Section A

1. $7 \equiv 1 \bmod 3$

$\Rightarrow 7^6 \equiv 1^6 \bmod 3$

$\therefore 7^6 \bmod 3 = 1$

option a) 1

2. When A runs 1000m; B runs 950m and C runs 931m

$\Rightarrow$ when B runs 950m; C runs 931m

$\therefore$ when B runs 1000m; C runs $\frac{931}{950} \times 1000$

ie $\underline{980\text{m}}$

$\therefore$ B can allow C 20m.

option d) 20m

3. Speed of the boat in still water is $x = 15$ km/h.

Speed of the current is $y = 3$ km/h.

Speed of downstream $= x + y = 15 + 3 = 18$ km/h.

Distance travelled = Speed × Time

$= 18 \text{ km/h} \times \frac{12}{60}$ hours.

$= \underline{3.6 \text{ km}}$

option d) 3.6 km

4. Given $A^2 - A + I = 0$

$\Rightarrow I = A - A^2 \longrightarrow \times A^{-1}$ on both sides.

$\Rightarrow I A^{-1} = A A^{-1} - A \cdot A \cdot A^{-1}$

$\Rightarrow A^{-1} = I - AI$

$\therefore A^{-1} = \underline{\underline{I - A}}$

option c) I - A.

---

5. We have $|KA| = K^n |A|$, where n is the order of the matrix.

$\therefore |3A| = 3^3 |A| = 27 \times 5 = \underline{\underline{135}}$

option d) 135

6, $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$; given A is a skew symmetric matrix.

$\therefore A = -A'$

$$\begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & -1 \\ a & b & c \\ 1 & 1 & 0 \end{bmatrix}$$

$\Rightarrow -a = -1$; $\boxed{b = 0}$; $c = -1$

$\Rightarrow \boxed{a = 1}$

$\therefore (a+b+c)^2 = 0$

option b) 0

7. c) Secular trend.

8. a) a rising trend.

9. $\int \frac{x^2+1}{x^2-1} dx$

$$\begin{array}{r} 1 \\ x^2-1\,\overline{)\,x^2+1} \\ \underset{-}{\ }x^2\underset{+}{-}1 \\ \hline 2 \end{array}$$

$= \int 1 + \frac{2}{x^2-1} dx$

$= x + \frac{1}{2} \times 2 \log \left|\frac{x-1}{x+1}\right| + c$

option a) $x + \log\left|\frac{x-1}{x+1}\right| + c$

10. $e^{y-x} \frac{dy}{dx} = 1$

$\Rightarrow e^{y} \cdot e^{-x} \frac{dy}{dx} = 1$

$\Rightarrow e^{y} dy = e^{x} dx$

$\Rightarrow e^{y} = e^{x} + c$

option a) $e^{y} = e^{x} + c$

11. option b) $E(X^2) - (E(X))^2$

12. option b) $t = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

13. option a) normal distribution

14. Given $a = 95$; $b = 2$

Trend equation is $y_t = a + bx$

$x = 2022 - 2016 = 6$.

$\therefore \quad y_t = 95 + 2(6) = 95 + 12 = 107$

option c) 107

---

15. $\text{Rate of Return} = \left(\frac{\text{Dividend}}{\text{Investment}}\right) \times 100$

$$\text{Number of Shares} = \frac{\text{Total Investment}}{\text{Price per Share}}$$

$$= \frac{9600}{80} = \underline{120}$$

$\therefore$ Dividend = Number of Shares × Dividend Rate

$= 120 \times 18 = ₹2,160$

$\therefore$ $\text{Rate of Return} = \left(\frac{2160}{9600}\right) \times 100 = 22.5\%$

option b) 22.5%

---

16. $$\text{EMI} = \frac{P + Pni}{n}$$

$$= \frac{25,000 + \frac{25,000 \times 11 \times 36}{100}}{36}$$

$\approx ₹923.61$

option d) Rs. 923.61

17. $P = \frac{R}{i} = \frac{1,000}{\left(\frac{5}{100}\right)} = \frac{100000}{5} = ₹20,000$

option d) Rs. 20,000.

18. $r_{eff} = \left(1 + \frac{r}{100p}\right)^p - 1$

$= \left(1 + \frac{6}{200}\right)^2 - 1$

$= (1.03)^2 - 1 = 1.0609 - 1 = 0.0609$

$\therefore$ $r_{eff} = 6.09\%$

option c) 6.09%.

19. Annual Depreciation $= \frac{\text{Cost} - \text{Scrap Value}}{\text{Useful Life}}$

$= \frac{50000 - 10000}{4} = 10,000$

$\therefore$ Depreciation rate (%) $= \frac{\text{Annual depreciation}}{\text{Total depreciation}} \times 100$

$= \frac{10,000}{40,000} \times 100$

$= 25\%$

(a) Both A and R are true and R is the

correct explanation of the assertion.

