

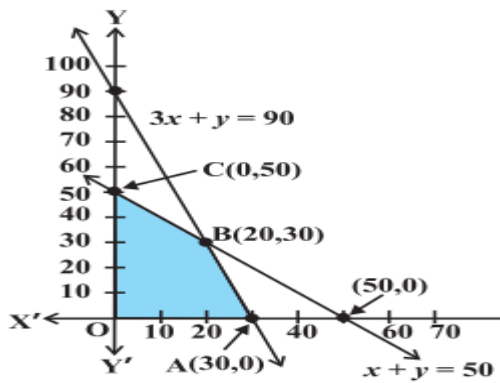
MARKING SCHEME of PB-1

SUBJECT: MATHEMATICS

CLASS: XII

SECTION-A

ANS.1	c)	1
ANS.2	(d)	1
ANS.3	c) 10.8 cm ² /s	1
ANS.4	(d)	1
ANS.5	c)	1
ANS.6	(a)	1
ANS.7	d)	1
ANS.8	b) 5π/6	1
ANS.9	(c)	1
ANS.10	(a) Continuous at x = 4	1
ANS.11	(a)	1
ANS.12	c)	1
ANS.13	b)	1
ANS.14	a)	1
ANS.15	(c)	1
ANS.16	(c) log (sec x + tan x)	1
ANS.17	d)	1
ANS.18	(b)	1
ANS.19	a) Both A and R are true and R is the correct explanation of A	1
ANS.20	(b) Both A and R are true but R is not the correct explanation of A.	1
SECTION-B		
ANS.21	<p>The area A of a circle with radius r is given by $A = \pi r^2$</p> $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow 100 = 2\pi r \times 5 \Rightarrow r = \frac{10}{\pi} \text{ cm.} \quad \left[\frac{dr}{dt} = 5 \text{ cm/s, } \frac{dA}{dt} = 100 \text{ cm}^2/\text{s} \right]$ <p style="text-align: center;">$\frac{10}{\pi}$</p> <p>Hence, radius of the circle is $r = \frac{10}{\pi}$ cm.</p>	<p>1</p> <p>1</p>
ANS.22	<p>The value of $\sin \left(\frac{\pi}{3} - \sin^{-1} \left(-\frac{1}{2} \right) \right)$</p> $= \sin \left(\frac{\pi}{3} - \left(-\frac{\pi}{6} \right) \right)$ $= \sin \frac{\pi}{2}$ $= 1$	<p>1</p> <p>1</p>
ANS.23	<p>For continuity at x = 1,</p> $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1) \Rightarrow \lim_{x \rightarrow 1^-} (4) = \lim_{x \rightarrow 1^+} (kx^2) = k$ $4 = k = k$ $\Rightarrow k = 4$	<p>1</p> <p>1</p>

<p>ANS.28</p>	<p>We observe that the feasible region is bounded.</p>  <p>The coordinates of the corner points are (0, 0), (30, 0), (20, 30) and (0, 50) respectively. maximum value of Z is 120 at the point (30, 0).</p>	<p>1/2 1 1 1/2</p>
<p>ANS.29</p>	$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8 \Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8 \Rightarrow \vec{a} ^2 - \vec{b} ^2 = 8$ $\Rightarrow (8 \vec{b})^2 - \vec{b} ^2 = 8 \quad [\text{Given that: } \vec{a} = 8 \vec{b}]$ $\Rightarrow 64 \vec{b} ^2 - \vec{b} ^2 = 8$ $\Rightarrow 63 \vec{b} ^2 = 8 \quad \text{therefore, } \vec{b} ^2 = \frac{8}{63}$ $\Rightarrow \vec{b} = \sqrt{\frac{8}{63}} \quad [\text{Magnitude of a vector is non-negative}]$ $\Rightarrow \vec{b} = \frac{2\sqrt{2}}{3\sqrt{7}}$ $\Rightarrow \vec{a} = 8 \vec{b} = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$	<p>1 1 1</p>
<p>ANS.30</p>	$\frac{dy}{dx} = \frac{x+1}{2-y} \quad \dots(1)$ <p>Separating the variables in equation (1), we get</p> $(2 - y) dy = (x + 1) dx \quad \dots\dots(2)$ <p>Integrating both the side and getting</p> $2y - y^2/2 = x^2/2 + x + C_1$ <p>Simplifying and getting</p> $x^2 + y^2 + 2x - 4y + C = 0 \quad (\text{where } C = 2C_1)$ <p>which is the general solution of equation (1)</p>	<p>1 1 1</p>
<p>ANS.31</p>	$\int \frac{dx}{1 + \tan x} = \int \frac{\cos x dx}{\cos x + \sin x}$ $= \frac{1}{2} \int \frac{(\cos x + \sin x + \cos x - \sin x) dx}{\cos x + \sin x}$	<p>1/2</p>

