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FIRST PRE BOARD EXAMINATION 2024-25 12PB24MAT02 MS

XII MATHEMATICS

ANSWER KEY

SECTION A [20 X 1 = 20]

1. d
2. c
3. b
4. c
5. c
6. a
7. a
8. d
9. d
10. b
11. c
12. c
13. d
14. d
15. a
16. d
17. b
18. a
19. a
20. a

SECTION B [5 X 2 = 10]

21.cos $\left(cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$ = sin $\left(sin^{-1}\frac{4}{5}\right)$ 1/2

$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$ 1/2

16 x ^{2} = 9 1/2

x = $\pm\frac{3}{4}$ 1/2

22 Let u $=\frac{x}{\sin x}$ v = sin x 1/2

$\frac{du}{dx} = \frac{sinx - x\cos x}{sin^2 x}$ $\frac{dv}{dx}$ = cos x 1

$\frac{du}{dv} = \frac{tanx - x}{sin^2 x}$ 1/2

23.tan $^{-1}$ ($x^2 + y^2$) = a

differentiating $2x+2y\frac{dy}{dx} = 0$ $1\frac{1}{2}$

$\frac{dy}{dx} = -\frac{x}{y}$ 1/2

OR

Let $y = (\sin x)^{\cos x}$

Taking log on both sides log y = cosx log sinx 1/2

Differentiating , $\frac{dy}{dx} = (\sin x)^{\cos x}\left(\frac{cos^2x}{\sin x} - sinx \,.\log(\sin x)\right)$ $1\frac{1}{2}$

24 . $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

Let $\vec{c} = \vec{a} + \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ 1

$|\vec{c}| = \sqrt{29}$ 1/2

$\hat{c} = \frac{1}{\sqrt{29}}(4\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$ 1/2

OR

$\vec{a}X\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = -17\hat{\imath} + 13\hat{\jmath} + 7\hat{k}$ $1\frac{1}{2}$

$\left|\vec{a}X\vec{b}\right| = \sqrt{507}$ 1/2

25. Given $|\vec{a}| = 10$, $\left|\vec{b}\right| = 2$ and $\vec{a}\,.\vec{b} = 12$

$\vec{a}\,.\vec{b}$ = a b cos θ = 12 1/2

cos $\theta = \frac{3}{5}$ 1/2

sin $\theta = \pm\frac{4}{5}$ 1/2

$\left|\vec{a}X\vec{b}\right|$ = a b sin θ = 16½

SECTION C [6 X 3 = 18]

26.Volume of cone = $\frac{1}{3}\pi r^2 h$, 1/2

given r = 6 h & $\frac{dV}{dt}$ = 12 cm ^{3} / sec

Then V = $12\pi h^3$ 1

$\frac{dV}{dt} = 36\pi h^2\frac{dh}{dt}$ 1

$\frac{dh}{dt} = \frac{1}{48\pi}\ cm/sec$ 1/2

27. $f(x) = \sin x + \sqrt{3}\cos x$

$f'(x) = \cos x - \sqrt{3}\sin x$ 1/2

$f'(x) = 0$, $\tan x = \frac{1}{\sqrt{3}}$ or $x = \frac{\pi}{6}$ 1

at $x = \frac{\pi}{6}$ $f''(x) = -2 < 0$ 1

Hence f(x) has maximum value at x = $\frac{\pi}{6}$½

28.Required equation of the line $is\ \vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$

Above equation and given equations are perpendicular

Hence $3x-16y+7z = 0$ and $3x + 8y - 5z = 0$

Solving $\frac{x}{24} = \frac{y}{36} = \frac{z}{72}$ or $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$

Equation of the required line is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

OR

$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and

$\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

Comparing with standard form $\vec{a1} = \hat{\imath} + \hat{\jmath}$ $\vec{b1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{a2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ $\vec{b2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ 1

$\vec{a2} - \vec{a1} = \hat{\imath} - \hat{k}$ $\vec{b1} \times \vec{b2} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$ 1

$|\vec{b1} \times \vec{b2}| = \sqrt{59}$ 1/2

Hence substituting in formula

Shortest distance = $\frac{10}{\sqrt{59}}$

29. $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$

Let $I = \int_0^{\frac{\pi}{4}} \log\,(1 + \tan x)\,dx$

Using $P_4 : \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore \quad I = \int_0^{\frac{\pi}{4}} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx \quad ...(1)$

$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}.\tan x}\right] dx$ $\left(\tan(a-b) = \frac{\tan a - \tan b}{1+\tan a \tan b}\right)$

$I = \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{1 - \tan x}{1 + 1.\tan x}\right] dx$ *(As* $\tan\left(\frac{\pi}{4}\right) = 1$*)*

$I = \int_0^{\frac{\pi}{4}} \log\left[\frac{1 - \tan x + 1 - \tan x}{1 + \tan x}\right] dx$

$I = \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1 + \tan x}\right] dx$