20. $Z = ax - by + 1900$

$Z = a(5) - b(0) + 1900 = 1950$

$\Rightarrow 5a = 50$

$\Rightarrow \boxed{a = 10}$

Also $Z = a(0) - b(5) + 1900 = 1550$

$-5b = -350$

$\Rightarrow \boxed{b = 70}$

$\therefore Z = 10(5) - 70(3) + 1900$ at $(5, 3)$.

$Z = \underline{\underline{1740}}$

(a) Both A and R are true and R is the correct explanation of the assertion.

Section B

21.
(a) Given: Cost of one variety of Rice = ₹ 161 per kg.

Cost of another variety = ₹ 179 per kg.

By alligation Mixture,

cost of Cheaper (c) ₹ 161 | cost of dearer (d) ₹ 179 — $\frac{1}{2}$ m

mean (m) ₹ x

d - m ₹ 179 - x | m - c x - 161

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{d-m}{m-c} = \frac{5}{4}$$ -½-

$$\Rightarrow \frac{179-x}{x-161} = \frac{5}{4}$$

$$\Rightarrow 4(179-x) = 5(x-161)$$

$$\Rightarrow 716-4x = 5x-805$$

$$\Rightarrow 9x = 1521$$ -½-

$$\therefore x = ₹169$$

$\therefore$ Mean price $=$ ₹169.

profit = Selling price - Mean price

$= 202.80 - 169 = 33.80$

$\therefore$ Profit percentage $= \frac{33.8}{169} \times 100 \approxeq 20\%$ -½-

OR

(b) $-2\frac{1}{2} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$

$$\left.\begin{aligned}&\Rightarrow -\frac{5}{2} \le \frac{x}{2} - \frac{4}{3} < \frac{1}{6}\\&\Rightarrow -\frac{5}{2} + \frac{4}{3} \le \frac{x}{2} < \frac{1}{6} + \frac{4}{3}\end{aligned}\right\}$$ -½-

$$\Rightarrow \frac{-15+8}{6} \le \frac{x}{2} < \frac{1+8}{6}$$

$$-\frac{7}{6} \leq \frac{x}{2} < \frac{9}{3} \qquad \times 2$$

$$\Rightarrow -\frac{7}{3} \leq x < \frac{18}{3}$$

ie $\boxed{-\frac{7}{3} \leq x < 6} \Rightarrow -2.33 \leq x < 6$ -½-

(i) when x is an integer

$x = \{-2, -1, 0, 1, 2, 3, 4, 5\}$ -½-

(ii) when x is a real number.

$x \in [-\frac{7}{3}, 6)$ -½-

---

22. To find the Last digit of 17^{17},

we find $17^{17} \bmod 10$.

$17 \equiv 17 \bmod 10$

$\Rightarrow 17 \equiv 7 \bmod 10$ -½-

$\Rightarrow 17^2 \equiv 7^2 \bmod 10$

$\Rightarrow 17^2 \equiv 9 \bmod 10$

$\Rightarrow (17^2)^2 \equiv 81 \bmod 10$ -½-

$\Rightarrow 17^4 \equiv 1 \bmod 10$

$\Rightarrow (17^4)^4 \equiv 1^4 \bmod 10$

$17^{16} \equiv 1 \mod 10$ -½-

$17^{16} \times 17 \equiv 1 \times 17 \mod 10$

$\Rightarrow \boxed{17^{17} \equiv 7 \mod 10}$

$\therefore$ Last digit of 17^{17} is 7. -½-

---

23. $A = [a_{ij}]_{3\times2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times2}$ where a_{ij} is

Given as $a_{ij} = \frac{1}{2}|-3i+j|$

$a_{11} = \frac{1}{2}|-3\times1+1| = \frac{1}{2}|-2| = 1$

$a_{12} = \frac{1}{2}|-3\times1+2| = \frac{1}{2}$

-½-

$a_{21} = \frac{1}{2}|-3\times2+1| = \frac{5}{2}$

$a_{22} = \frac{1}{2}|-3\times2+2| = 2$

-½-

$a_{31} = \frac{1}{2}|-3\times3+1| = 4$

$a_{32} = \frac{1}{2}|-3\times3+2| = 7/2$

-1-

$\therefore A = \begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \\ 4 & 7/2 \end{bmatrix}$

24.

(a) $\int \frac{\log(\log x)}{x} dx$

put $\log x = t$ -½-

$\frac{1}{x} = \frac{dt}{dx}$

$\therefore \int \frac{\log(\log x)}{x} dx = \int \log t \, dt$ ILATE I II

$= \int \log t \, (1) \, dt$ -½-

$= \log t \int 1 \, dt - \int \frac{d}{dt}[\log t] \int 1 dt \cdot dt$

$= t \log t - \int \frac{1}{t} \times t \, dt$ -½-

$= t \log t - t + C$

$= \log x \cdot \log(\log x) - \log x + C$ -½-

OR

(b) $\int \frac{1}{\sqrt{5x^2 - 2x}} dx$

Consider $5x^2 - 2x = 5\left(x^2 - \frac{2}{5}x\right)$

$= 5\left[(x - 1/5)^2 - (1/5)^2\right]$ -½-

Substitute $x - 1/5 = t \Rightarrow dx = dt$ -½-

$$\therefore \int \frac{dx}{\sqrt{5x^2-2x}} = \int \frac{dt}{\sqrt{5\left[t^2-(1/5)^2\right]}}$$