$$= \frac{1}{2} \int dx + \frac{1}{2} \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

$$= \frac{x}{2} + \frac{C_1}{2} + \frac{1}{2} \int \frac{\cos x - \sin x}{\cos x + \sin x} dx \quad \dots (1)$$

Now, consider $I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

Put $\cos x + \sin x = t$ so that $(\cos x - \sin x) dx = dt$

Therefore $I = \int \frac{dt}{t} = \log |t| + C_2 = \log |\cos x + \sin x| + C_2$

Putting it in (1), we get

$$\int \frac{dx}{1 + \tan x} = \frac{x}{2} + \frac{C_1}{2} + \frac{1}{2} \log |\cos x + \sin x| + \frac{C_2}{2}$$

$$= \frac{x}{2} + \frac{1}{2} \log |\cos x + \sin x| + \frac{C_1}{2} + \frac{C_2}{2}$$

$$= \frac{x}{2} + \frac{1}{2} \log |\cos x + \sin x| + C, \left(C = \frac{C_1}{2} + \frac{C_2}{2} \right)$$

OR

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \quad \dots (1)$

Then, by P_4

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 \left(\frac{\pi}{2} - x\right)}{\sin^4 \left(\frac{\pi}{2} - x\right) + \cos^4 \left(\frac{\pi}{2} - x\right)} dx = \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad \dots (2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx = \int_0^{\frac{\pi}{2}} dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

Hence $I = \frac{\pi}{4}$

SECTION-D

ANS.32

The given equation is $y = |x + 3|$.

The corresponding values of x and y are given in the following table:

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x + 3|$ as follows:

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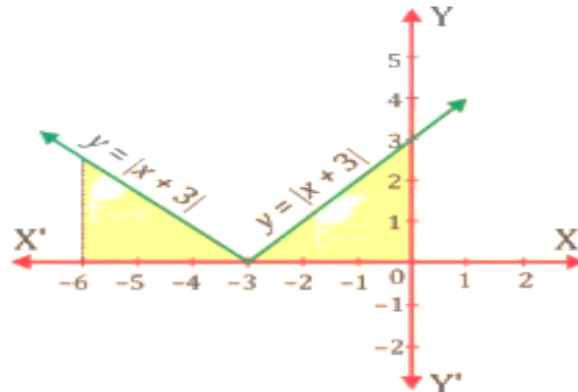
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It is known that:

$$(x + 3) \leq x \leq -3 \text{ and } (x + 3) \geq 0 \text{ for } -3 \leq x \leq 0.$$



$$\begin{aligned} \int_{-6}^0 |x + 3| dx &= - \int_{-6}^{-3} (x + 3) dx + \int_{-3}^0 (x + 3) dx \\ &= \left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} - \left[\frac{x^2}{2} + 3x \right]_{-3}^0 \\ &= \left[\left(\frac{(-3)^2}{2} + 3(-3) \right) - \left(\frac{(-6)^2}{2} + 3(-6) \right) \right] - \left[0 - \left(\frac{(-3)^2}{2} + 3(-3) \right) \right] \\ &= \left[\frac{9}{2} \right] - \left[-\frac{9}{2} \right] = 9 \end{aligned}$$

