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PB-1 EXAMINATION (2024-25)
SUBJECT: -MATHEMATICS (041)
CLASS XII

MAX MARKS=80 **TIME: 3 HRS**

General Instructions:
1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. Section **A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section **B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section **C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section **D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section **E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

Select the correct option (question 1 – question18).

Q.1 General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is

(a) $e^{-y} = \frac{1}{2}e^{2x} + C$ (b) $\frac{1}{e^y} + \frac{1}{2}e^{2x} + C$
(c) $-e^{-y} = \frac{1}{2}e^{2x} + C$ (d) $e^y = \frac{1}{2}e^{2x} + C$

Q.2 A and B are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is
(a) 16 (b) 4 (c) 6 (d) $\frac{1}{16}$

Q.3 The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of Change of its surface area when edge is 3 cm is
a)10.8 cm b) 10.8 cm^2 c) 10.8 cm^2/s (d) 10.8 cm/s

Q.4 Integral of $\cot^2 x\, dx$ equals to
(a) $\cot x - x + C$ (b) $\cot x + x + C$ (c) $-\cot x + x + C$ (d) $-\cot x - x + C$

Q.5 The degree of the differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is

(a) 1 (b) 2 (c) 3 (d) 4

Q.6 If $P(A \cap B) = 70\%$ and $P(B) = 85\%$, then $P(A/B)$ is equal to

(*a*) 14/17 (*b*)17/20 (*c*) 7/8 (*d*) 1/8

Q.7 The area enclosed by the circle $x^2 + y^2 = 8$ is

(a) 16π sq units (b) 22π sq units (c) $8\pi^2$ sq units (d) 8π sq units

Q.8 $\cos^{-1}(\cos 7\pi/6)$ is equal to

a) $7\pi/6$ b) $5\pi/6$ c) $\pi/3$ d) $\pi/6$

Q.9 The matrix A satisfies the equation $\begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A is

(*a*) $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$

(*c*) $\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$

Q.10 The function $y = |x - 4|$ is

(a) Continuous at x = 4

(b) Differentiable at $x = 4$

(c) Both continuous and differentiable at $x = 4$

(d) Neither continuous nor differentiable at $x = 4$

Q.11 If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \cdot \vec{b}| = 12\sqrt{3}$ then the value $|\vec{a} \times \vec{b}|$ is

(*a*) 12 (*b*) $12\sqrt{3}$ (*c*) 6 (*d*) $4\sqrt{3}$

Q.12 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 5A - 7I$ is

(*a*) a zero matrix (*b*) an identity matrix (*c*) diagonal matrix (*d*) none of these

Q.13 The objective function of LPP is

(a) a constraint (b) a function to be optimised

(c) a relation between the variables (d) None of these

Q.14 The diagonal elements of a skew symmetric matrix are

(a) all zeroes (b) are all equal to some scalar $k(\neq 0)$

(c) can be any number (d) none of these

Q.15 Unit vectors along vector $\hat{i} + 2\hat{j} - 2\hat{k}$ are

(*a*) $\pm(\hat{i} + 2\hat{j} - 2\hat{k})$ (*b*) $\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$ (*c*) $\pm\left(\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right)$ (*d*) none of these

Q.16 If f ' (x) = sec x, the f(x) is

(a) sec x tan x (b) sec x + tan x

(c) log (sec x + tan x) (d) log (sec x - tan x)

Q.17 The objective function for a given linear programming problem is Z = ax + by –5. If Z attains same value at (1, 2) and (3, 1), then

(a) a + 2b = 0 (b) a + b = 0 (c) a = b (d) 2a – b = 0

Q.18 Direction ratios of a line passing through the points (2, 1, 0) and (3, 2, –1) are

(*a*) (1, 1, –1) (*b*) 1, 1, –1 (*c*) < 5, 3, –1 > (*d*) none of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true**.**

Q.19 **Assertion** (A) : In set $A = \{a, b, c\}$ relation R in set A, given as $R = \{(a, c)\}$ is transitive.