$$= \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{t^2-(1/5)^2}}$$ -1/2-

$$= \frac{1}{\sqrt{5}} \log\left|t + \sqrt{t^2-(1/5)^2}\right| + c$$

$$= \frac{1}{\sqrt{5}} \log\left|x - 1/5 + \sqrt{x^2-\frac{2x}{5}}\right| + c$$ -1/2-

25.

Given: $R = ₹360$

$i = 6\%$ p.a

$= \frac{6}{2}\%$ per period.

$i = 3\%$ per. period. -1/2-

$$\therefore \quad P = R + \frac{R}{i}$$ -1/2-

$$= 360 + \frac{360}{3/100}$$ -1/2-

$$= 360 + 120 \times 100$$

$$= 12000 + 360$$

$P = ₹12,360$ -1/2-

Section C

26. Given: $f(x) = 2x^3 + 9x^2 + 12x + 20$

Differentiating w.r.t. x,

$f'(x) = 6x^2 + 18x + 12$

For critical points,

$f'(x) = 0$

-1-

$\Rightarrow 6x^2 + 18x + 12 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow x^2 + 2x + x + 2 = 0$

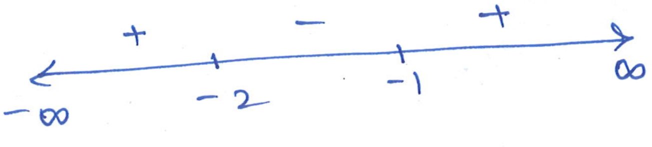
$x[x+2] + 1[x+2] = 0$

$\Rightarrow (x+1)(x+2) = 0$

-1-

$\therefore x = -1$ or -2

Using wavy curve method



Intervals	Sign of $f'(x)$	Conclusion
$(-\infty, -2)$	$f'(x) > 0$	f is increasing
$(-2, -1)$	$f'(x) < 0$	f is decreasing
$(-1, \infty)$	$f'(x) > 0$	f is increasing

$\frac{1}{2}$

$\therefore$ $f(x)$ is increasing in $(-\infty, -2) \cup (-1, \infty)$ and decreasing in $(-2, -1)$. — ½ —

27.

Years (t)	No. of Tourists (y)	$x = t - 2007$	x^2	xy
2004	18	−3	9	−54
2005	20	−2	4	−40
2006	23	−1	1	−23
$\boxed{2007}$	25	0	0	0
2008	24	1	1	24
2009	28	2	4	56
2010	30	3	9	90
	$\Sigma y = 168$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 53$

— 1 —

$$a = \frac{\Sigma y}{n} = \frac{168}{7} = 24$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{53}{28} = 1.89$$

— 1 —

$\therefore$ Trend equation $y_t = a + bx$

$$y_t = 24 + 1.89x$$

— 1 —

$\therefore$ For the year 2014 ; $x = 2014 - 2007 = 7$

$\therefore$ $y_t = 24 + 1.89(7) = 24 + 13.23 = \underline{\underline{37.23 \text{ millions}}}$

28. Given: $\mu = 18.5$ thousand kg.

$n = 14$

$\bar{x} = 17.85$ thousand kg

$S = 1.955$

-½-

$H_0: \mu = 18.5$ (There is no significant difference between the population mean and sample mean) -½-

$H_1: \mu < 18.5$ (There is a significant difference between the population mean and sample mean)

Test statistic $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ -½-

$$t = \frac{17.85 - 18.5}{1.955} \times \sqrt{14}$$

$$= \frac{-0.65\sqrt{14}}{1.955}$$

$$= \frac{-0.65 \times 3.74}{1.955}$$ -½-

$$t \approx -1.245$$

$|t| = 1.245$; given degree of freedom $= n-1$
$= 13$

Clearly $|t| < t_{13}(0.05)$ } Given $t_{13}(0.05) = 2.16$ ½

∴ Accept the null hypothesis.

The deviation in the mean breaking strength of the steel rods is not statistically significant at the 0.05 Level. ½

29.

Given: P = Rs. 20,00,000

n = 15 years = 15 × 12 = 180

i = 12% per annum } ½

$= \frac{12}{1200} = 0.01$ per period.

$$EMI = \frac{Pi}{1-(1+i)^{-n}}$$ ½

$$= \frac{20,00,000 \times 0.01}{1-(1+0.01)^{-180}}$$ ½

$$= \frac{20,000}{1-0.1668} = \frac{20,000}{0.8332}$$ 1

$$= ₹\, 24,003.84$$ ½

30.

(a) Given : Current population = 1,00,000

Let P is the population at time t.

