MODERN VIDYA NIKETAN SCHOOLS

MVN Satyameva Jayate

PRE-BOARD 1 - 2024

CLASS: XII

SUBJECT: MATHEMATICS

Time: 3 Hour **Max. Marks: 80**

General Instructions:

All the questions are compulsory.

SECTION - A

1. A function $f: R \to R$ defined as $f(x) = x^2 - 4x + 5$

   (a) Injective but not surjective (b) Subjective but not injective

   (c) Both injective and surjective (d) Neither injective nor subjective

2. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$ then the value of x is

   (a) -1 (b) 0 (c) 1 (d) 2

3. Derivative of $e^{\sin^2 x}$ w.r.t $\cos x$

   (a) $\sin x\, e^{\sin^2 x}$ (b) $\cos x\, e^{\sin^2 x}$ (c) $-2\cos x\, e^{\sin^2 x}$ (d) $-2\sin^2 x \cos x\, e^{\sin^2 x}$

4. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has local minima at x equal to

   (a) 2 (b) 1 (c) 0 (d) -1

5. $\int \frac{1}{x(\log x)^2} dx$ is equal to

   (a) $2\log(\log x) + c$ (b) $\frac{-1}{\log x} + c$ (c) $\frac{(\log x)^3}{3} + c$ (d) $\frac{3}{(\log x)^3} + c$

6. $\int_{-1}^{1} x|x|\, dx$ is equal to

   (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{-1}{6}$ (d) 0

7. The degree of differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is

   (a) 4 (b) 3 (c) 2 (d) not defined

8. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalas matrix then the value of $a+2b+3c+4d$ is

(a) 0 (b) 5 (c) 10 (d) 25

9. If $A=\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I-A+A^2-A^3+\ldots\ldots$ is

(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 1 \\ -4 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10. For what value of k the function given below is continuous at $x=0$

$$f(x)=\begin{cases} \dfrac{\sqrt{4+x}-2}{x} & x\neq 0 \\ k & x=0 \end{cases}$$

(a) 0 (b) $\frac{1}{4}$ (c) 1 (d) 4

11. The general solution of the differential equation $xdy+ydx=0$

(a) $xy=c$ (b) $x+y=c$ (c) $x^2+y^2=c^2$ (d) $\log y=\log x+c$

12. The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx}=y,\ y>0$

(a) $\frac{1}{x}$ (b) x (c) y (d) $\frac{1}{y}$

13. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=2$ and $|\vec{b}|=2$ and $\vec{a}.\vec{b}=\sqrt{3}$. The angle between $2\vec{a}$ and $-\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

14. The vector equation of line passing through the point (1, –1, 0) and parallel to y-axis is

(a) $\vec{r}=\hat{i}-\hat{j}+\lambda(\hat{i}-\hat{j})$ (b) $\vec{r}=\hat{i}-\hat{j}+\lambda\hat{j}$ (c) $\vec{r}=\hat{i}-\hat{j}+\lambda\hat{k}$ (d) $\vec{r}=\lambda\hat{j}$

15. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) 3

16. The probability distribution of a random variable x is

x	0	1	2	3	4
$p(x)$	0.1	k	$2k$	k	0.1

where k is some unknown constant.

The probability that the random variable x takes the value 2 is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) 1

17. If the direction cosines of a line are $<\frac{1}{c}, \frac{1}{c}, \frac{1}{c}>$ then

(a) $0<c<1$ (b) $c>2$ (c) $c=\pm\sqrt{2}$ (d) $c=\pm\sqrt{3}$

18. If A and B are event such that $P(A/B)=P(B/A)\neq 0$ then

(a) $A\subset B$ but $A\neq B$ (b) $A=B$ (c) $A\cap B=\phi$ (d) $P(A)=P(B)$

Assertion Reason based questions

Question No. 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark. Two statements are given on labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true

19. Assertion (A): The relation $R=\{(x,y): x+y$ is prime number and $x,y\in N\}$ is not reflexive relation.

Reason (R): The number $2x$ is composite for all natural n.

