R.4

ROLL NUMBER	

SET	A

INDIAN SCHOOL MUSCAT
PREBOARD-2 EXAMINATION 2025
APPLIED MATHEMATICS (241)

CLASS: 12
DATE: 9.01.2025

TIME ALLOTTED:3 HRS
MAX. MARKS: 80

General Instructions:
Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is not allowed.

SECTION – A 20 x 1 = 20

1. Skewness of normal distribution is
   A. negative B. zero C. positive D. undefined
2. In a 50m race P can give a start of 5m to Q and a start of 14 m to R. In the same race how much start can Q give to R?
   A. 9 m B.10 m C. 12 m D.11 m
3. If $x = 3at^2$ and $y = 2at$ then $\frac{dy}{dx} =$

A. $\frac{1}{3t}$ B. t C. $\frac{-1}{3t}$ D. $\frac{a}{t}$

4. For matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ to be symmetric

A. $a = \frac{3}{2}$ and $b = \frac{-2}{3}$ B. $a = \frac{1}{3}$ and $b = \frac{3}{2}$

C. $a = \frac{-2}{3}$ and $b = \frac{3}{2}$ D. $a = \frac{3}{2}$ and $b = \frac{1}{3}$

5. If $a, b \in R$ and $a > b > 0$, then

A. $\frac{1}{a} > \frac{1}{b}$ B. $\frac{1}{a} < \frac{1}{b}$ C. $\frac{1}{a} \geq \frac{1}{b}$ D. $\frac{1}{a} \leq \frac{1}{b}$

6. The central limit theorem states that if the sample size_______

A. increases sampling distribution must approach normal distribution

B. decreases then the sample distribution must approach normal distribution

C. increases then the sampling distribution much approach an exponential distribution

D. decreases then the sampling distribution much approach an exponential distribution

7. Match the following columns to complete the sentence and choose the correct option

Trend component	Pattern of variation	Time period of variation
I. Secular trend	a. is a regular periodic variability	i. over a period more than a year
II. Cyclical trend	b. has smooth, regular variations	ii. within a period of one year
III. Seasonal trend	c. has oscillatory variation	iii. over a long term period

(A) I – a – ii ; II – b – iii ; III – c – I
(B) I – b – iii

A. I – a – ii ; II – b – iii ; III – c – I B. I – b – iii ; II – c – i ; III – a – ii

C. I – b – ii ; II – c – i ; III – a – iii D. I – b – ii ; II – a – iii ; III – c – i

8. A money lender charges interest at the rate ₹10 per ₹100 per half year, payable in advance. The nominal rate of interest(r) compounded half yearly in this case is

A. 10% B. 5% C. 22.22% D. 11.11%

9. Optimization of objective function of a LPP means

A. to maximise the objective function B. to minimise the objective function

C. both A and B D. None of these

10. $\int \frac{x}{\sqrt{x^2+1}} dx =$

A. $2\sqrt{x^2+1} + c$ B. $\sqrt{x^2+1} + c$ C. $\frac{1}{2}\sqrt{x^2+1} + c$ D. $\sqrt{x^2+1} + c$

11. It is given that at x = 1, the function $f(x) = x^3 - 12x^2 + kx + 7$ attains a maximum value, then the value of k is

A. 10 B. 12 C. 21 D. 13

12. The mortality rate for a certain disease is 7 in 1000. It is reported that there have been 2 deaths on account of this disease in a group of 400. If this distribution follows Poisson variate then the variance of the distribution is

A. 8 B. 1.4 C. 2.8 D. 0.8

13. Country A has an average farm size of 191 acres, while Country B has an average farm size of 199 acres. Assume the data were attained from two samples with standard deviations of

38 and 12 acres and sample sizes of 8 and 10, respectively. To infer that the average size of the farms in the two countries is different at $\alpha = 0.05$, assuming that the populations are normally distributed, the degree of freedom is

A. 16 B. 17 C. 7 D. 9

14. $\int_{-24}^{24}(x^{23} + x^{25})dx =$

A. 48 B. 0 C. -1 D. -48

15. If the cash equivalent of a perpetuity of ₹1200 payable at the end of each quarter is ₹ 96,000, then the rate of interest convertible quarterly.