The population doubled in 25 years.

-½-

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \frac{dP}{P} = k\,dt$$

-½-

$$\log P = kt + c$$

When $t=0$; Let $P = P_0$

$$\therefore \log P_0 = c$$

-½-

$$\therefore \log\left(\frac{P}{P_0}\right) = kt$$

When $t = 25$ yrs; $P = 2P_0$

-½-

$$\therefore \log\left(\frac{2P_0}{P_0}\right) = 25k$$

$$\Rightarrow k = \frac{1}{25}\log 2 = \frac{0.6931}{25} \approx 0.0277$$

$\therefore$ Now when $P = 5,00,000$

-½-

$$\log\left(\frac{5,00,000}{1,00,000}\right) = (0.0277)t$$

$\log 5 = 0.0277 t$

$\therefore \quad t = \frac{1.609}{0.0277} = \frac{16090}{277}$

-½-

$\therefore \quad t \approx 58$ years

OR

(b) Let P be the principal.

rate $r = 5\%$. -½-

when $t=0$; $P = ₹1,000$.

Given: $\frac{dp}{dt} = 5\% \text{ of } P$

$\Rightarrow \frac{dp}{dt} = \frac{5}{100} P$ -½-

$\Rightarrow \frac{dp}{p} = \frac{1}{20} dt$

-½-

$\log P = \frac{1}{20} t + c$

when $t=0$; $c = \log 20,000$

$\therefore \log P = \frac{1}{20} t + \log 20,000$

. When $t = 10$; what is P?

-½-

$$\therefore \log P = \frac{10}{20} + \log 20{,}000$$

$$\Rightarrow \log\left(\frac{P}{20{,}000}\right) = \frac{1}{2} = 0.5$$

-½-

$$\frac{P}{20{,}000} = e^{0.5}$$

$$\Rightarrow P = 20{,}000\, e^{0.5}$$

$$P = 20{,}000 \times 1.648$$

½-

$$\therefore P = ₹32,960.000$$

---

31. Given: Maximize $Z = x + y$

Subject to the constraints,

$x + y \leq 3$

$y - 2x \leq 1$

$x \leq 2$

$x \geq 0, \; y \geq 0$

Consider $x + y = 3$; $y - 2x = 1$

½

x	0	3
y	3	0

x	0	$-\frac{1}{2}$
y	1	0

checking $(0,0)$ condition,

$0 \leq 3$ is true , $0 \leq 1$ is true, $0 \leq 2$ is true.

$\therefore$ $(0,0)$ is part of the solution region for all the constraints.

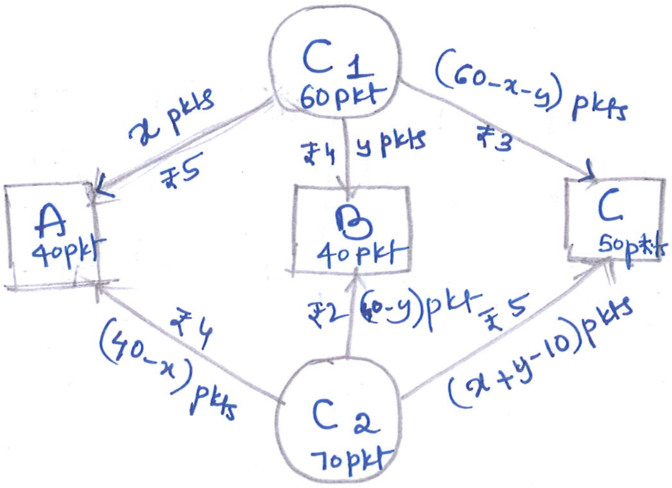
$\frac{1}{2}$

Corner Points	Value of $Z = x+y$
$O(0,0)$	0
$A(0,1)$	1
$B(2/3, 7/3)$	3
$C(1,3)$	4 $\longrightarrow$ M
$D(2,0)$	2

