vectors

$\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$

OR

Find  $|\vec{a} \times \vec{b}|$ , if  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

25. Find  $\lambda$  when the projection of  $\vec{p} = \lambda\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{q} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is 4 units.

**SECTION C**

**[ 6 X 3 = 18 ]**

26. Sand is pouring from a pipe at the rate of  $12 \text{ cm}^3/\text{sec}$ . The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing with the height is 4 cm

27. Prove that  $f(x) = \sin x + \sqrt{3} \cos x$  has maximum value at  $x = \frac{\pi}{6}$

28. Find the vector equation of the line passing through the point (1,2,-4) and perpendicular

to the two lines  $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

OR

Find shortest distance between two lines  $\vec{r} = \hat{i} + \hat{j} + \lambda (2\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu (3\hat{i} - 5\hat{j} + 2\hat{k})$

29. Find  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$

OR

Evaluate  $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$

30. Maximise  $Z = x + y$  subject to the constraints

$$x + 4y \leq 8, \quad 2x + 3y \leq 12, \quad 3x + y \leq 9, \quad x \geq 0 \text{ and } y \geq 0$$

31. A letter is known to have come either from 'TATA NAGAR' or from 'CALCUTTA'.

On the envelop, just two consecutive letters TA is only visible. What is the probability that the letter came from 'TATA NAGAR'?

OR

Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice.

**SECTION D**

[ 4 X 5 = 20 ]

32. Find the area of the region bounded by the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

OR

Find the area of the region bounded by the curve  $y = \sqrt{16 - x^2}$  and X - axis

33. If  $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$  then find BA and use this to solve the

system of equations,  $x - y = 3$ ,  $2x + 3y + 4z = 17$ ,  $y + 2z = 7$

34. If  $(x - a)^2 + (y - b)^2 = c^2$ , for some  $c > 0$ , prove that  $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$  is a constant

independent of a and b

OR

If  $y = e^{a \cos^{-1} x}$ ,  $-1 \leq x \leq 1$ , show that  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

35. Find the foot of the perpendicular from the point  $(2, 3, -8)$  with respect to the line

$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ . Also find the perpendicular distance from the given point to the line

**SECTION E**

**[ 4 X 3 = 12 ]**

36. The total cost  $C(n)$  of manufacturing  $n$  earphone sets per day in the house of Spark Electronic Limited is given by  $C(n) = 400 + 4n + 0.0001n^2$  dollars.

Each earphone set is sold at  $q = 10 - 0.0004n$  dollars, where  $n \geq 0, q \geq 0$ .

The daily profit in dollars is determined by the equation  $P(n) = qn - C(n)$

Based on the above information answer the following questions:

- (i) The marginal cost,  $M(n)$  is the change in total production cost that comes from making or producing one additional unit. It is determined by the instantaneous rate of change of the total cost. Find the marginal cost  $M(n)$  of 10 earphone sets.
- (ii) What quantity of daily production maximizes the profit

37. A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.



From this situation answer the following:

- (i) Let the target is hit by A and B: the target is hit by B and, C: the target is hit by A and C. Then, what is the probability that A, B and, C all will hit,
  - (ii). Referring to (i), what is the probability that B, C will hit and A will lose?
  - (iii) With reference to the events mentioned in (i), what is the probability that ‘any two of A, B and C will hit?’
- OR**
- (iv) What is the probability that ‘none of them will hit the target’?

38. Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin’s sister Raji observed and noted the possible outcomes of the throw every time belongs to set  $\{1,2,3,4,5,6\}$ . Let A be the set of players while B be the set of all possible outcomes.



$A = \{S, D, B = \{1,2,3,4,5,6\}$

- (i) Verify that  $R : B \rightarrow B$  be defined by  $R = \{(x, y) : y \text{ is divisible by } x\}$  is an equivalence relation or not
- (ii) Let  $R: B \rightarrow B$  be defined by  $R = \{(1,1), (1,2), (2,2), (3,3), (4,4), (5,5), (6,6)\}$ , then verify  $R$  is reflexive, symmetric or transitive
- (iii) Raji wants to know the number of functions from  $A$  to  $B$ . How many number of functions are possible?

**OR**

- (iv) Raji wants to know the number of relations possible from  $A$  to  $B$ . How many numbers of relations are possible?

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