Using $\log\left(\frac{a}{b}\right) = \log a - \log b$

$I = \int_0^{\frac{\pi}{4}} [\log 2 - \log(1 + \tan x)]\,dx$

$I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\,dx$

$I = \frac{\pi}{8} \log 2$

OR

I $= \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}$ dx

$= \int \frac{1 - 3\sin^2 x\cos^2 x}{\sin^2 x\cos^2 x}$dx using the formula of $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

= splitting and simplifying

I = tan x – cot x - 3x + C

30. Given constraints are $x + 4y \leq 8$, $2x + 3y \leq 12$, $3x + y \leq 9$, $x \geq 0$ and $y \geq 0$

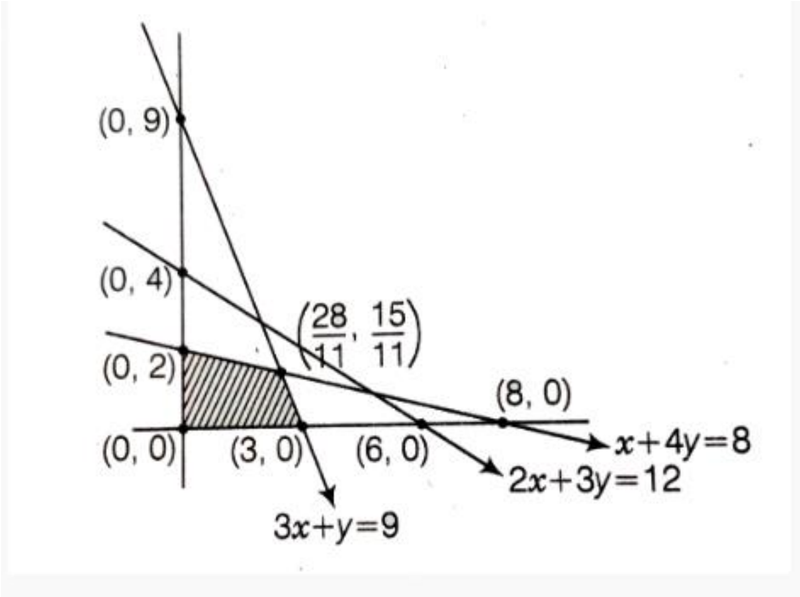


Figure 2 M

Max value of Z = x+ y at (28/11, 15/11) is given by 33/11 1 M

31. Let E_1 be the event that letter is from TATA NAGAR and E_2 be the event that letter from CALCUTTA 1/2

Let A be the event that on the letter, two consecutive letters are visible 1/2

$P(A/E_1) = 2/8$ $P(A/E_2) = 1/7$ 1

Since if letter is from TATA NAGAR two consecutive letters visible are

{ TA,AT,TA,AN,NA,AG,GA,AR}

Using Baye's theorem $P(E_1/A) = 7/11$ 1

OR

Let X is the random variable score obtained when a die is thrown twice.

X = 1,2,3,4,5,6 1

S ={(1,1),(1,2),(2,1),(2,2),(1,3),(2,3),(3,1),(3,2),(3,3)………(6,6) } ½

Required distribution is $1\frac{1}{2}$

X	1	2	3	4	5	6
P(X)	1/36	3/36	5/36	7/36	9/36	11/36

SECTION D (5 X 4 = 20)

32 . Given ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ figure 1 M

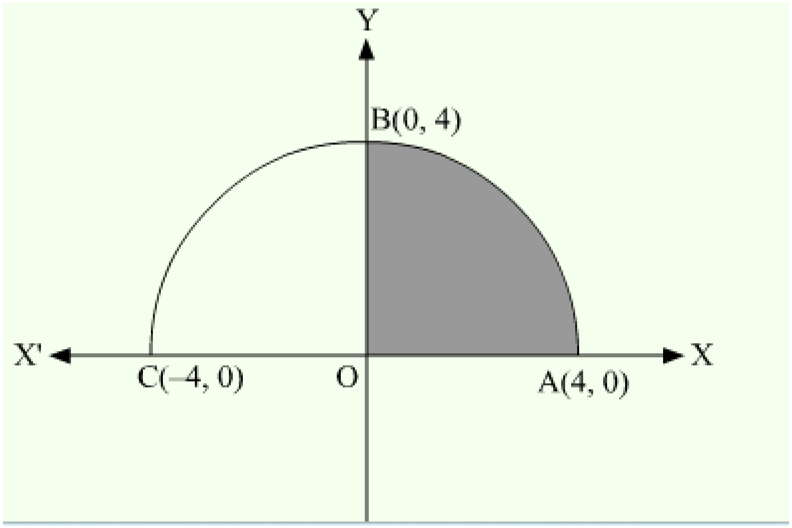
$Y = \frac{3}{4}\sqrt{16 - x^2}$ 1

Required area= 4 area in the first quadrant 1/2

$= 4\int_0^4 \frac{3}{4}\sqrt{16-x^2}dx = 12\pi$ sq units $2\frac{1}{2}$

OR

Given curve $y = \sqrt{16-x^2}$



Required area = 2 shaded area = $2\int_0^4 \sqrt{16-x^2}\,dx$

The area of the region CABC = 2(Area of the region OABO)