OR

The given equation of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

Area bounded by ellipse = 4 × Area of OAB

$$\text{Area of ABCD} = \int_0^4 y dx$$

$$= \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} dx = \frac{3}{4} \int_0^4 \sqrt{16 - x^2} dx$$

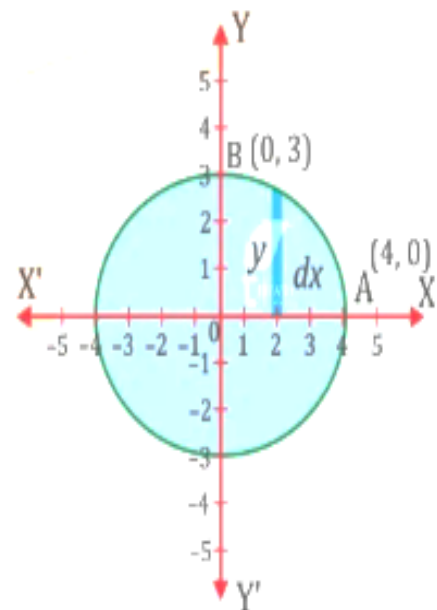
$$= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4$$

$$= \frac{3}{4} [2\sqrt{16 - 16} + 8 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0)]$$

$$= \frac{3}{4} \left[\frac{8\pi}{2} \right]$$

$$\Rightarrow \frac{3}{4} \left[\frac{8\pi}{2} \right] = 3\pi$$

Therefore, area bounded by the ellipse = 4 × 3π = 12π units



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ANS.33

$$\vec{r} = (i + 2j + 3k) + \lambda(i - 3j + 2k) \text{ and } \vec{r} = 4i + 5j + 6k + \mu(2i + 4j + k)$$

It is known that the shortest distance between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \quad \dots (1)$$

Comparing the given equations with $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$, we have

$$\vec{a}_1 = i + 2j + 3k$$

$$\vec{b}_1 = i - 3j + 2k$$

$$\vec{a}_2 = 4i + 5j + 6k$$

$$\vec{b}_2 = 2i + 3j + k$$

Therefore, $\vec{a}_2 - \vec{a}_1 = (4i + 5j + 6k) - (i + 2j + 3k) = 3i + 3j + 3k$ and

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = (-3 - 6)i - (1 - 4)j + (3 + 6)k = -9i + 3j + 9k$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-9)^2 + (3)^2 + (9)^2} = \sqrt{81 + 9 + 81} = 3\sqrt{19}$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (-9i + 3j + 9k) \cdot (3i + 3j + 3k) = -9 \times 3 + 3 \times 3 + 9 \times 3 = 9$$

Substituting all the values in equation (1), we get

$$d = \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$$

Therefore, the shortest distance between the two given lines is $\frac{3}{\sqrt{19}}$ units.

OR

$$\text{Equations of the given lines } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

It is known that the shortest distance between the two lines,

$$\frac{x+x_1}{a_1} = \frac{y+y_1}{b_1} = \frac{z+z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \quad \dots (1)$$

Comparing the given equations, we have

$$x_1 = -1, \quad y_1 = -1, \quad z_1 = -1$$

$$a_1 = 7, \quad b_1 = -6, \quad c_1 = 1$$

$$x_2 = 3, \quad y_2 = 5, \quad z_2 = 7$$

$$a_2 = 1, \quad b_2 = -2, \quad c_2 = 1$$

Then

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6) = -16 - 36 - 64 = -116$$

$$\text{and } \sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2} = \sqrt{(-6 + 2)^2 + (1 + 7)^2 + (-14 + 6)^2}$$

$$= \sqrt{16 + 36 + 64} = \sqrt{116} = 2\sqrt{29}$$

Substituting all the values in equation (1), we have

$$d = \frac{-116}{2\sqrt{29}} = \frac{-58}{\sqrt{29}} = -2\sqrt{29}$$

Since, distance is always non-negative, so, the distance between the given lines is $2\sqrt{29}$ units.