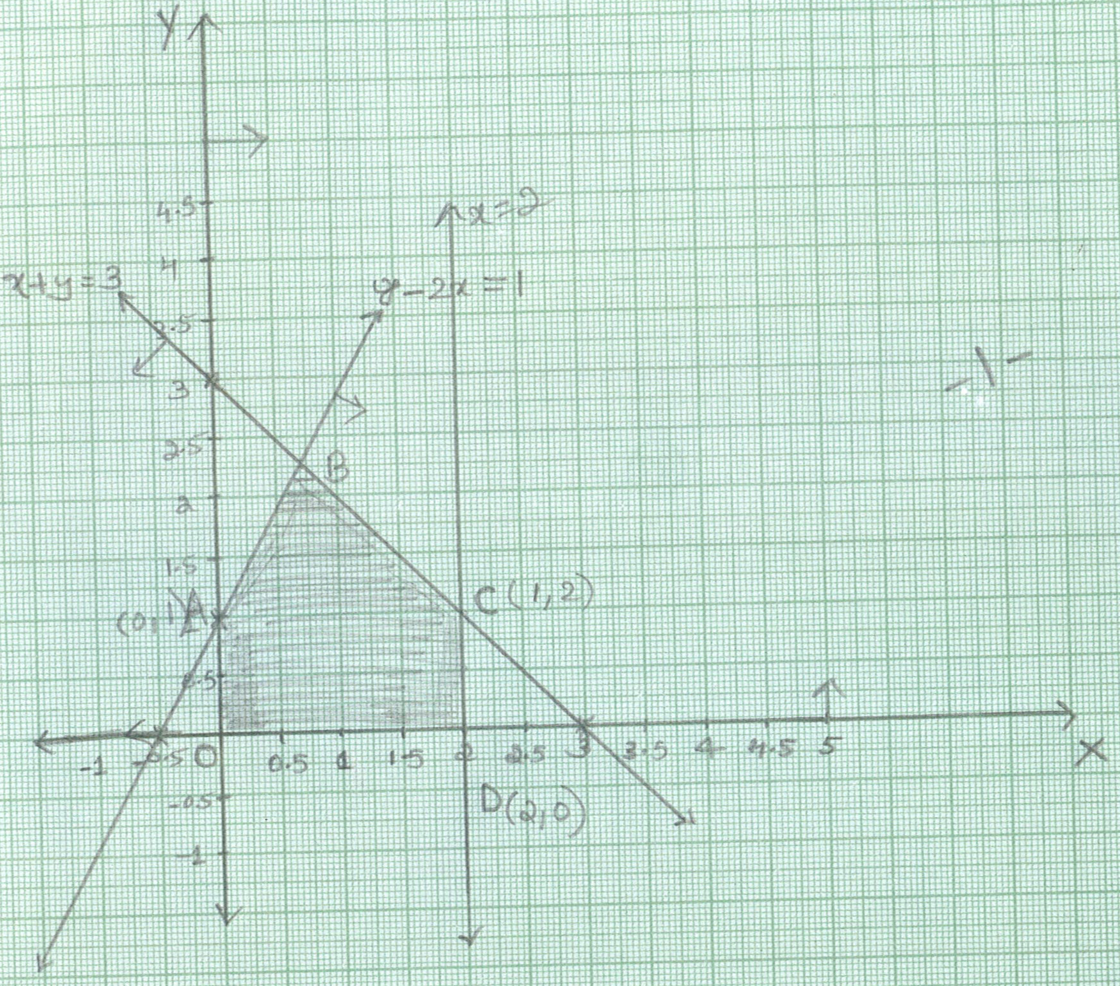
$\therefore$ Value of Z is maximum at $x=1$ and $y=3$. and the maximum value is 4.

OR

(b)



31. a)


Y
x=2
x+y=3
y−2x=1
4.5
4
3.5
3
2.5
2
1.5
1
0.5
B
C(1,2)
(0,1)A
O
-1
-0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
X
-0.5
-1
D(2,0)

-1-

Mathematical formulation of the given problem as an LPP is

Minimize $Z = 5x+4y+3(60-x-y)+4(40-x)$
$+2(40-y)+5(x+y-10)$

$Z = 5x+4y+180-3x-3y+160-4x$
$+80-2y+5x+5y-50$ -1-

i.e ~~Minim~~ Minimize $Z = 3x+4y+370$

Subject to the constraints,

$x+y \le 60$

$x \le 40 \;, \; y \le 40$ -1-

$x+y \geq 10$

$x, y \geq 0$.

Section D

32. (a)

$$AB = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6-1-1 & 2-3+1 & -2-1+3 \\ -3+2+1 & -1+6-1 & 1+2-3 \\ 3-1-2 & 1-3+2 & -1-1+6 \end{bmatrix}$$

-1-

$$AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4I$$

$$\therefore \quad \underline{\underline{AB = 4I}} \Rightarrow A\left[\frac{B}{4}\right] = I \therefore A^{-1} = \frac{B}{4}$$

-1-

The given system of equations are

$$2x - y + z = -1$$

$$-x + 2y - z = 4$$

$$x - y + 2z = -3$$

-1-

can be written in the matrix form as

$$AX = B \quad \text{where}$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}; \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \; B = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$

-1-

$$\therefore \quad X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4/4 \\ 8/4 \\ -4/4 \end{bmatrix}$$

-1-

$$\therefore \quad x = 1 \; ; \; y = 2 \; ; \; z = -1$$

OR

(b) $C = xI + yO + z$

Given:

$$12 = 2x + 3y + z$$

$$13 = 6x + 2y + z$$

$$15 = 5x + 3y + z$$

The above system of equations can be written in the matrix form as $AX = B$

where $A = \begin{bmatrix} 2 & 3 & 1 \\ 6 & 2 & 1 \\ 5 & 3 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 12 \\ 13 \\ 15 \end{bmatrix}$ $-1-$ $\frac{1}{2}$

To find x, y, z; $AX = B$

$$\Rightarrow X = A^{-1} B$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 1 \\ 6 & 2 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= 2(2-3) - 3(6-5) + 1.(18-10)$$ $-1-$

$$= -2 - 3 + 8 = 3 \neq 0$$

$\therefore A^{-1}$ Exists.