Thus,

$$\text{Required area} = 2\int_0^4 (y)\,dx$$
$$= 2\int_0^4 \left(\sqrt{16-x^2}\right) dx$$
$$= 2\left(\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right)_0^4$$
$$= 2\left[\left(\frac{4}{2}\sqrt{16-4^2} + 8\sin^{-1}\frac{4}{4}\right) - \left(\frac{0}{2}\sqrt{16-0^2} + 8\sin^{-1}\frac{0}{4}\right)\right]$$
$$= 2\left[\left(2\sqrt{16-16} + 8\sin^{-1}1\right) - (0+0)\right]$$
$$= 2\left[\left(0 + 8\times\frac{\pi}{2}\right)\right]$$
$$= 2[(4\pi)]$$
$$= 8\pi$$

Thus, Area $= 8\pi$ sq. units

33. $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

BA =6 I 1

Given system can be written as $\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\17\\7\end{bmatrix}$ 1

$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}^{-1}\begin{bmatrix}3\\17\\7\end{bmatrix}$ 1

$= \frac{1}{6}\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix}$ 1

$= \begin{bmatrix}2\\-1\\4\end{bmatrix}$ 1

x = 2, y = -1 z = 4

34.$(x-a)^2 + (y-b)^2 = c^2$

$\frac{dy}{dx} = \frac{a-x}{y-b}$ $1\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\frac{c^2}{(y-b)^3}$ $1\frac{1}{2}$

Substituting above values $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = -c$, a constant 2

OR

y = $e^{a\cos^{-1}x}$

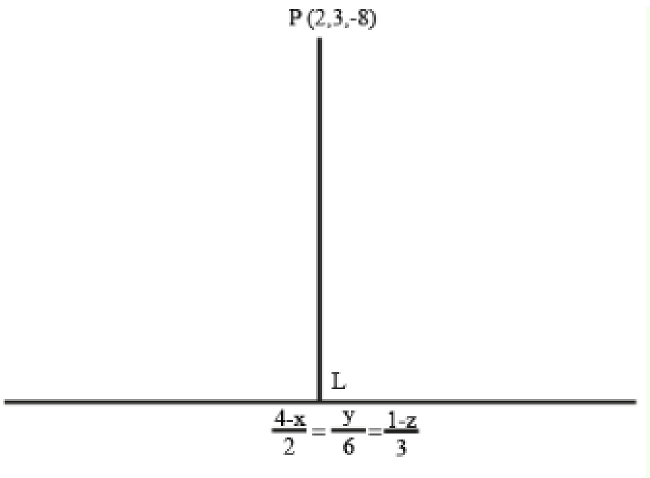
$\frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$ 2

Squaring $(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$ 2

Again differentiating we have $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

35 $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ can be written as $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$.

$x = -2\lambda + 4$ $y = 6\lambda$ $z = -3\lambda + 1$



Let L (-2 λ +4 , 6λ , -3λ + 1)

Since PL is perpendicular to the given line 2 (-2 λ +4)+6 x 6λ , +(-3λ + 1)3 = 0

Solving λ = 1

L (2,6,-2) which is the foot of the perpendicular.

Distance PL = 3 $\sqrt{5}$units ,using distance formula.

SECTION E **[3 X 4 = 12]**

36.(i) Total cost C(n) = 400 + 4n + $0.0001n^2$

Marginal cost = C' (n) = 4 + 0.0002n

For n = 10, marginal cost = 4.002 dollars 2 M

(ii) profit P(n) = qn – C(n)but q = 10− 0.0004n

Hence P(n) = 6 n − $0.0005n^2$−400

Differentiating P'(n) = 6 − 0.001n

P'(n) = 0 then n = 6000

P'' (n) < 0 at n = 6000

To maximize the profit daily production must be 6000. 2 M

37. $P(E_1) = \frac{4}{5} P(E_2) = \frac{3}{4} P(E_3) = \frac{2}{3}$

(i) $\frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{5}$ 1

(ii) $\frac{3}{4} \cdot \frac{2}{3} \frac{1}{5} = \frac{1}{10}$ 1

(iii) $\frac{4}{5} \cdot \frac{3}{4} \frac{1}{3} + \frac{4}{5} \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{3}{4} \frac{2}{3} = \frac{13}{30}$ 2

OR

(iv) $\frac{1}{5} \frac{1}{4} \frac{1}{3} = \frac{1}{60}$

38. (i) reflexive,transitive but not symmetric 1

(ii) reflexive and transitive
1

(iii) 6^2 **OR** (iv) 2^{12} 2

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