<p>ANS.34</p>	<p>Given that: $y = e^{a \cos^{-1} x}$, therefore,</p>	
	<p>Differentiating with respect to x, we have</p>	
	$\frac{dy}{dx} = \frac{d}{dx} e^{a \cos^{-1} x} = e^{a \cos^{-1} x} \frac{d}{dx} a \cos^{-1} x$	<p>½</p>
	$\Rightarrow \frac{dy}{dx} = e^{a \cos^{-1} x} a \cdot \frac{-1}{\sqrt{1-x^2}} = -\frac{ay}{\sqrt{1-x^2}}$	<p>½</p>
	<p>Squaring both the sides, we have</p>	
	$\left(\frac{dy}{dx}\right)^2 = \frac{a^2 y^2}{1-x^2}$	
	$\Rightarrow (1-x^2) \left(\frac{dy}{dx}\right)^2 = a^2 y^2$	<p>1</p>
	<p>Differentiating again with respect to x, we have</p>	
	$(1-x^2) \cdot 2 \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \frac{d}{dx} (1-x^2) = a^2 2y \frac{dy}{dx}$	
	$\Rightarrow \frac{dy}{dx} \left[2(1-x^2) \frac{d^2 y}{dx^2} + \frac{dy}{dx} (-2x) \right] = 2a^2 y \frac{dy}{dx}$	<p>½</p>
$\Rightarrow 2 \frac{dy}{dx} \left[(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} \right] = 2a^2 y \frac{dy}{dx}$		
$\Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = a^2 y$	<p>1½</p>	
$\Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$	<p>1</p>	
<p>OR</p>		
<p>We have $y = \sin^{-1} x$. Then</p>		
$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$	<p>1</p>	
$\sqrt{1-x^2} \frac{dy}{dx} = 1$		
$\frac{d}{dx} \left(\sqrt{1-x^2} \cdot \frac{dy}{dx} \right) = 0$	<p>1</p>	
$\sqrt{1-x^2} \cdot \frac{d^2 y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx} \left(\sqrt{1-x^2} \right) = 0$	<p>1</p>	
$\sqrt{1-x^2} \cdot \frac{d^2 y}{dx^2} - \frac{dy}{dx} \cdot \frac{2x}{2\sqrt{1-x^2}} = 0$	<p>1</p>	
$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$	<p>1</p>	

ANS.35 Let first, second and third numbers be denoted by x , y and z , respectively. Then, according to given conditions, we have $x + y + z = 6$
 $y + 3z = 11$
 $x + z = 2y$ or $x - 2y + z = 0$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

Here $|A| = 1(1+6) - (0-3) + (0-1) = 9 \neq 0$. Now we find $\text{adj } A$

$$\begin{aligned} A_{11} &= 1(1+6) = 7, & A_{12} &= -(0-3) = 3, & A_{13} &= -1 \\ A_{21} &= -(1+2) = -3, & A_{22} &= 0, & A_{23} &= -(-2-1) = 3 \\ A_{31} &= (3-1) = 2, & A_{32} &= -(3-0) = -3, & A_{33} &= (1-0) = 1 \end{aligned}$$

Hence

$$\text{adj } A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Thus

$$A^{-1} = \frac{1}{|A|} \text{adj } (A) = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Since

$$X = A^{-1} B$$

$$X = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 42-33+0 \\ 18+0+0 \\ -6+33+0 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Thus

$$x = 1, y = 2, z = 3$$

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OR

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4+4) + 3(-6+4) + 5(3-2) = 2(0) + 3(-2) + 5(1) = -6 + 5$$

= -1 ≠ 0; Inverse of matrix exists.

Find the inverse of matrix:

Cofactors of matrix:

$$A_{11} = 0, A_{12} = 2, A_{13} = 1$$

$$A_{21} = -1, A_{22} = -9, A_{23} = -5$$

$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\text{adj. } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, matrix of equation can be written as:

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

And, $X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 1$, $y = 2$ and $z = 3$.

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