$A_{11} = -1$; $A_{12} = -(1)$; $A_{13} = 8$

$A_{21} = -(0)$; $A_{22} = -3$; $A_{23} = -(-9)$

$A_{31} = 1$; $A_{32} = -(-4)$; $A_{33} = -14$

-1-

$$\therefore \text{adj } A = \begin{bmatrix} -1 & -1 & 8 \\ 0 & -3 & 9 \\ 1 & 4 & -14 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & -3 & 4 \\ 8 & 9 & -14 \end{bmatrix}$$

-1/2-

$$\therefore X = \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ -1 & -3 & 4 \\ 8 & 9 & -14 \end{bmatrix} \begin{bmatrix} 12 \\ 13 \\ 15 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -12+0+15 \\ -12-39+60 \\ 96+117-210 \end{bmatrix}$$

-1-

$$= \frac{1}{3} \begin{bmatrix} 3 \\ -9 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/3 \\ -9/3 \\ 3/3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}}}$$

$\therefore x = 1$; $y = -3$; $z = 1$

-1-

33.

(i) the relation representing the total length of fencing wire is

$x+x+y=200$ -1-

$\Rightarrow \boxed{2x+y=200}$

(ii) Area of the garden as a function of x :

$A = xy$

$\Rightarrow A = x(200-2x)$ -1-

$A = 200x - 2x^2$

(iii) To find the maximum area, we use second derivative test.

$A = 200x - 2x^2$

Diff w.r.t. x,

$$\frac{dA}{dx} = 200 - 4x$$ -1-

put $\frac{dA}{dx} = 0 \Rightarrow 200 = 4x$

$\boxed{x = 50}$

$\therefore y = 200 - 2(50) = 100$

Differentiate $\frac{dA}{dx}$ w.r.t. x again, -1-

$$\frac{d^2A}{dx^2} = -4 < 0$$

$\therefore$ Area is maximum at $x = 50$ -1-

$\therefore$ Area $= 50 \times 100 = \underline{\underline{5000}}$ sq. ft.

34. (a)

Given: $p = 5\% = \frac{5}{100}$; $P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$

$n = 100$

$\lambda = np = 100 \times \frac{5}{100} = 5$ -1/2-

$e^{-5} = 0.006738.$

(i) $P(X=0) = \frac{e^{-5}(5)^0}{0!} = e^{-5} = \underline{\underline{0.006738}}$ -1-

(ii) $P(X=5) = \frac{e^{-5} \times 5^5}{5!} = \underline{\underline{\frac{0.006738 \times 5^5}{120}}} \approx 0.1754$ -1-

(iii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= \frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} + \frac{e^{-5} \times 5^2}{2!} + \frac{e^{-5} \times 5^3}{3!}.$$ -1-

$$P(X \leq 3) = e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{6}\right)$$ 1/2

$$= 0.006738\left(\frac{36 + 75 + 125}{6}\right)$$

$$= 0.001123\,(236)$$ 1

$$\approx \underline{\underline{0.265028}}$$

OR

(b) Given: $\mu = 14$

$SD(\sigma) = 2.5$ 1/2

We have, $Z = \dfrac{X - \mu}{\sigma}$

(i) $P(12 < X < 15)$

$$= P\left(\frac{12-14}{2.5} < Z < \frac{15-14}{2.5}\right)$$ 1/2

$$= P\left(\frac{-2}{2.5} < Z < \frac{1}{2.5}\right)$$

$$= P(-0.8 < Z < 0.4)$$ 1/2

$$= F(0.4) - F(-0.8)$$

$$= 0.1554 - [1 - F(0.8)] = 0.6554 - 1 + 0.7881$$

$= \underline{\underline{0.4435}}$ ½

$\therefore$ Number of students $= 1000 \times 0.4435$

$\approx \underline{\underline{444}}$ students.

(ii) Above 18

$$P(X > 18) = P\left(Z > \frac{18-14}{2.5}\right)$$

$$= P\left(Z > \frac{4}{2.5}\right)$$ ½

$$= P(Z > 1.6)$$ ½

$$= 1 - P(Z < 1.6)$$

$$= 1 - F(1.6)$$

$$= 1 - 0.9452$$ ½

$$= \underline{\underline{0.0548}}$$

$\therefore$ Number of students $= 1000 \times 0.0548$

$\approx \underline{\underline{55}}$ ½

(iii) below 8.

$$P(X < 8) = P\left(Z < \frac{8-14}{2.5}\right) = P\left(Z < \frac{-6}{2.5}\right)$$

$= P(Z < -2.4)$

$= F(-2.4) = 1 - F(2.4)$ ½

$= 1 - 0.9918$

$= 0.0082$

∴ Number of Students $= 1000 \times 0.0082$

≈ 8 ½

35.