Reason(R): A singleton relation is transitive

Q.20 **Assertion**:-:$\frac{d}{dx}(e^{\cos x}) = e^{\cos x}(-\sin x)$

Reason:-$\frac{d}{dx}(e^{x}) = e^{x}$

SECTION-B

Q.21 Radius of a variable circle is changing at the rate of 5 cm/s. What is the radius of the circle at a time when its area is changing at the rate of 100 cm^2/s?

Q.22 Find the value of $Sin(\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right))$

Q.23 Find the value of k, so that the function $f(x)$ = $f(x) = \begin{cases} kx^2, & \text{if } x \geq 1 \\ 4, & \text{if } x < 1 \end{cases}$, is continuous at $x = 1$.

Q.24 Find λ, if . $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$

Q.25 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

SECTION-C

Q.26 A family has two children. What is the probability that both the children are boys given that at least one of them is a boy ?

Q.27 Differentiate $x^{\sin x}$, x > 0 w.r.t. x.

Q.28 Solve the following linear programming problem graphically: Maximise Z = 4x + y subject to the constraints: $x + y \le 50$

$$3x + y \le 90$$
$$x \ge 0$$
$$y \ge 0$$

Q.29 Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$.

Q.30 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}$, $y \neq 2$

Q.31 Find the integral of $\displaystyle\int \frac{1}{1+\tan x}\,dx$

OR

Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,dx$

SECTION-D

Q.32 Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\,dx$.

OR

Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

Q.33 Find the shortest distance between the lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$

and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

OR

Find the shortest distance between the lines

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \quad \text{and} \quad \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

Q.34 If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$.

OR

If $y = \sin^{-1} x$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 0$.

Q.35 The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

OR

If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

SECTION-E

Q.36 An organization conducted bike race under 2 different categories- boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B= $\{b_1, b_2, b_3\}$, C = $\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions. On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relations possible from B to G .How many such relations are possible? [1Mark]

(ii) Write the smallest equivalence relation on G. [1Mark]

(iii) (a) Ravi defines a relation from B to B as $R_1 = \{(b_1, \boldsymbol{b_2}), (\boldsymbol{b_2}, \boldsymbol{b_1})\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes-

(A) reflexive but not symmetric (B) reflexive and symmetric but not transitive.

[2Marks]

OR

(b) If the track of the final race (for the biker b_1) follows the Curve $x^2 = 4y$; (where$0 \le x \le 20\sqrt{2}$ & $0 \le y \le 200$), then state whether the track represents a one-one and onto function or not. (Justify). [2Marks]

Q.37 The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(a) The rate of growth of the plant with respect to sunlight is ______ .

(i) $4x - \frac{1}{2}x^2$ (ii)$4 - x$ (iii)$x - 4$ (iv)$x - \frac{1}{2}x^2$

(b) What is the number of days it will take for the plant to grow to the maximum height?

(i)4 (ii)6 (iii)7 (iv)10

(c) What is the maximum height of the plant?

(i)12 cm (ii)10 cm (iii)8 cm (iv)6 cm

(d) What will be the height of the plant after 2 days?

(i)4 cm (ii)6 cm (iii)8 cm (iv)10 cm

Q.38 Nisha and Arun appeared for first round of an competitive examination for two vacancies. The probability of Nisha's selection is 1/6 and that of Arun's selection is 1/4. Based on the above information, answer the following questions:

(a)The probability that at least one of them is selected, is

i) 1/8 ii) 3/7 iii) 2/7 iv) 3/8

(b) The probability that both of them are selected is-

i) 1/6 ii) 1/12 iii) ½ iv) 1/24

(c) The probability that none of them is selected, is :

i) 3/8 ii) 1/3 iii) 2/7 iv) 5/8

(d) The probability that only one of them is selected, is :

i) 2/7 ii) 5/8 iii) 2/3 iv) 1/3

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