AN EDUCATIONAL INSTITUTE

SUBJECT:MATHS **MAX. MARKS : 40**
DATE :23/11/24 **DURATION : 60 MIN**

PBMT – 04

UNIT – 4 VECTOR & 3D
Ch – 10 Vector algebra
Ch – 11 3D

General Instruction:

This Question Paper has 5 Sections A-E.

1. **Section A** has 6 MCQs carrying 1 mark each.
2. **Section B** has 3 questions carrying 02 marks each.
3. **Section C** has 3 questions carrying 03 marks each.
4. **Section D** has 1 questions carrying 04 marks each.
5. **Section E** has 3 questions carrying 05 marks each .

Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

SECTION – A
Questions 1 to 6 carry 1 mark each.

1. The lines $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k})$ *and* $\vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(6\hat{\imath} + 9\hat{\jmath} - 18\hat{k})$ are
(a) coincident (b) skew (c) intersecting (d) parallel

2. The equation of the line in vector from passing through the point (-1,3,5) and parallel to the line $\frac{x-3}{2} = \frac{y-4}{3}, z = 2$ is

(a) $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k} + t(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$
(b) $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k} + t(2\hat{\imath} + 3\hat{\jmath})$)
(c) $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} - 2\hat{k} + t(-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$
(d) $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + t(-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$

3. If the position vectors of P and Q are $\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and $5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ respectively ,then the cosine of the angle between $\overrightarrow{PQ}$ and y – axis is
(a) $\frac{4}{\sqrt{162}}$ (b) $\frac{11}{\sqrt{162}}$ (c) $\frac{5}{\sqrt{162}}$ (d) $-\frac{5}{\sqrt{162}}$

4. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$
(a) three (b) infinite (c) two (d) one

5. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$ and $\vec{a}$ is perpendicular to $\vec{b}$ .If $\vec{c}$ makes angle α *and* β with $\vec{a}$ *and* $\vec{b}$ respectively , then cos α + cos β =
(a) -1 (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

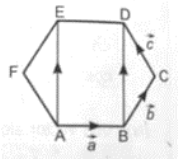
6. Assertion (A) : Acute angle between the vectors $\hat{\imath} + \hat{\jmath}$ - k and $2\hat{\imath}$ + 3k is $\cos^{-1}\left(\frac{1}{\sqrt{39}}\right)$.

Reason (R) : For vectors $\vec{a}$ and $\vec{b}$, the acute angle between them is, $\cos^{-1}\left(\frac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|}\right)$.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

SECTION – B
Questions 7 to 9 carry 2 mark each.

7. ABCDEF is a regular hexagon, $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{BC} = \vec{b}$ and $\vec{CD} = \vec{c}$, find $\overrightarrow{AE}$.



8. An aeroplane is flying along the line $\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$ where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

9.If $\vec{a} = 2\hat{\imath}$ - $\hat{\jmath}$ and $\vec{b} = 3\hat{\imath} + 2\hat{k}$, find $|\vec{a} + \vec{b}|$.

Or

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b}$ = -1 , find the angle between $\vec{a}$ and $\vec{b}$.

SECTION – C
Questions 10 to 12 carry 3 mark each.

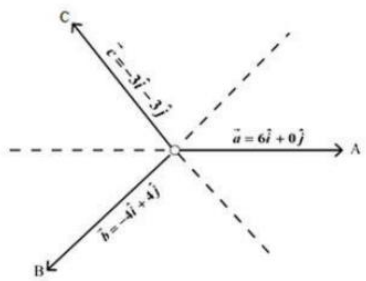
10. If $\vec{a}$ and $\vec{b}$ are unit vectors ,then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\sqrt{2}\,\vec{a} - \vec{b}$ is a unit vector ?

11. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude ,then prove that the vector $(2\vec{a} + \vec{b} + 2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$.Also ,find angle between $\vec{a}$ and $(2\vec{a} + \vec{b} + 2\vec{c})$.

12. The scalar product of the vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

SECTION – D
Questions 13 carry 4 mark each.

13. Read the following passage and answer the questions given below: Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.
Team A pulls with force $F1 = 6\hat{\imath} + 0\hat{\jmath}kN$,
Team B pulls with force $F2 = -4\hat{\imath} + 4\hat{\jmath}kN$,
Team C pulls with force $F3 = -3\hat{\imath} - 3\hat{\jmath}kN$



(i) What is the magnitude of the force of Team A ?
(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

OR

(iii) In what direction is the ring getting pulled?

SECTION – E
Questions 14 to 16 carry 5 mark each

14. An aeroplane is flying along the line $\vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$ where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

OR

Show that the lines $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect . Also, find the coordinates of the point of intersection.

15. Find the value of 't' for which the following lines are perpendicular to each other :

$\frac{x-5}{5t+2} = \frac{2-y}{5} = \frac{1-z}{-1}$; $\frac{x}{1} = \frac{y+\frac{1}{2}}{2t} = \frac{z-1}{3}$ .Hence ,find whether the lines intersect or not .

16.Find the image of the point (2, -1 ,5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.Also find the equation of the line joining the given point and its image .Find the length of that line segment also .

End