20. Assertion (A): Domain of $y=\cos^{-1}x$ is $[-1,1]$.

Reason (R): The range of the Principal value branch of $y=\cos^{-1}x$ is $[0, \pi]-\{\pi/2\}$

SECTION - B

21. Find the value of k if $\sin^{-1}\left\{k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}=\frac{\pi}{3}$

22. Verify whether the function f defined by

$$f(x)=\begin{cases} x\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$ is continuous at $x=0$ or not

OR

Check for differentiability of the function f defined by $f(x)=|x-6|$ a the point $x=6$.

23. If $y = \cos^3(\sec^2 2t)$. Find $\frac{dy}{dt}$

OR

If $x^y = e^{x-y}$. Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

24. Find the interval in which the function $f(x) = x^4 - 4x^2 + 10$ is strictly decreasing.

25. Find : $\int \frac{1}{x(x^2-1)}$

SECTION - C

26. Evaluate $\int \frac{2x^2+3}{x^2(x^2+9)} dx \; x \neq 0$

27. Evaluate $\int \sqrt{\frac{x}{1-x^3}} dx, \; x \in (0,1)$

OR

Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx$

28. Solve the differential eq^n $ye^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y^2 \right) dy, y \neq 0.$

OR

Solve $\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \leq x < \frac{\pi}{2} \right)$

29. Solve the following linear programming problem graphically.

Minimize $z = x + 2y$

subject to constraints: $x + 2y \geq 100, 2x - y \leq 0 \; , 2x + y \leq 200, x, y \geq 0$

30. If A and B are two independent events, such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find $P(A)$ and $P(B)$.

31. If $(a+bx)e^{\frac{y}{x}} = x$ then Prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx} \right)^2$

OR

If $(x-a)^2+(y-b)^2=c^2$ then prove that $\dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}}{\dfrac{d^2y}{dx^2}}$ is constant and independent of a and b.

SECTION - D

32. Let $A=\{x\in z: 0\le x\le 15\}$ show that $R=\{(a,b): a,b\in A, |a-b| \text{ is multiple of } 7\}$ is an equivalence relation find the set of all element related to 1. Also write the equivalence class [2].

33. If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$ find A^{-1}. Hence solve the system of equations

$x-2y=10$

$2x-y-z=8$

$-2y+z=7$

OR

If $A=\begin{bmatrix}2 & 1 & -3\\ 3 & 2 & 1\\ 1 & 2 & -1\end{bmatrix}$. Find A^{-1} and hence solve the following system of equation.

$2x+y-3=13$

$3x+2y+z=4$

$x+2y-z=8$

34. Using Integration find the area of bounded by the ellipse $9x^2+25y^2=225$ the lines $x=-2, x=2$ and the axis.

OR

Sketch the graph of $y=x|x|$ and hence find the area bounded by this curve, x-axis and ordinates $x=-2$ and $x=2$ using integration.

35. Find the equation of the line which bisects the line segment joining points $A(2, 3, 4)$ and $B(4, 5, 8)$ is perpendicular to the lines $\dfrac{x-8}{3}=\dfrac{y+19}{16}=\dfrac{z-10}{7}$ and $\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}$

Case-Study Based

36. A potter made a mud vessel where the shape of the pot is based on $f(x)=|x-3|+|x-2|$ where $f(x)$ represented the height of the pot. Based on the above information. Answer the following questions.

(i) When $x > 4$ what will be the height in term of x. (1)

(ii) Will the slope vary with x value (1)

(iii) What is $\frac{dy}{dx}$ at $x = 3$. (2)

OR

If the potter is trying to make a pot using the function $f(x) = [x]$ will he get a pot or not? Why?

37. $P(x) = -5x^2 + 125x + 37{,}500$ is the total profit function of a company where x is the production of the company.

Based on the above information answer the following questions.

(i) What will be the production when the profit is maximum.

(ii) In which interval the profit is strictly increasing.

OR

What will be production of the company when the profit is ₹38250.

38. A departmental store sends bill to charge its customer once a month. Past experience show that 70% of its customers pay their first month bill in time. The store also found that the customers who pays the bill is time has the probability of 0.8 of paying in time next month and the customers who does not pay in time has the probability of 0.4 of paying in time the next month.

Based an above information answer the following questions.

i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$; $P(E_2)$ (1)

ii) Let A denotes the event of customer paying second month's bill in time then find $P(A/E_1)$ and $P(A/E_2)$. (1)

iii) Find the probability of customer paying second month's bill in time. (2)