A. 4% B. 5% C. 5.5% D. 6.5%

16. Old hens can be bought for ₹ 20 each and young ones at ₹50 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week. Each egg being worth ₹3. A hen costs ₹10 per week to feed. A man has only ₹800 to spend for buying the hens. He wants to maximise his weekly profit. If he purchases 'x' old hens and 'y' young hens, then the objective function of the LPP is

A. $5y - x$ B. $x + y$ C. $20x + 50y$ D. $9x + 15y$

17. $(5^{48} - x) mod\ 24$ is 0 then x =

A. 2 B. 0 C. 1 D. 23

18. The ratio in which a grocer mix two varieties of pulses costing ₹ 50 and ₹ 65 per kg respectively so that he gets a resulting mixture worth ₹ 55 per kg is

A. 2 : 1 B. 1 : 2 C. 10 : 11 D. 11 : 13

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

A. Both A and R are true and R is the correct explanation of the assertion
B. Both A and R are true but R is not the correct explanation of the assertion
C. A is true, but R is false
D. A is false, but R is true

19. **Assertion (A):** If an LPP attains its maximum value at two corner points of the feasible region then it attains maximum value at infinitely many points.
Reason (R): If the value of the objective function of a LPP is same at two corners then it is same at every point on the line joining two corner points.

20. **Assertion A :** The effective rate which is equivalent to nominal rate of 6% compounded semi-annually is 6.9%
Reason R : Effective rate is calculated as $\left(1 + \frac{r}{100m}\right)^m - 1$

SECTION – B (5 x 2 = 10)

21. Find the last two digits in the product 4321 × 3215

22. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and I is identity matrix of order 2 x 2, find **k** such that

$$A^2 = 5A + kI$$

23. If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

OR

Find the interval in which $f(x) = \frac{x}{logx}$ is strictly increasing.

24. Two pipes can fill a tank in 10 minutes and 14 minutes respectively. A third pipe can empty the tank at the rate of 10 litres per minute. If all the pipes are working together can fill the empty tank in 8 minutes, then what is the capacity of the tank.

25. Find the difference between the effective rate of interest, which is equivalent to the nominal rate of 10% per annum, compounded monthly and semi-annually.
[Given : $1.0083^{12} = 1.1047$]

OR

A startup company invested ₹1,50,000 in shares for 4 years. The value of the investment was ₹1,90,000 at the end of the second year, ₹1,75,000 at the end of 3rd year and on maturity the final value stood at ₹2,25,000. Calculate the CAGR on the investment.
[Given : $1.5^{0.25} = 1.107$]

SECTION – C (6 x 3 = 18)

26. A man goes 12 km down streams and comes back to the starting point by swimming non-stop in 3 hours. If the speed of the stream is 3 km/hr, find the speed with which the man can swim in still water.

27. If $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \\ -5 & 0 \end{bmatrix}$, verify if (AB)' = B' A'

OR

Solve the following system of equations using Cramer's rule
$6x + y - 3z = 5,\quad x + 3y - 2z = 5,\quad 2x + y + 4z = 8$

28. A company has been producing steel tubes of mean inner diameter of 2 cm. A sample of 10 tubes gives mean inner diameter of 2.01 cm and a variance of 0.004cm^2. Using t-test statistic discuss if there is a significant difference between the two means.
[Given: $t_{9,\,0.05} = 2.262$]

29. A machine cost a company ₹40,000 and its effective life is estimated to be 10 years. A sinking fund is created for replacing the machine at the end of the lifetime when it's scrap releases ₹5000 only. Calculate what amount should be retained out of profits at the end of each year to accumulate at 5 % per annum with compound interest for 10 years to replace the machinery by a new one which is estimated to be 25% more than the present one.
[Given : $1.05^{10} = 1.62889$]

30. Aman buys a laptop for ₹80,000. The laptop has a scrap value of ₹10,000 at the end of 5 years. Using linear depreciation, calculate the annual depreciation. Also construct the annual depreciation schedule.