Loan amount = Rs. 10,00,000.

Interest rate $= 8\%$. p.a $= \frac{8}{4}\%$ per quarter. -1-

∴ Interest $= 10,00,000 \times \frac{2}{100} = ₹20,000$

Sinking fund $A = R\left[\frac{(1+i)^n - 1}{i}\right]$ -1-

$R = ?$; $n = 5 \times 4 = 20$; $i = \frac{6\%}{4} = \frac{3}{2}\%$

$i = 0.015$ -1-

∴ $R = \frac{A\,i}{(1+i)^n - 1} = \frac{10,00,000 \times 0.015}{(1.015)^{20} - 1}$

$$R = \frac{15,000}{1.346852-1}$$ -1-

$$= \frac{15,000}{0.346852} \approx \frac{15,000}{0.3469}$$

$R \approx$ ₹43,240.1

$\therefore$ Total quarterly cost = 20,000 + 43,240

= ₹63,240 -1-

Section E

36.

(i) We have downstream Speed = $(x+y)$ km/h

and upstream speed = $(x-y)$ km/h.

from the question,

$$t_{down} = \frac{12}{x+y} \; ; \; t_{up} = \frac{12}{x-y}$$ -½-

$$\therefore \quad t_{up} - t_{down} = 6$$

$$\frac{12}{x-y} - \frac{12}{x+y} = 6$$

$$2(x+y) - 2(x-y) = x^2 - y^2$$ -1/2-

$$\Rightarrow \boxed{4y = x^2 - y^2} \Rightarrow y^2 = x^2 - 4y$$

(ii) For the downstream journey, the new speed = $2x+y$ km/h.

and upstream speed = $2x-y$ km/h. ½

According to the question,

$$\frac{12}{2x-y} - \frac{12}{2x+y} = 1$$

$$12(2x+y) - 12(2x-y) = 4x^2 - y^2$$ ½

$$\Rightarrow \boxed{24y = 4x^2 - y^2} \longrightarrow ②$$

(iii) (a) From ① and ②

$$24y = 4x^2 - (x^2 - 4y)$$

$$\Rightarrow 24y = 3x^2 + 4y$$ ½

$$\Rightarrow 20y = 3x^2 \Rightarrow \boxed{x^2 = \frac{20}{3}y} \longrightarrow ③$$

Substituting ③ in ②

$$24y = 4\left[\frac{20}{3}\right]y - y^2$$ ½

$$\Rightarrow y^2 + y\left[24 - \frac{80}{3}\right] = 0$$

$$\Rightarrow y^2 + y\left[-\frac{8}{3}\right] = 0$$ -1-

$$y\left[y-\frac{8}{3}\right] = 0 \Rightarrow y = 0 \text{ or } \boxed{y = \frac{8}{3} \text{ km/h}}$$

OR

(b) Speed of boat in still water = 5 km/h.

Speed of water current = 1 km/h -½-

Speed of downstream = $(x+y) = 5+1 = 6$ km/h.

Speed of upstream = $x-y = 5-1 = 4$ km/h.

-½-

$T_{down} + T_{up} = 1$

$$\Rightarrow \frac{x}{6} + \frac{x}{4} = 1 \Rightarrow \frac{2x+3x}{12} = 1$$ -½-

$$\Rightarrow 5x = 12$$

$$x = \frac{12}{5} = \underline{\underline{2.4 \text{ km/h}}}$$ -½-

37. Given

	M_1	M_2	M_3	Profit
Half sleeves	1	2	8/5	₹1
Full sleeves	2	1	8/5	₹1.50
Time :	≤40	≤40	≤40	

(i) objective function: Let x and y represent the number of half sleeves and full sleeves shirts resp

Maximize $Z = x + 1.5y$

-1-

Subject to the constraints,

$x + 2y \leq 40$

$2x + y \leq 40$

-1-

$\frac{8}{5}x + \frac{8}{5}y \leq 40 \Rightarrow x + y \leq 25$

$x, y, \geq 0$

(ii)

