INFINITY THINK BEYOND.....

AN EDUCATIONAL INSTITUTE

SUBJECT: MATHS **MAX. MARKS : 40**

DATE : 20/11/24 **DURATION : 90 MIN**

PBMT – 03

UNIT – 3 CALCULUS

Ch – 5 Continuity & Differentiability

Ch – 6 Applications of Derivative, Ch – 7 Integration

Ch – 8 Application of Integration

Ch – 9 Differential equations

General Instruction:

This Question Paper has 5 Sections A-E.

1. **Section A** has 6 MCQs carrying 1 mark each.
2. **Section B** has 3 questions carrying 02 marks each.
3. **Section C** has 3 questions carrying 03 marks each.
4. **Section D** has 1 questions carrying 04 marks each.
5. **Section E** has 3 questions carrying 05 marks each .

Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A
Questions 1 to 6 carry 1 mark each.

1. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$.

(a)2 and 3 (b) 3 and 3 (c) 2 and 2 (d) 2 and not defined

2. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2 \cos x$ is

(a) $\log x$ (b) $-\log x$ (c) x (d) $\frac{1}{x}$

3. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx}$ =

(a) e^{y-x} (b) e^{x+y} (c) $-e^{y-x}$ (d) $2e^{x-y}$

4. If the function $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous , then the value of k is

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

5. The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3 + 5x = 0$ are

(a) 3 ; 6 (b) 3 ; 3 (c) 3;9 (d) 6;3

6. Assertion (A) : The function $f(x) = x^2 - x$ is increasing in the interval $\left(\frac{1}{2}, \infty\right)$

Reason (R) : For above function $f'(x) = 2x+1$.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

SECTION – B
Questions 7 to 9 carry 2 mark each.

7. If $x = at^2$, y = 2at then find $\frac{d^2y}{dx^2}$.

8. Find the particular solution of the differential equation ($2x^2 + y$) $\frac{dx}{dy}$ = x given that $when\ x = 1$, $y = 2$.

9.Find the general solution of the following differential equation $2xe^{\frac{y}{x}}dy + (x - 2ye^{\frac{y}{x}}) dx = 0$.

SECTION – C
Questions 10 to 12 carry 3 mark each.

10. Find the area bounded between the curve $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

11. If $y = (\cos x)^{(\cos x)^{(\cos x)\cdots\cdots\infty}}$ then show that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \log(\cos x) - 1}$.

12. Integrate the function $\frac{x^2}{1 - x^4}$ w.r.t. x .

OR

Integrate the function $\frac{2x}{(x^2+1)(x^2+2)}$ w.r.t to x .

SECTION – D
Questions 13 carry 4 mark each.

13. Read the following and answer the questions: Relation between the height of the plant (y in cm) with respect to exposure to sun light is governed by the following equation, $Y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

(i) Find the rate of growth of plant w.r.t sunlight.
(ii) What is the maximum height of the plant?
(iii) What will be the height of the plant after two days?

OR

(iii) If the height of the plant is $\frac{7}{2}$ cm, the number of days it has been exposed to the sunlight.

SECTION – E
Questions 14 to 16 carry 5 mark each

14. Show that the function $f(x) = |x - 1| + |x + 1|$, $\forall\ x \in R$, is not differentiable at the points $x = -1$.

15. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window

OR

Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, with its vertex at one end of the major axis.

16. Evaluate the following: $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x}$ dx .

End