31. The random variable X can take only values 0, 1 and 2. Given that $P(X=0) = P(X=1) = p$ and $E(X^2) = E(X)$, find the value of p.

OR

Assuming that half the population are consumers of chocolate, so that the chance of an individual being a consumer is $\frac{1}{2}$ and if 100 investigators each take 10 individuals to see whether they are consumers, how many investigators should you expect to report that 3 people or less were consumers.

SECTION D (4 x 5 = 20)

32. The income of a group of 10,000 people were found to be normally distributed with mean ₹750 per month and standard deviation ₹50. Show that this group has about 95% income exceeding ₹668 and 5% had income exceeding ₹832. Also find the lowest income among the richest 100?

[Given: $P(0 < z < 1.64) = 0.4495$, $P(z < 2.33) = 0.99$]

33. Under the monopoly, the quantity sold and market price are determined by demand function. If the demand function for a profit maximising monopolist is $p = 274 - x^2$ and the marginal cost is equal to $4 + 3x$, find the consumer's surplus.

OR

The rate of growth of a population is proportional to the number present. If the population of a city doubled in past 25 years and the present population is 1,00,000, when will the city have a population of 5,00,000 .

[Given : $\log_e 2 = 0.6931$ and $\log_e 5 = 1.609$]

34. Fit a straight line trend by method of least squares to find the trend values using the following data.

Year:	2010	2012	2013	2014	2015	2016	2019
Sales (in lakhs)	65	68	70	72	75	67	73

OR

Find the trend values by taking four yearly moving averages for the following data: Also show the smoothening of data on the graph.

Year:	2015	2016	2017	2018	2019	2020	2021	2022
Sales(in lakhs)	108	112	110	120	140	120	100	135

35. A pharmaceutical company manufactures two drugs - drug A and drug B. The process involves two steps - synthesis and testing. Each lot of drug A requires 15-man hours for synthesis and 3-man hours for testing. Each lot of drug B requires 5-man hours for synthesis and 2-man hours for testing. For synthesizing and testing, the maximum man hours available per week are 390 and 24 respectively. The company makes a profit of Rs 3500 on each lot of drug A and Rs 8000 on each lot of drug B.
Formulate the linear programming problem and using corner point method find out how many lots of drug A and drug B should be manufactured each week to maximize the profit.

SECTION E

Case based Questions (3 x 4 = 12)

36. Anil amortizes a loan of ₹15,00,000 for renovation of his house by 8 years mortgage at the rate of 12% per annum compounded monthly. Find

i) the equated monthly installment.

ii) the principal outstanding at the beginning of 40th month

iii) the interest paid in the 40th month.

OR

iii) total interest paid to the lender.

[Given $(1.01)^{96} = 2.5933$,

$(1.01)^{57} = 1.7633$.]

37. Consider two families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family. The recommended daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a child and that of protein is 45 grams for a man, 55 grams for a woman and 33 grams for children.

Based on the above information, answer the following questions.

a) Represent the requirement of calories and proteins for each person in matrix form.

b) Evaluate the requirement of calories of family A using matrix algebra.

OR

b) Evaluate the requirement of calories of family B using matrix algebra.

38. The life span of a certain flowering plant is around 6 years, t years after the sapling is planted the plant produces r gram of flowers each day. The relation between r and t can be approximated as

$$r = \frac{t^3}{3} - 6t^2 + 32t, \quad 0 \le t \le 6$$

(i) What will be the yield per day given 3 years after planting the sapling.

(ii) Find the interval of t in which r decreases.

(iii) What will be the maximum yield per day?

OR

(iii) Using the result of part (ii), find the year in which height will be maximum. Justify your answer.