Consider

$x + 2y = 40$

x	0	40
y	20	0

$0 \leq 40$

$2x + y = 40$

x	0	20
y	40	0

$0 \leq 40$

$x + y = 25$

x	0	25
y	25	0

$0 \leq 25$

1/2

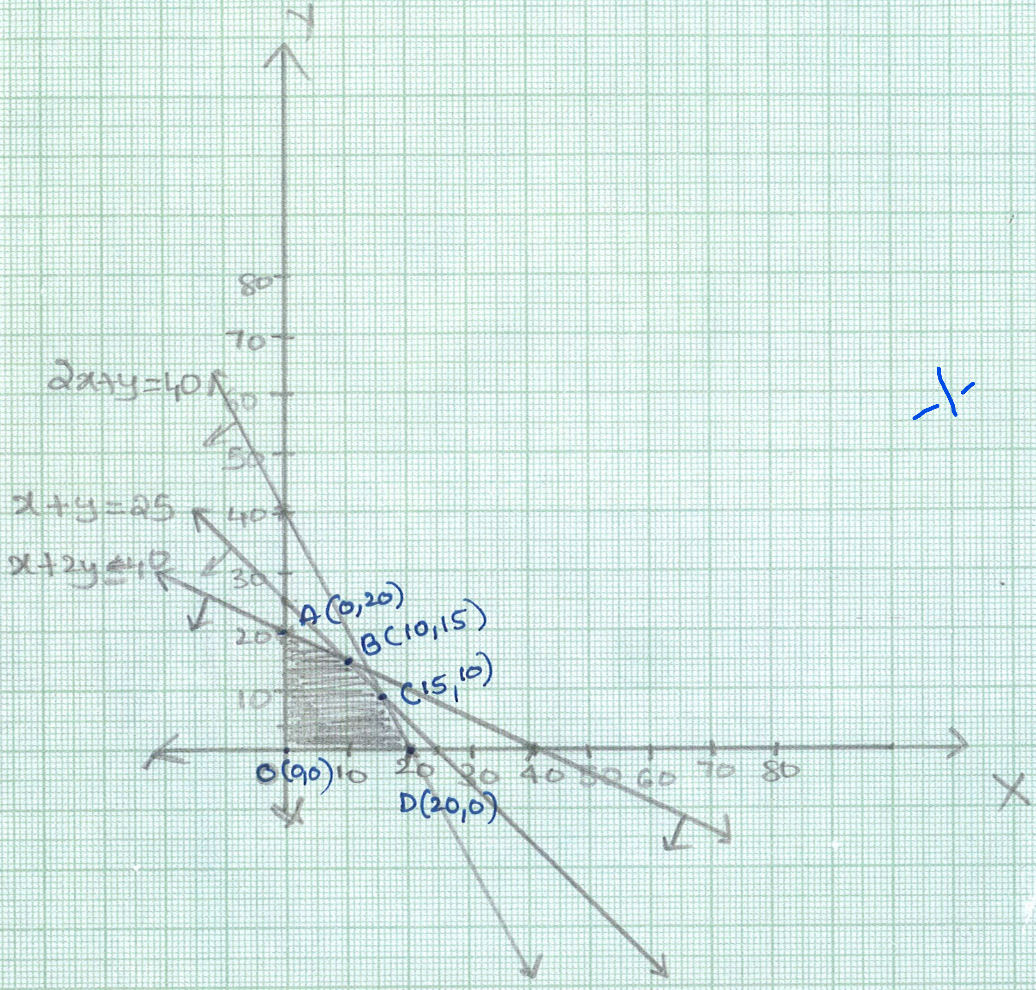
is true for all the 3 constraints.

(0,0) is part of Solution region.

1/2

Corner Points	Values of $Z = x + 1.5y$
A(0,20)	30
B(10,15)	$\boxed{32.5}$ — M
C(15,10)	30
D(20,0)	20

37.(b)


Y
80
70
2x+y=40
60
50
x+y=25
40
x+2y≤40
30
A(0,20)
20
B(10,15)
10
C(15,10)
O(0,0)
10
20
30
40
50
60
70
80
D(20,0)
X

-1-

$\therefore$ Profit is maximum at $x = 10$ and $y = 15$ with $Z = 32.5$.   1- marks for graph.

38.

Given : $P(X=0) = 0.08$

$P(X=1) = 0.11$

$P(X=2) = 0.27$

$P(X=3) = 0.33$

(i) $P(X=4) = ?$

We have $\Sigma P = 1$ $\frac{1}{2}$

$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$

$\Rightarrow 0.08 + 0.11 + 0.27 + 0.33 + P(X=4) = 1$

$\Rightarrow P(X=4) = 1 - 0.79$ $\frac{1}{2}$

$\Rightarrow P(X=4) = \underline{\underline{0.21}}$

(ii) $P(X \geqslant 2) = P(X=2) + P(X=3) + P(X=4)$ $\frac{1}{2}$

$= 0.27 + 0.33 + 0.21$

$= \underline{0.81}$ $\frac{1}{2}$

(iii) $E(X) = \Sigma x P(x)$ $\frac{1}{2}$

(iii)

X	0	1	2	3	4
P(X)	0.08	0.11	0.27	0.33	0.21
X P(X)	0	0.11	0.54	0.99	0.84

½

$E(x) = \sum x P(x) = 0.11 + 0.54 + 0.99 + 0.84$ ½

$E(X) = \underline{\underline{2.48}}$ ½

OR

Variance $= E(x^2) - (E(x))^2$ ½

$= [0.11 + 1.08 + 2.97 + 3.36] - [2.48]^2$

$= 7.52 - 6.1504$ ½

$= \underline{\underline{1.3696}}$

Standard deviation $(\sigma) = \sqrt{\text{Variance}}$ ½

$= \sqrt{1.3696}$

$= \underline{\underline{1.1702}}$ ½